Constitutive Modeling of Reinforced Concrete Panel Behavior under Cyclic Loading

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SUMMARY:
A new constitutive model – named the Fixed Strut Angle Model (FSAM) – is proposed for simulating the cyclic shear behavior of reinforced concrete panel elements. The main inherent assumption underlying the formulation of the FSAM is that upon cracking of concrete in a reinforced concrete panel, crack directions in concrete do not rotate, as observed consistently in reinforced concrete panel and wall tests. Refined material constitutive relationships were incorporated in the formulation of the model. Detailed correlation studies were conducted to compare the model predictions with results of cyclic reinforced concrete panel tests available in the literature. The model was shown to capture, with reasonable accuracy, overall behavioral attributes of reinforced concrete panel elements subjected to cyclic shear effects; including cyclic shear stress vs. shear strain behavior, shear stress capacity, shear stiffness, cyclic stiffness degradation, pinching, ductility, and failure mode. The model has also yielded promising results on local deformation predictions.

Keywords: reinforced concrete, panel, analytical model, shear, cyclic

1. INTRODUCTION

With the introduction of performance-based methodologies for design and evaluation of reinforced concrete structures subjected to earthquake actions, analytical modeling of the behavior of individual reinforced concrete members under generalized and cyclic loading effects has recently gained substantial importance among researchers. A reliable prediction of the nonlinear inelastic earthquake response of such structural systems inherently requires the use of analytical models that can accurately capture the hysteretic behavior of individual structural members under generalized loading conditions.

In seismic-resistant design of reinforced concrete (RC) buildings, in particular, use of structural walls has been proven to be a very feasible alternative for resisting earthquake actions. Analytical modeling of the inelastic response of structural wall systems can be accomplished by using microscopic (finite element) or macroscopic (behavioral) models. For finite element modeling, although a number of cyclic constitutive models have been proposed for simulating the nonlinear responses of constitutive panel elements of the finite element model (e.g., Stevens et al. (1991), Palermo and Vecchio (2003), Mansour and Hsu (2005), Gerin and Adebar (2009)), these models are not included in commonly-used structural analysis platforms due to complexities in their implementation. Therefore, development of new constitutive models, which can provide sufficiently accurate response predictions, yet are simple in formulation, would be desirable for implementation into widely-used analysis programs.

Therefore, in this study, a new constitutive panel model was proposed – based on interpretation and simplification of previous modeling approaches – for simulating the behavior of RC panel elements under generalized, in-plane, reversed-cyclic loading conditions. The proposed constitutive model is presented as a feasible candidate for implementation into a two-dimensional finite-element analysis formulation, for efficient and practical response prediction for structural walls experiencing coupled nonlinear flexural and shear responses, using the finite element modeling approach.

2. DESCRIPTION OF THE FIXED STRUT ANGLE MODEL (FSAM)

3.1. Model Formulation

The Fixed Strut Angle Model (FSAM) is a constitutive model for simulating the behavior of RC panel elements under generalized, in-plane, reversed-cyclic loading conditions. As assumed by other RC panel models available in the literature, in the Fixed Strut Angle model, the strain field acting on concrete and reinforcing steel components of an RC panel is assumed to be equal to each other, implying perfect bond assumption between concrete and reinforcing steel bars. Further, reinforcing steel bars are assumed to develop zero shear stresses perpendicular to their longitudinal direction, implying no dowel action on reinforcement. While the reinforcing steel bars develop only uniaxial stresses under uniaxial strains in their longitudinal direction, the behavior of concrete is defined using stress–strain relationships in biaxial directions, and the orientation of those biaxial directions is governed by the state of cracking in concrete.

In the uncracked state of concrete, the stress–strain behavior of concrete is represented with a rotating strut approach (similar to the Modified Compression Field Theory, Vecchio and Collins (1986) and the Rotating Angle Strut and Tie Model, Pang and Hsu (1995)). The strain field imposed on concrete is transformed into principal strain directions, which are assumed to coincide with principal stress directions, and uniaxial stress–strain relationships for concrete are applied along the principal strain directions in order to obtain the principal stresses in concrete. Although the stress–strain relationships used for concrete in principal directions are fundamentally uniaxial in nature, they also incorporate biaxial softening effects including compression softening and biaxial damage. At this stage of the behavior, monotonic stress–strain relationships for concrete are used, since it is reasonable to assume that concrete behavior follows a monotonic (virgin) stress–strain relationship, prior to first cracking under a biaxial state of stress (Fig. 1(a)).

