Modal Displacement Based Seismic Design of Asymmetric-Plan Structures

P.Wilkinson, O.Lavan & A.Rutenberg Faculty of Civil and Environmental Engineering, Technion – Israel Institute of Technology Technion City, Haifa 32000, Israel



SUMMARY:

This paper presents the formulation and validation of two displacement based seismic design methods referred to collectively as Modal Displacement Based Design (MDBD). The first method, called 'Modal Pushover Design' (MPD), is iterative at the single degree of freedom (SDF) system response spectra (RS) level and the second, called 'Direct Modal Pushover Design' (DMPD), is non-iterative. Results are presented for single story asymmetric plan wall structures. Advantages of MDBD methods include direct consideration of torsional and higher mode effects in the design procedure and smaller reliance on empirical capacity design equations for establishing design shear and elastic moment capacities.

Keywords: displacement based seismic design, asymmetric structures

1. INTRODUCTION

The physical and fiscal threat earthquakes present to the safety of communities located in seismic regions, is increasing with the growth of the population living in the built environment and with heavier investment into buildings and infrastructure. Developing practical, robust seismic design procedures which produce economical designs for asymmetric plan structures would contribute to the mitigation of seismic hazards. Both structural and non-structural earthquake damage in buildings corresponds more directly to deformations than to forces. For asymmetric structures, which exhibit significantly nonlinear behavior, a displacement based design approach appears more likely to produce more uniformly safer designs than traditional linear force based design methods.

As is known, the Direct Displacement Based Design (DDBD)¹ procedure is a non-iterative method which considers inelastic displacements explicitly. DDBD uses design code limits on structural deformations directly in the design process as peak response targets. In this study (results not shown) DDBD was found to perform very well for small strength eccentricities ($|e_V| < 15\%$ of the overall building dimension perpendicular to the direction of loading) for the structure shown in Fig.3.1. As DDBD considers only the first 'inelastic mode' shape, empirical factors based on the results of parametric studies must be used to account for torsional and higher mode effects in asymmetric plan building response.

This paper presents two design procedures for asymmetric plan wall structures which directly consider torsional and higher mode effects. The first, called 'Modal Pushover Design' or 'MPD' for short, is applicable to any building structure for which the MPA² method provides acceptably accurate response

prediction. MPD is iterative at the level of single degree of freedom (SDF) systems using response spectra (RS). The second is a non-iterative method called 'Direct Modal Pushover Design' or 'DMPD' for short. The performance of the procedures was evaluated using Nonlinear Dynamic Analysis (NLDA) with a suite of 20 historic ground motions scaled to match their response spectra to an elastic design response spectrum shown in Fig.3.2.

The advantages of MDBD are its direct consideration of torsional and higher mode effects in the design procedure and its smaller reliance on empirical capacity design equations for establishing design shear and elastic moment capacities. DMPD also has the advantage of not requiring iteration.

The motivation for MDBD is the need for robust design solutions for irregular structures which:

- (1) are designed to respond inelastically and
 - a. have centers of strength (c_V) which cannot be located at the centers of mass (c_M) or where
 - b. the c_M is clearly not the optimal location for the c_V in terms of utilized ductility capacity.
- (2) are designed to respond elastically or with low ductility (e.g. $\mu < 2$) having response significantly influenced by torsion even when the c_V is located at the c_M .

In this paper results are presented only for torsionally restrained³ single story one way asymmetric plan structures subjected to a single component of ground motion.

2. MDBD FOR ONE STORY ASYMMETRIC-PLAN WALL STRUCTURES

2.1. Design for linear response

The equation governing the linear response of building structures to seismic ground motion excitation in the z (horizontal) direction may be written as

$$\boldsymbol{M}\ddot{\boldsymbol{u}}(t) + \boldsymbol{C}\dot{\boldsymbol{u}}(t) + \boldsymbol{K}\boldsymbol{u}(t) = -\boldsymbol{M}\boldsymbol{l}\ddot{\boldsymbol{u}}_{\boldsymbol{g}\boldsymbol{z}}(t)$$
(2.1.1)

where M, C and K are the mass, damping and stiffness matrices respectively, u(t) is the relative displacement vector in global coordinates and the overdots denote the order of the derivatives with respect to time. l is the influence vector describing rigid body motion corresponding to a unit ground displacement in the direction of the excitation and $\ddot{u}_{gz}(t)$ is the ground acceleration vector in the z direction.