When the value of the principal tensile strain in concrete exceeds the monotonic cracking strain of concrete for the first time, the first crack is formed, and for following loading stages, the principal strain direction corresponding to first cracking in concrete is assigned as the first "Fixed Strut" direction for the panel. After formation of this first crack, while principal directions of the applied strain field continues to rotate based on the applied strain field, the principal stress directions in concrete are assumed to be along and perpendicular to the first Fixed Strut direction. This is somewhat similar to the fixed crack angle approach by Pang and Hsu (1996); although in that approach, direction of the cracks was assumed to coincide with the fixed angle following the principal stress directions of the applied loading. The present model assumes that the first crack (or strut) direction coincides with the principal stress directions in concrete. This physically implies zero shear aggregate interlock along a crack, which was an inherent assumption of the original model formulation (Ulugtekin, 2010). Since the direction of the first strut is fixed, a uniaxial hysteretic stress–strain relationship for concrete can now be applied in principal stress directions (parallel and perpendicular to the first strut), and history variables in the concrete stress-strain relationship can be easily tracked and stored in the two fixed directions. For calculation of concrete stresses in principal directions, the applied strain field in concrete should be transformed into strain components that are parallel and perpendicular to the first fixed strut direction, instead of principal strain directions (Fig. 1(b)).

The analysis is continued in the form of a single fixed strut mechanism until the formation of the second crack, after which the second strut will develop in the panel model. During the first fixed strut stage of the analysis, the model tracks the concrete stress–strain behavior along the first strut direction, and when the strains along the first strut direction first exceeds the cyclic cracking strain (which depends on both the monotonic cracking strain and the plastic strain upon reversal from a compressive stress state), the second crack is formed. In case of the zero aggregate interlock assumption, the second crack has to develop in perpendicular direction to the first crack, according to a stress-based cracking criterion, since the first strut direction is a principal stress direction and the concrete stress–strain relationship is assumed to be uniaxial along the first strut direction. It should be mentioned that although other cracking criteria may be used for defining the direction of the second crack (e.g., associating the formation and/or direction of the second crack with principal strains), this was found to be the simplest and mechanically-consistent approach. After formation of this second crack, the second "Fixed Strut" will develop in the direction of the second crack (in perpendicular direction to the first strut), and for further loading stages, the concrete mechanism consists of two independent struts, working as interchanging compression and tension struts in the two Fixed Strut directions, based on the applied strain field. While principal directions of the applied strain field continues to rotate, the principal stress directions in concrete are assumed to be along the two Fixed Strut directions, again implying zero shear stresses (zero shear aggregate interlock) developing along the two cracks. Since the direction of both struts are fixed, the uniaxial hysteretic stress–strain relationship for concrete can be applied in principal stress directions (parallel to the first and second strut directions), and history variables in the concrete stress-strain relationship can be tracked and stored in the two fixed directions. Again, for calculation of concrete stresses in the two principal directions, the applied strain field in concrete should be transformed into strain components that are parallel to the first and second fixed strut directions, instead of principal strain values (Fig. 1(c)).

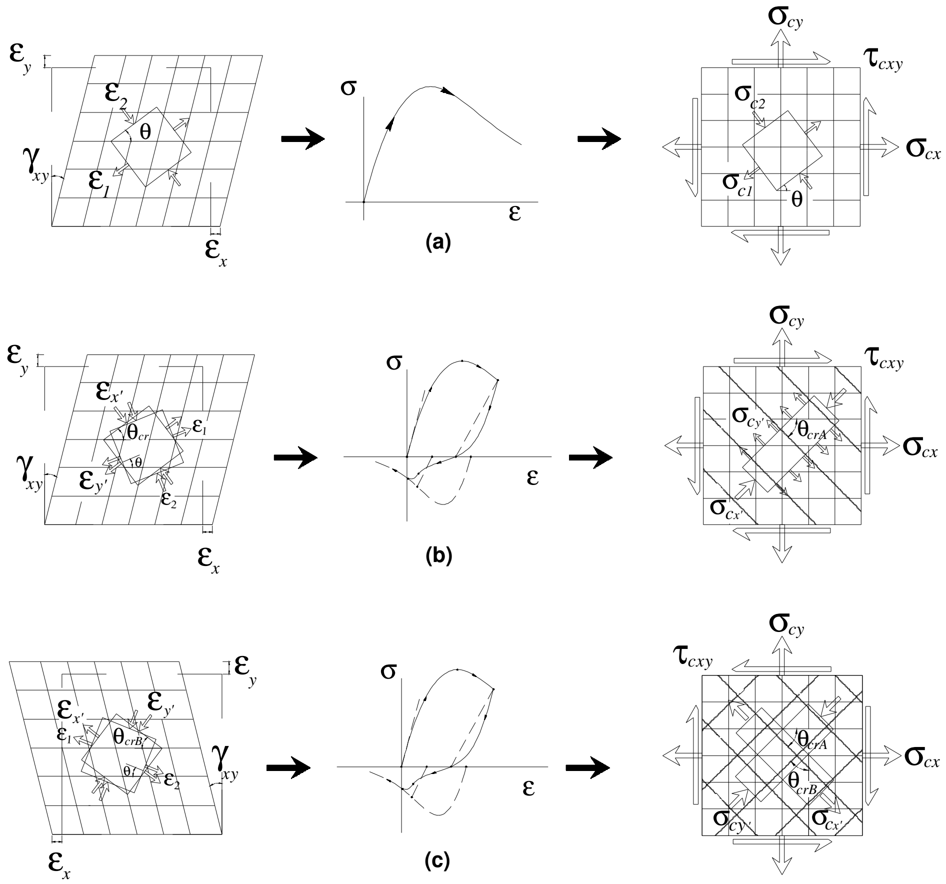

Figure 1. Concrete biaxial behavior in the Fixed Strut Angle Model: (a) uncracked behavior, (b) behavior after formation of first crack, (c) behavior after formation of second crack

Details of the FSAM are provided in the thesis by Ulugtekin (2010). As described above, the main inherent assumption underlying the formulation of the original FSAM is that the principal stress directions in concrete coincide with crack directions, implying zero shear stresses action along cracks, and therefore zero shear aggregate interlock. In an RC member, sliding along crack surfaces is known to develop an aggregate interlocking action, resulting in shear stress along the crack, the magnitude of which is affected by the crack width and the amount of slip deformation along the crack. However, in the proposed model, shear strains along crack surfaces, developing due to the deviation between principal strain directions and the principal stress (crack) directions in concrete, are considered as shear slip (sliding) deformations and are assumed not to develop shear stresses due to aggregate interlocking along crack surfaces. This inherent assumption of the model is based on interpretation of existing panel tests in the literature. Available test results in the literature typically indicate that for an RC panel, after formation of cracks, the principal stress direction in concrete does not change significantly with loading, although the principal strain direction on a panel may undergo significant variation (e.g., Stevens et al., 1991). The principal stress directions in concrete being insensitive to loading may imply that after formation of cracks, the principal stress directions in concrete follow approximately the fixed crack directions, indicating that shear stresses along a crack (and thus shear aggregate interlocking along a crack) has marginal influence on the panel behavior.

However, this assumption also allows the flexibility to incorporate a suitable cyclic shear aggregate interlock constitutive model (shear stress versus shear strain along a crack) in the FSAM, since the formulation of the model allows calculating shear strains along a crack. In the model formulation presented in this paper, a simple friction-based constitutive model was adopted to represent shear aggregate interlock effects, as described in the following section, since the zero-aggregate interlock assumption generally results in significant overestimation of sliding shear strains along crack surfaces for panels with inclined reinforcement or non–equal reinforcement ratios in x and y directions. The present model formulation also assumes that no shear stress is resisted by the reinforcing steel bars, indicating no dowel action on the reinforcement.

3.2. Material Constitutive Models Implemented

Refined and state-of-the-art constitutive models were implemented in the model formulation, for describing the cyclic stress-strain behavior of concrete and reinforcing steel. The advanced constitutive relationship proposed by Chang and Mander (1994) (Fig. 2(a)) was implemented for concrete, since it allows details calibration of the monotonic and hysteretic parameters, for improved representation of concrete stress-strain behavior. This constitutive model provides a direct and flexible approach to incorporate important material behavioral features (e.g., hysteretic behavior in tension, progressive gap closure, tension stiffening effects) into the analysis. The constitutive model adopted for reinforcing steel used is the well-known Menegotto and Pinto (1973) relationship (Fig. 2(b)). The model formulation incorporates cyclic degradation of the curvature of the unloading and reloading curves and thus allows the Bauschinger's effect to be represented. This constitutive model, although simple in formulation, has been shown to accurately simulate experimental behavior.

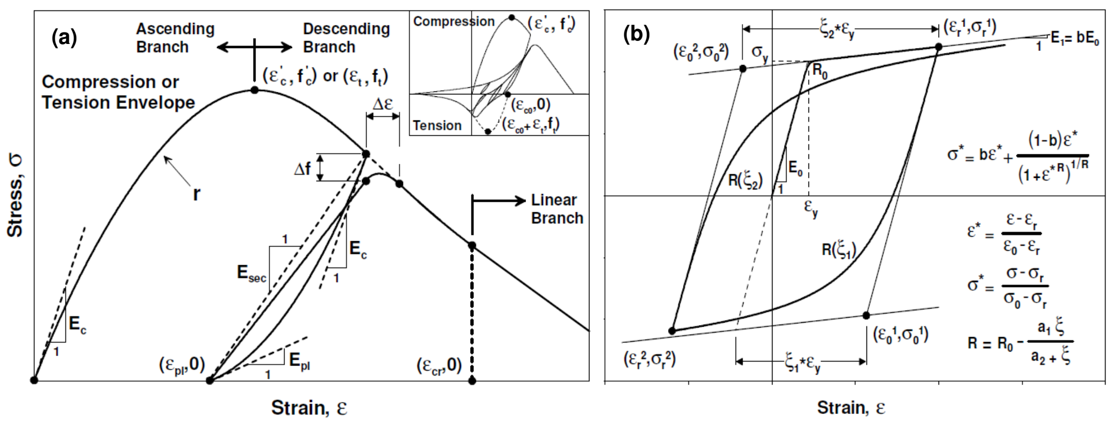

Figure 2. Constitutive material models: (a) concrete (b) reinforcing steel

In the implementation, the original formulation of the Chang and Mander model was modified to represent behavioral features of concrete under biaxial loading; via inclusion of parameters representing compression softening (defined by Vechio and Collins, 1993), hysteretic biaxial damage (defined by Mansour et al., 2002) , and tension stiffening effects (defined by Belarbi and Hsu, 1994). Details on these behavioural features and parameters are provided in the thesis by Ulugtekin (2010).