It can be shown⁴ that the contribution of the nth mode to the total displacement can be written as

$$\boldsymbol{u}_{\boldsymbol{m}\boldsymbol{n}} = \Gamma_{\boldsymbol{n}} \boldsymbol{\phi}_{\boldsymbol{n}} \boldsymbol{S}_{d\boldsymbol{n}} \tag{2.1.2}$$

where
$$\Gamma_n = L_n / M_n$$
 where $L_n = \boldsymbol{\phi}_n^T \boldsymbol{M} \boldsymbol{l}$ and $M_n = \boldsymbol{\phi}_n^T \boldsymbol{M} \boldsymbol{\phi}_n$ where $\boldsymbol{\phi}_n$ is the nth mode shape.

The peak displacement response of the structure to z direction excitation, $-Ml\ddot{u}_{gz}(t)$, can be estimated as a modal combination of u_{mn} including all N modes contributing significantly to the response. Applying the square root of the sum of the squares (SRSS) modal combination rule to each term in ϕ_n seperately

$$\boldsymbol{u_m} = \sqrt{\sum_{n=1}^{N} (\boldsymbol{u_{mn}}^2)} = \sqrt{\sum_{n=1}^{N} (\Gamma_n \boldsymbol{\phi}_n S_{dn})^2}$$
(2.1.3)

Defining $\gamma_{Dn} = S_{dn}/S_{d1}$ and $u_{lm,cr} = a_{w,cr}u_m$ where $u_{lm,cr}$ is the local maximum displacement (corresponding to the design code deformation limit state) of the critical wall, in its local coordinates, and $a_{w,cr}$ is the global to local coordinate transformation vector for the critical wall. Eqn. 2.1.3 may be rearranged to obtain S_{d1} as a function of γ_{Dn} , $u_{lm,cr}$, Γ_n and the mode shapes ϕ_n as

$$S_{d1} = \frac{u_{lm,cr}}{\sqrt{\sum_{n=1}^{N} (a_{w,cr} \Gamma_n \phi_n \gamma_{Dn})^2}}$$
(2.1.4)

Eqn. 2.1.4 can be used to set the design spectral displacement for a SDF substitute structure representing 1^{st} mode response for a displacement-based design procedure for linear response which directly considers higher mode response. Only the *relative* wall stiffnesses are required to calculate the mode shapes and relative periods.

2.2. Design for nonlinear response

Eqn. 2.1.4 can also be used in design for nonlinear response. This is applicable only when the approximation (additional to that of estimating peak seismic responses using modal combination rules) inherent in neglecting the modal coupling due to yielding is acceptable². In the nonlinear case S_{d1} may be determined iteratively from inelastic constant ductility design spectra or an elastic design spectrum using R- μ -T relationships (see e.g. refs. in Chopra, 2007) or estimated directly using 'secant mode' shapes as presented in section 2.4.2.

2.3. Overview of procedures

Two methods are presented. The first (MPD) is an 'inverse MPA' procedure which achieves the target deformation limit when analyzed by MPA and is iterative only at the SDF system level. The second (DMPD) uses estimations instead of iteration. Both design procedures are applied in the context of a given structural geometry and design code limit states including seismicity and performance criteria. The seismic demand is described by the site's design spectrum and the performance criteria are described by material strain and interstory drift ratio limits.

Similarly to DDBD, yield and limit displacements of all walls are estimated from material and section properties and element curvature distributions. Relative element strengths are then chosen by the design engineer and relative elastic stiffnesses (to nominal yield) are calculated. In DMPD secant stiffnesses (to peak ductility) are estimated. Modal pushover curves are used to define the yield displacements of the SDFs for each mode. In DMPD 'secant mode' shapes calculated using the estimated secant stiffnesses are used to define the design level displacements of the SDF systems.

The fundamental period T_1 corresponding to the critical wall achieving its deformation limit is computed iteratively in MPD and estimated directly in DMPD. Each wall's total response including all significant 'modal' contributions is set equal to that wall's limit displacement. Each wall's limit displacement corresponds to a unique c_M displacement through the 'modes'. The minimum of these c_M displacements is associated with the critical wall's limit displacement according to Eqn.2.1.4. This c_M displacement is used in defining the design spectral displacement of the SDF substitute structure representing the 1st 'mode' response of the MDF structure.

2.4. MDBD step-by-step procedure outlines

MPD is presented in 2.4.1 and DMPD in 2.4.2. Some of the standard DDBD formulas used are not presented here.