The original formulation of the FSAM described by Ulugtekin (2010) adopted the zero shear aggregate interlock assumption along the cracks. In this study, a simple friction-based aggregate interlock constitutive model is implemented. The proposed cyclic shear aggregate interlock model starts with linear loading/unloading behavior, relating the sliding shear strain along a crack to the shear stress, via a simple linear elastic relationship between the sliding shear strain and the resultant shear stress along the crack surface. However, the shear stress is restrained to zero value when the concrete normal stress perpendicular to the crack is tensile (crack open); and is bounded via the product of a friction coefficient and the concrete normal stress perpendicular to the crack, when the concrete normal stress is compressive (crack closed). The linear unloading/reloading slope of the shear stress vs. sliding strain relationship was taken as a fraction of the concrete elastic modulus (a value $0.4E_c$ was adopted, representing the elastic shear modulus of concrete), and a value of 0.2 was assumed for the friction coefficient. Under constant compressive stress in concrete perpendicular to the crack, this model yields an elasto–plastic aggregate interlock behaviour under cyclic loading, similar to the cyclic stress–strain behaviour of reinforcing steel. It must be mentioned that the friction coefficient needs to be further calibrated with experimental data on panel or wall specimens experiencing sliding shear failures along cracks.

3. EXPERIMENTAL VALIDATION OF THE MODEL

In this paper, the results of six cyclic panel tests from two experimental programs were used for experimental calibration and validation of the proposed FSAM. The first of these two test programs, referred to by Stevens et al. (1991), was conducted using the “Shell Element Tester” facility at the University of Toronto (Fig. 3(a)); and the other referred to by Mansour and Hsu (2005), was performed using the “Universal Element Tester” facility at the University of Houston (Fig. 3(b)).

(a) (b)

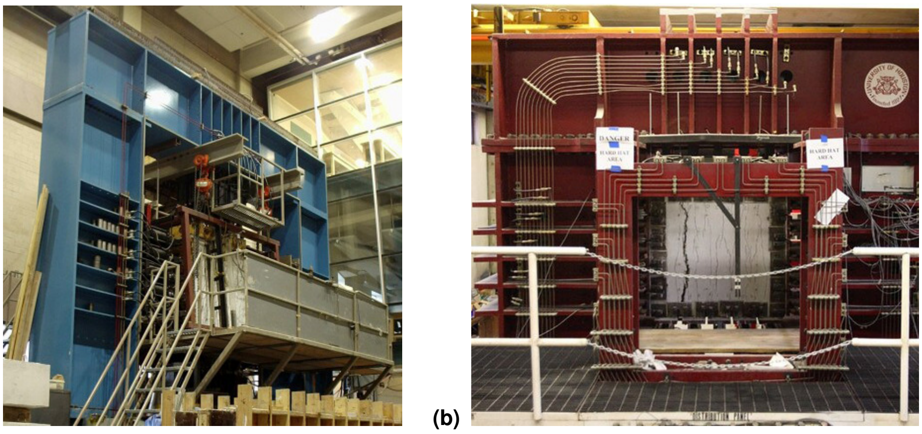

Figure 3. RC panel tests: (a) Stevens et al. (1991), (b) Mansour and Hsu (2005)

Cyclic loading tests on three RC panel specimens were referred to by Stevens et al. (1991). All tests were conducted under stress control. The test panels were square, with 1625x1625 mm dimensions and 285 mm thickness. The testing equipment was configured for loading principal (normal) stresses on the specimens. All panel reinforcement was arranged orthogonally in x and y directions. Properties of the panel specimens and loading conditions are listed in Table 3.1. In this test program, there were two different parameters investigated; the loading type and reinforcement ratio. While specimens SE8 and SE10 were used to examine the effect of loading type on panel response, specimens SE8 and SE9 were used to investigate the effect of reinforcement ratio.

Mansour and Hsu (2005) referred to cyclic tests conducted on 12 panel specimens under strain control. The test program was aimed to investigate the effects of reinforcement ratio and reinforcement orientation on panel behavior. Only three panel specimens (the CA series), having orthogonal reinforcement in x and y directions were considered within the scope of this paper. All three of these panel specimens had 1397x1397 mm dimensions and 178 mm thickness. Properties of the three panel specimens investigated are presented in Table 3.2. The only variable among these three specimens is the reinforcement ratio.

For detailed comparison of the test results with the model predictions, the model was calibrated to represent measured material properties, as well as the geometry and reinforcement characteristics of the test specimens. The monotonic parameters of the constitutive material models (f_y, f'_c, and ε_{co} in Tables 3.1 and 3.2) were calibrated to represent the results of uniaxial tests conducted on concrete cylinder specimens and rebar coupon samples; whereas the cyclic parameters, as well as the parameters representing compression softening, biaxial damage, and tension stiffening, were calibrated per the original empirical relationships defined in the constitutive models implemented. No adjustment was made on the constitutive parameters to improve the correlation between the test results and the model predictions. The only parameter defined arbitrarily was the shear aggregate interlock friction coefficient, which was set equal to a value of 0.2, for all specimens investigated. The model formulation, together with the constitutive models, was implemented in MATLAB, together with a displacement-controlled incremental-iterative nonlinear analysis solution strategy, to compare the model results with the experimentally-obtained responses for the panel specimens considered. Only selected response comparisons are presented in this paper, whereas detailed comparison of the test results with analytical predictions for all specimens, including those with inclined reinforcement, is underway.

Table 3.1. Panel specimen parameters, Stevens et al. (1991)

Panel Specimen:	SE8	SE9	SE10
Loading Type:	$\sigma_x = 0$ $\sigma_y = 0$ τ_{xy} : *Reversed Cyclic*	$\sigma_x = 0$ $\sigma_y = 0$ τ_{xy} : *Reversed Cyclic*	$\sigma_x = -\lvert\tau_{xy}/3\rvert$ $\sigma_x = -\lvert\tau_{xy}/3\rvert$ τ_{xy} : *Reversed Cyclic*
ρ_x	0.03	0.03	0.03
ρ_y	0.01	0.03	0.01
$f_{y,x}$	492 MPa	422 MPa	422 MPa
$f_{y,y}$	479 MPa	422 MPa	479 MPa
f'_c	37 MPa	44 MPa	34 MPa
ε_{co}	0.0026	0.0026	0.0023
f_{ct}	2.0 MPa	2.2 MPa	2.0 MPa
ε_t	0.0001	0.0001	0.00013

Table 3.2. Panel specimen parameters, Mansour and Hsu (2005)

Panel Specimen:	CA2	CA3	CA4
Loading Type:	$\sigma_x = 0$ $\sigma_y = 0$ τ_{xy} : *Reversed Cyclic*	$\sigma_x = 0$ $\sigma_y = 0$ τ_{xy} : *Reversed Cyclic*	$\sigma_x = 0$ $\sigma_y = 0$ τ_{xy} : *Reversed Cyclic*
ρ_x	0.0077	0.017	0.027
ρ_y	0.0077	0.017	0.027
$f_{y,x}$	424 MPa	425 MPa	453 MPa
$f_{y,y}$	424 MPa	425 MPa	453 MPa
f'_c	45 MPa	44.5 MPa	45 MPa
ε_{co}	0.0025	0.0024	0.0028
f_{ct}	2.2 MPa	2.2 MPa	2.2 MPa
ε_t	0.00008	0.00008	0.00008

The experimentally-measured shear stress vs. shear strain responses for panel specimens SE8, SE9, and SE10 investigated by Stevens et al. (1991) are compared with the analytical model predictions in Fig. 4 (a), (b), and (c), respectively. The comparisons indicate that the model provides reasonably accurate shear stress vs. strain response predictions, for varying reinforcement ratios and loading conditions. Overall, a good level of agreement is achieved between the test data and model results in terms of shear stress capacity, stiffness, ductility, shape of the unloading/reloading loops, and pinching characteristics of the response. The most apparent discrepancy between the test results and the model predictions is observed for Specimen SE8, where the model overestimates the shear stress capacity by approximately 30% (Fig. 4(a)). After sensitivity studies, it was observed that reducing the shear aggregate interlock friction coefficient from the original value of 0.2 to a value of 0.1 improved the response prediction for this specimen (Fig. 4(d)). Overall, a friction coefficient value of 0.2 yields reasonably accurate model predictions for all of the panel specimens presented here, except Specimen SE8, for which using a coefficient of 0.1 provides a better shear stress capacity prediction.