2.4.1. MPD: Iterative design procedure using MPA

- 1) Obtain the structure's geometry and material properties
- 2) Select a limit state which may govern the design:
 - (i) Seismicity defined by design code constant μ spectra for the site
 - (ii) Performance targets defined by code interstory drift and material strain limits.
- 3) Estimate yield and limit displacements $u_{y,wi}$ and $u_{d,wi}$ and allowable ductilities μ_{wi} for all walls using standard DDBD formulas
- 4) Select a relative lateral strength distribution (this defines the strength eccentricity e_V)

$$M'_{y,wi} = \xi_{wi} \rho'_{wi} L^2_{wi}$$
 and $V'_{y,wi} = M'_{y,wi} / h_{wi}$ (2.4.1)

where $M'_{y,wi}$ and $V'_{y,wi}$ are the relative wall moment and shear capacities to nominal yield respectively and $\xi_{wi} = \alpha f_y b_{wi}$ where b_{wi} , h_{wi} and L_{wi} are the *i*th wall's thickness, effective height and length respectively, and f_y is the design yield strength of the longitudinal reinforcing steel and α is a constant (≈ 0.4) which depends on section properties for low levels of axial load. ξ_{wi} relates the *i*th wall's relative yield moment to ρ'_{wi} which is wall *i*'s relative longitudinal steel reinforcing ratio ($\rho'_{wi} = \rho_{wi}/\rho_{w1}$).

The distribution of lateral strength between the walls may be based on either

- a. a target strength eccentricity e_V (which may be determined, for example, by using an estimate of the maximum likely global overturning moment demand to estimate an average ρ_{wi} . Code ρ limits, b_{wi} and $\beta = L_{w1}/L_{w2}$ may then be used to find a target minimum e_V) or
- b. a rational reinforcing steel distribution ρ'_{wi}
- 5) Calculate relative elastic wall stiffnesses (to nominal yield) k'_{wi}

$$k'_{wi} = V'_{y,wi}/u_{y,wi} \tag{2.4.2}$$

- Form relative elastic stiffness matrix K_{rel} in global coordinates (using 3dofs @ c_M as in Fig. 3.1 below)
- 7) Calculate elastic mode shapes ϕ_n and normalize them by the translational term in the direction of the excitation (pure rotational modes have a zero participation factor and so may be ignored). Calculate modal participation factors Γ_n , relative periods T_n ', effective modal masses $m_n = \Gamma_n^2 M_n$ and heights h_n
- 8) Develop modal base shear roof displacement $(V'_{bn} u_{rn})$ curves by conducting a pushover analysis on the MDF structural model using the relative wall stiffnesses k'_{wi} and strengths $V'_{y,wi}$ and a unique invariant load vector for each mode defined as

$$\boldsymbol{p}_n = \boldsymbol{s}_n^* = \mathbf{M}\boldsymbol{\phi}_n \tag{2.4.3}$$

9) Approximate the pushover curves bilinearly without changing the initial elastic stiffness by defining the relative yield shear V'_{yn} , the yield displacement u_{yn} and a post yield stiffness. The post yield stiffness should match the MDF system $V'_{bn} - u_{rn}$ general post yield stiffness and would ideally

also match the hysteretic rules in the RS generation. In this study the post yield stiffness was set to achieve equal areas under the MDF system curve and the approximate bilinear curve over the displacement range $0 \le u_{rn} \le u_{\lim n}$ where $u_{\lim n}$ is the c_M displacement corresponding to the first wall reaching its limit displacement.

10) For each mode define a nonlinear SDF system having the same yield shear as defined for the bilinear approximation of the MDF system pushover curve and the spectral yield displacement defined by Eqn.2.4.4.

$$D_{yn} = \frac{u_{yn}}{\Gamma_n} \tag{2.4.4}$$

11) Define the ratio of SDF system elastic periods $T'_n = 2\pi \sqrt{\frac{m_n D_{yn}}{v'_{yn}}}$ and let $\gamma_{Tn} = T'_n / T'_1$ (2.4.5)

12) Iteratively determine the T₁ corresponding to the critical wall achieving its design displacement

a) Calculate initial values of T_n^0 and D_n^0 for oi = 0

i.
$$D_1^0 = \frac{u_{lim_1}}{\Gamma_1}$$
 (2.4.6)

where u_{lim1} is the c_M translation extracted from the 1st mode pushover analysis data corresponding to the 1st instance when a wall achieves its displacement limit.