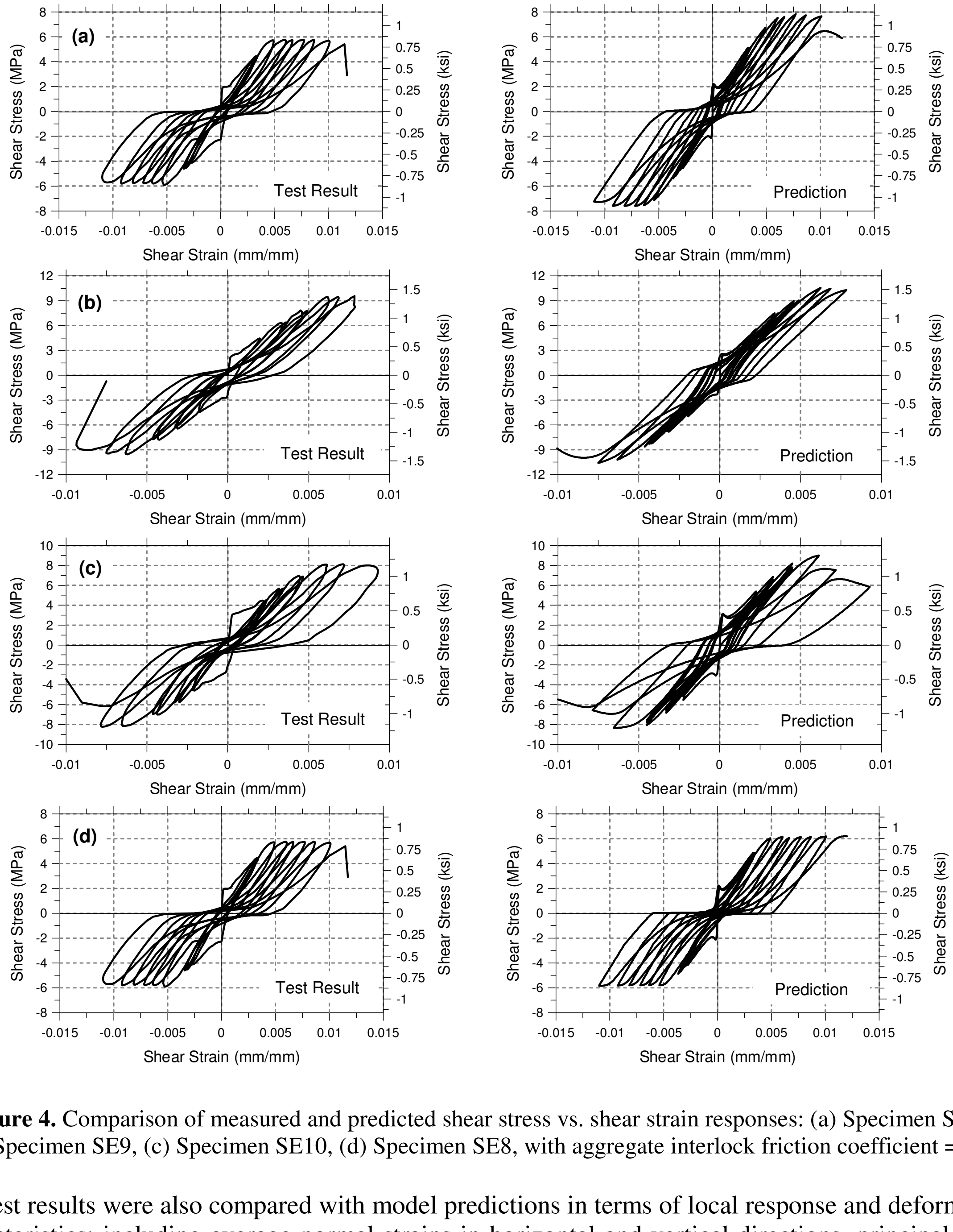

Figure 4. Comparison of measured and predicted shear stress vs. shear strain responses: (a) Specimen SE8, (b) Specimen SE9, (c) Specimen SE10, (d) Specimen SE8, with aggregate interlock friction coefficient = 0.1

The test results were also compared with model predictions in terms of local response and deformation characteristics; including average normal strains in horizontal and vertical directions, principal strain directions, and principal stress directions. Average normal strain histories in x and y directions (in the directions of the reinforcing bars) measured by displacement transducers attached to Specimen SE9, are compared with the model predictions in Fig. 5. The data presented in the Fig. 5 relates the average normal strain to the load step (or data point) number during testing and analysis. Comparison of the analytical and test results reveals that the model captures the general ascending trend in the average normal strain values with reasonable accuracy, albeit with increasing discrepancies at later stages of loading, especially in the y direction. The normal strains measured and predicted for this specimen are below the yield strain value, which also agrees with the concrete crushing failure mode observed during testing of this specimen, due to high reinforcement ratios in both directions (Table 3.1).

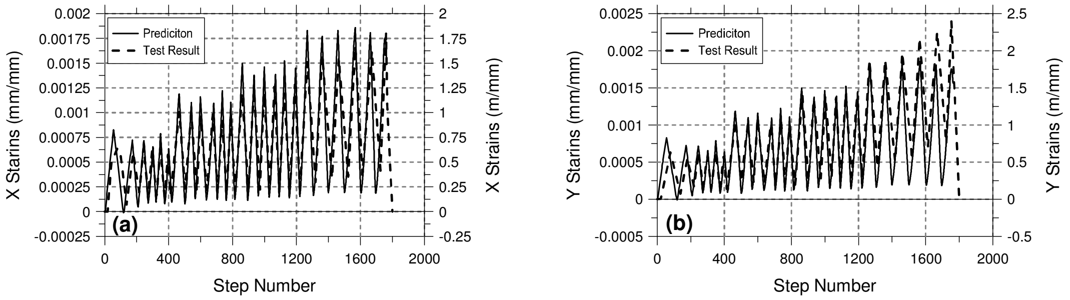

Figure 5. Average normal strain histories for Specimen SE9: (a) in x direction, (b) in y direction

Experimentally-measured principal strain direction histories are compared with model predictions in Fig. 6(a). Overall, the model predicts the variation in principal strain directions with reasonable accuracy, with slight underestimation of the principal strain directions at the beginning of the analysis (for relatively small shear strain values). Figure 6(b) compares the principal stress direction in concrete vs. shear stress behavior predicted by the model, with the envelope of the test results, since the plot presented by Stevens et al. (1991) showing the cyclic test results was too congested for digitizing purposes. Concrete principal stress direction in the model becomes the fixed direction of the strut working in compression, since the compression strut is subjected to zero shear stress from aggregate interlock, due to tensile stresses in concrete in the perpendicular direction. Although there exists variation in the test results with shear stress, the upper and lower bounds of the measured principal stress directions do not vary significantly with the magnitude of the shear stress, and are in good agreement with model predictions, validating the fixed-strut-angle modeling approach used.

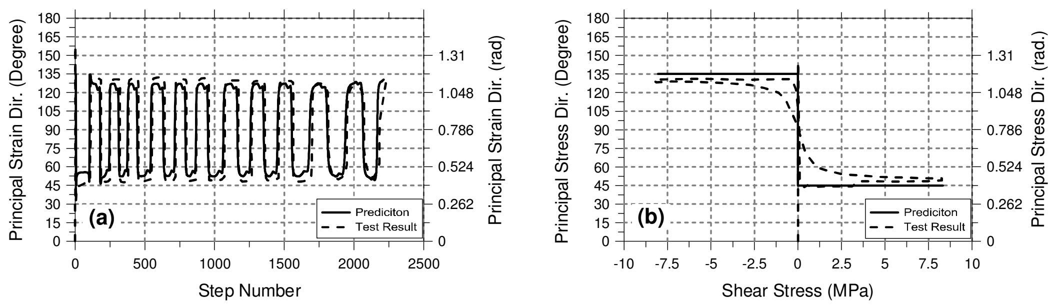

Figure 6. Principal strain and stress directions for Specimen SE10: (a) Principle strain direction history, (b) Principal stress direction in concrete vs. shear stress behavior

The experimentally-measured shear stress vs. shear strain responses for panel specimens CA2, CA3, and CA4 investigated by Mansour and Hsu (1991) are compared with the analytical model predictions in Fig. 7. Again, the model provides reasonably accurate shear stress vs. strain response predictions, for varying reinforcement ratios. An acceptable level of agreement is observed between model and test results in terms of shear stress capacity, stiffness, ductility, shape of the unloading/reloading loops, and pinching characteristics of the response. Furthermore, the behavior characteristics and failure modes observed during the tests, including yielding of reinforcement and crushing of concrete, were observed to be consistent with the analytically-predicted responses.