ii. Using $\mu_1 = \frac{D_1^0}{D_{y_1}}$ enter the constant $\mu = \mu_1$ design RS at D_1^0 to estimate T_1^0 (initial elastic period) and calculate the corresponding higher mode periods $T_n^0 = \gamma_{Tn} T_n^0$

iii. Using
$$\mu_n = \frac{\gamma_{Tn} D_1^0}{D_{yn}}$$
 enter the constant $\mu = \mu_n$ design RS at T_n^0 to estimate D_n^0

- b) Outer iteration loop: for oi = 1, 2..., $T_n^{oi} = T_n^{oi-1}$
 - i. Inner iteration loop: for ii=1, 2...,

Calculate
$$\mu_n^{ii} = \frac{D_n^{ii-1}}{D_{yn}}$$
, then retrieve D_n^{ii} from design RS using T_n^{oi} and μ_n^{ii}
Repeat this step i. until max $_{n=1:N} \left| 1 - \frac{D_n^{ii}}{D_n^{ii-1}} \right|$ is acceptably close to zero

ii. Extract the modal contributions to wall displacements from the pushover analysis results corresponding to $u_{rn} = \Gamma_n \phi_n D_n^{ii}$. Calculate total wall displacement responses $u_{m,wi}^{oi}$ using an appropriate modal combination rule. Compare wall displacements to displacement limits and compute performance indices:

$$pi_{wi}^{oi} = \frac{u_{m,wi}^{oi}}{u_{d,wi}} - 1 \quad \text{and} \quad pim^{oi} = \max(pi_{wi}^{oi})$$
 (2.4.7)

iii. If any of the predicted peak wall displacements are unacceptably large or if the maximum performance index pim^{oi} is too low, adjust the periods of the SDF systems:

$$T_1^{oi} = T_1^{oi-1} \left(\frac{1}{pim^{oi}}\right)^p$$
 and $T_n^{oi} = \gamma_{Tn} T_1^{oi}$ (2.4.8)

where ($P \approx 1$ is a parameter controlling convergence) and repeat steps i.- ii. using D_n^{ii} and T_n^{oi} until the critical wall's displacement is close enough to its displacement limit.

13) Calculate the design initial elastic stiffness of the 1st mode SDF system $k_1 = \frac{4\pi^2 m_1}{T_1^2}$ and the associated steel reinforcing ratio of wall one: $\rho_{w1} = \frac{k_1}{\Gamma_1^2 \phi_1^T K_{rel} \phi_1}$. Then $\rho_{wi} = \rho'_{wi} \rho_{w1}$

- 14) Calculate wall design shears and moments $M_{y,wi} = \xi_{wi} \rho_{wi} L_{wi}^2$ and $V_{y,wi} = M_{y,wi} / h_{wi}$
- 15) Calculate the design base shear by $V_b = \sum_{i=1}^{2} V_{y,wi}$
- 16) Repeat steps 2-16 for other limit states which may govern the design

2.4.2. DMPD: Direct Modal Pushover Design

Steps 1-11 are the same as for MPD.

- 12) Estimate the T_1 corresponding to the critical wall achieving its design displacement
 - a) Estimate relative secant wall stiffnesses to peak ductility $k'_{e,wi}$:

$$k'_{e,wi} = k'_{wi}/\mu^{*}_{wi}$$
(2.4.9)
where $\mu^{*}_{wi} = \max(1, u_{m,wi}/u_{y,wi})$
where $u_{m,wi} = a_{wi}\phi_1 u_{d1}$ where $u_{d1} = \min_{i=1:2} \left(\frac{u_{d,wi}}{1 + x_{wi}\phi_{\theta_1}}\right)$

- b) Form relative secant stiffness matrix \mathbf{K}_{sec} using $k'_{e,wi}$ and calculate modal terms $\boldsymbol{\phi}_{en}$, Γ_{en} and m_{en} and h_{en} where the subscript 'e' refers to 'effective' stiffness.
- c) Estimate the ratio of modal spectral displacement demands $\gamma_{Dn} = D_n/D_1$:
 - i. Estimate limit displacement D_1 using Eqn.2.4.6 with Γ_{e1} instead of Γ_1
 - ii. Using $\mu_1 = \frac{D_1}{D_{y_1}}$ enter the constant $\mu = \mu_1$ design RS at D_1 to estimate T_1 (initial elastic period) and calculate the corresponding higher mode periods $T_n = \gamma_{Tn} T_1$
 - iii. Using $\mu_n = \frac{\gamma_{Tn}D_1}{D_{yn}}$ enter the constant $\mu = \mu_n$ design RS at T_n to estimate D_n
 - iv. Let $\gamma_{Dn} = D_n/D_1$
- d) Calculate the design 1^{st} mode spectral displacement D_1 using the minimum value for all walls of:

$$D_1 = \min_{i=1:N_w} \left(\frac{u_{d,wi}}{\sqrt{\sum_{n=1}^N (a_{wi} \Gamma_{en} \phi_{en} \gamma_{Dn})^2}} \right)$$
(2.4.10)

e) Repeat steps c) ii-iii once using γ_{Dn} instead of γ_{Tn} in step iii to find T_n , μ_n and D_2

Steps 13-16 are the same as for MPD.

3. STRUCTURAL SYSTEMS AND MODELING

MPD and DMPD designs were generated for ranges of three key parameters. These were the fundamental period of vibration T₁, the ratio of wall lengths $\beta = L_{z1}/L_{z2}$ and strength eccentricity $e_V = \frac{\sum_{i=1}^{N_{wz}} (V'_{y,wi} x_{wi})}{\sum_{i=1}^{N_{wz}} V'_{y,wi}}$ where x_{wi} is the distance from the ith wall to the c_M as indicated in Fig. 3.1 and N_{wz} is the total number of walls aligned parallel to the z direction. For each value of T₁ a range of single-story one-way asymmetric-plan RC wall structure designs was generated by varying the ratio of the lengths of

walls w_1 and w_2 shown in Fig. 3.1 within the limits $1 \le \beta \le 4$. A range of strength eccentricities from $-0.4X \le e_V \le 0$ was considered where X is the overall building dimension perpendicular to the ground motion. Each wall was modeled as an elastic perfectly plastic (EPP) spring having an initial relative elastic stiffness k'_{wi} .

The procedures specify design moments and shears for each wall in the intended plastic hinge regions (PHRs) and elastic regions. Walls 3 and 4 were designed to achieve the same total base shear capacity as in the z direction. The design moments for elastic regions should be increased by material overstrength factors as should the design shears for PHRs and elastic regions. The purpose of these capacity increases is to ensure no yielding occurs except in the intended plastic hinge regions and that no brittle shear failures occur.





Figure 3.1. One way asymmetric wall structure

Figure 3.2. Design ($_{DES}$) & mean matched ($_{MM}$) acceleration and displacement constant μ RS (ξ =5%)

Twenty ground motions from the LA10in50 ensemble were selected and scaled using a computer program called SeismoMatch⁵ to match their RS to a chosen elastic design spectrum having a corner period of 4s and a corner spectral displacement of 0.6m similar to that used in ref. 6.

The peak design spectral acceleration was 1g. Inelastic constant ductility (μ) response spectra were generated for displacement ductility demands of 2, 3, 4, 6 and 8 using a computer program called Matlab⁷. The mean matched elastic spectra are compared to the design spectrum in Fig. 3.2 above. Constant μ mean RS of the 20 ground motions for $\mu = 4$ and 8 are also shown Fig 3.2.

4. EVALUATION OF PROPOSED METHODS

The MPD and DMPD designs for the range of structural parameters considered were analyzed using NLDA and the maximum percentage differences between predicted mean peak wall displacements and design displacements of all walls are shown in Fig. 4.1 below. The percentage performance index was defined as

$$PI = \left(\max_{i=1:N_w} \left| \frac{u_{m,wi}}{u_{d,wi}} \right| - 1 \right) * 100 \text{ over all walls}$$

$$(4.1)$$

where $u_{m,wi}$ is the mean peak displacement of wall *i* and $u_{d,wi}$ is the ith wall's design displacement.

A value of zero corresponds to the critical wall achieving its deformation limit exactly. A single ultimate limit state was designed for and design wall displacements were governed by the critical of a 2% drift limit and a maximum allowable ultimate reinforcing steel strain of $40\varepsilon_{sy}$. The drift of the flexible wall governed most cases.

PI was also evaluated using MPA (results not shown) and MPD achieved the design displacement exactly for the critical wall. Hence the level of approximation of MPD is the same as MPA and MPD is therefore expected to perform well when MPA is appropriate⁸.