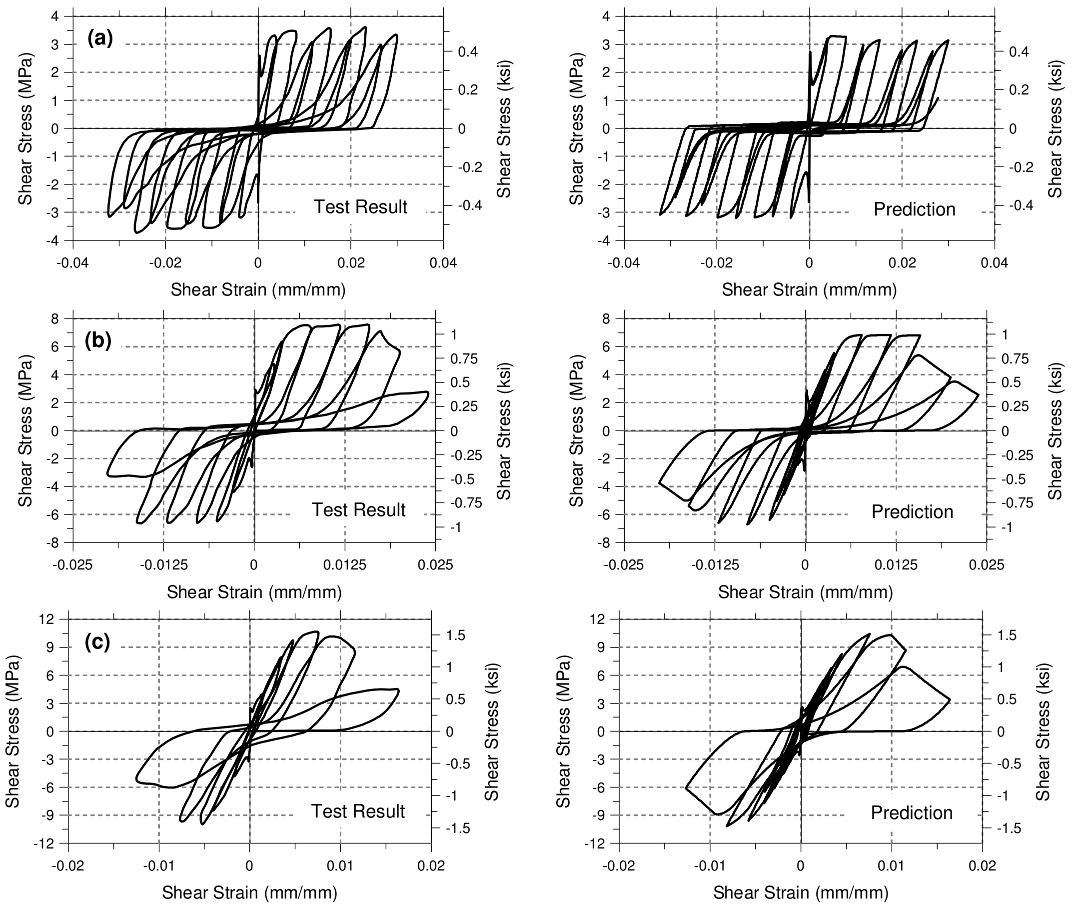

Figure 7. Comparison of measured and predicted shear stress vs. shear strain responses: (a) Specimen CA2, (b) Specimen CA3, (c) Specimen CA4

Average normal strain histories in x and y directions (in the directions of the reinforcing bars) measured using displacement transducers attached on Specimen CA2, are compared with the model predictions in Fig. 8. The model captures the average normal strain histories in both directions, with reasonable accuracy. The measured and predicted average normal strains on this specimen are large post–yield strains; which agrees with the reinforcement yielding failure mode observed during testing of this specimen, due to relatively low reinforcement ratios used in both directions (Table 3.2).

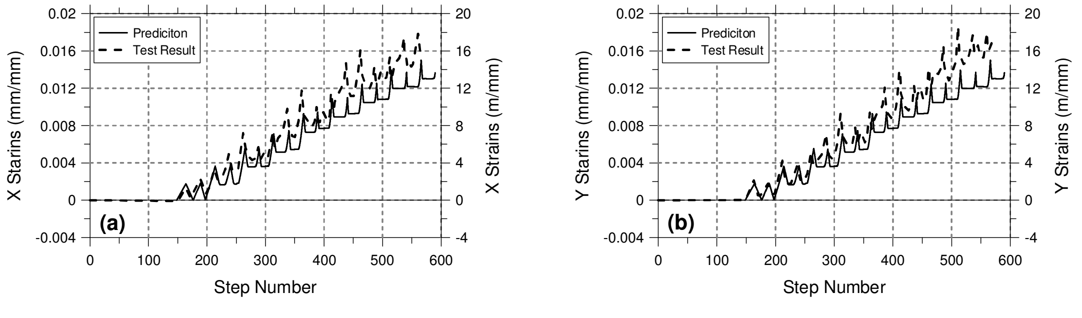

Figure 8. Average normal strain histories for Specimen CA2: (a) in x direction, (b) in y direction

4. SUMMARY AND CONCLUSIONS

A new constitutive model – named the Fixed Strut Angle Model (FSAM) – was proposed for simulating the cyclic shear behavior of RC panel elements. The FSAM, although simple in formulation, is capable of providing reasonably accurate predictions of the nonlinear shear behavior and axial–shear response coupling of RC panels, subjected to generalized and reversed cyclic loading conditions. The main inherent assumption underlying the formulation of the FSAM is that upon cracking of concrete in a panel, crack directions in concrete do not rotate with subsequent loading. A simple friction-based constitutive relationship representing shear aggregate interlock behavior along crack surfaces is also implemented in the model formulation.

Detailed correlation studies were conducted to compare the model predictions with results of selected cyclic RC panel tests available in the literature. The model was shown to capture, with a reasonable level of accuracy, overall behavioral attributes of RC panels; including cyclic shear stress vs. shear strain behavior, shear stress capacity, initial stiffness, cyclic stiffness degradation, pinching, ductility, and failure mode. The model has also provided reasonably accurate local response and deformation predictions; including average longitudinal strain histories in horizontal and vertical directions, principal strain direction histories, and principal stress direction vs. shear stress behavior. The proposed constitutive model is expected to be a feasible candidate for implementation into a two-dimensional finite-element analysis formulation, for efficient and practical seismic response prediction of RC structural walls with various geometries, aspect ratios, and reinforcement details.

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