Fig.4.1 presents values of PI for the range of fundamental periods $0.5s \le \sim T_1 \le 3.0s$, wall length ratios $1 \le \beta \le 4$ and the maximum and minimum strength eccentricities $e_V = -0.4X$ and 0. Each data point is computed as the mean of the 20 values obtained from the ground motions in the selected ensemble. There are four graphs; one for each approximate value of fundamental period. In each graph β is plotted on the horizontal axis representing geometrical asymmetry resulting from differing wall lengths and PI is plotted on the vertical axis. There are six lines on each graph; three for $e_V = -0.4X$ and three for $e_V = 0$.

The results of three design methods are presented in each graph; two versions of MPD and one of DMPD. One version of MPD employed constant μ RS generated using the elastic perfectly plastic hysteretic rule (MPD_{EPP RS}) and the other version employed NLDA of the SDF systems having bilinear hysteretic rules using the selected ensemble of ground motions (MPD_{SDF NLDA}). The first of these versions corresponds to the procedure outlined in section 2.4.1 but the second employed NDA of the modal bilinear SDF systems to compute D_n instead of the inner iteration loop in step 12-b-i which uses RS.

It can be seen from Fig.4.1 that the MDBD methods outlined in 2.4.1 generally performed well producing designs achieving a peak critical wall displacement from 20% below to 10% above the limit displacement over the range of strength and stiffness asymmetry considered. It should be noted that in designing structures having $\beta \approx 1$ it is unlikely that large strength eccentricities would be desirable. Conversely for highly asymmetric structures it is unlikely that a zero strength eccentricity would be achievable due to code limits on steel reinforcement ratios.

The discrepancies between the hysteretic characteristics of the bilinear SDF systems defined in step 10 in 2.4.1 and those of the SDF systems used in generating the constant μ RS are an additional source of approximation. The EPP rule was used in generating the RS shown in Fig.3.2 however the bilinear approximations of the MDF system pushover curves had post yield stiffness to elastic stiffness ratios *, r* , of up to 0.6. The average r over all T₁and β for the minimum e_V was 0.007 and increased to 0.09 for the maximum e_V. In particular for T₁ in the 0.5 - 1.0s ranges for $1 \le \beta \le 2$ for the maximum strength eccentricity (shown by purple dashed line with asterisk markers) r averaged approximately 0.29.

To evaluate the influence of this discrepancy in hysteretic characteristics, MPD designs were generated using NDA of the modal SDF systems with the selected suite of ground motions (in step 12) b) i) instead of using the RS generated using the EPP rule. As shown in Fig. 4.1 MPD using NDA on the SDF systems (labeled MPD_{SDF NLDA}) was found to produce designs achieving performance index values between 10% below to 10% above the design limit displacement. The discrepancy between hysteretic characteristics was thereby confirmed to be a relatively significant source of error which is not intrinsic to the MPD method. If constant μ RS were available for any bilinear *r* value then the procedure in 2.4.1 could produce designs performing as well as those produced by the MPD_{SDF NLDA} procedure.



5. CONCLUSIONS

Single story one way asymmetric wall structures were designed using two new methods referred to collectively as Modal Displacement Based Design (MDBD). The first method, called Modal Pushover Design (MPD), is an iterative procedure (at the SDF-RS level) which produces designs achieving the target performance level when analyzed by MPA. The second method is called Direct Modal Pushover Design (DMPD) and is non-iterative. For each mode considered in DMPD an equivalent SDF system yield displacement is defined (as for MPD) using the MDF system modal pushover curve. However,

unlike for MPD, SDF system design displacements are defined in DMPD using MDF system element secant stiffnesses (to estimated peak displacement ductility demand).

DMPD performed almost as well as MPD (using EPP RS) for low strength eccentricities and generally performed better for high strength eccentricities. The performance of the seismic designs was compared by NLDA and for many of the investigated structures the procedures were found to perform reasonably well in terms of achieving the design code deformation limits and utilizing ductility capacity while minimizing design base shear.

MPD using NLDA of the SDF systems to compute D_n instead of iterating on RS (generated using the EPP hysteretic rule) was found to produce designs achieving mean peak critical wall displacements between 10% above and below the limit displacement for all ranges of the three key parameters investigated. This implies that if constant μ RS for the range of bilinear stiffness factor $0 \le r \le 1$ were available then MPD could produce designs achieving similar levels of performance to the procedure based on NLDA of the SDF systems.

It is intended that future research includes extending MDBD to multistory buildings having irregularity in plan and elevation.

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