# Generalized scaling law for settlements of dry sand deposit

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**SUMMARY:**
A series of dynamic tests under two different centrifugal accelerations of 25 g and 50 g are conducted to verify the generalized scaling law. The model ground constitutes of a flat dry sand layer. With the scaling law, a prototype ground is scaled down to 1/100. A sinusoidal input acceleration of frequency 1.0 Hz, maximum amplitude 0.5 g, and duration 14 sec in prototype scale is applied to the model ground. Each model is exposed to the identical input motion sequentially 10 times. In total nine accelerometers are installed in the model. Surface settlements are measured by laser displacement transducers. Settlements at three different depths (300, 200 and 50 mm – model scale - from the surface) are measured by settlement gauges. Measured settlements after the initial shake in prototype scale show agreements between the two models when the intensity of shaking is nearly identical.

*Keywords: Centrifuge modelling, Scaling law, Dynamic, Settlements, Dry sand*

## 1. INTRODUCTION

In the centrifuge model testing, although structural models have to be small and simplified, the prototype behaviour is approximated in accordance with scaling laws (e.g., Garnier et al., 2007), and it qualitatively represents prototype behaviour. One of the major obstacles for application of physical modelling results to performance-based design practice is that a specific prototype cannot be tested due to restrictions associated with experimental conditions, such as the size of the model container and scaling effects on materials. For 1-g model testing, to overcome these restrictions, the size of the experimental facility has become larger and larger so that real-scale models can be tested [e.g., E-defense (Tokimatsu et al., 2007), NEES@UC San Diego Large High Performance Outdoor Shake Table (Einde et al., 2004)]. However, for geotechnical structures, development of larger research facility may still have limitations because, even with such a large facility, physical modelling with foundations and surrounding ground has to be reduced due to factors inherent in a large facility, such as the capacity of the shake table and budget.

Demands for the testing of large prototypes are increasing under the restrictions mentioned above. To resolve such demands and restrictions, Iai et al. (2005) generalized the scaling law by combining the scaling law for centrifuge testing with the one for 1-g dynamic-model testing. They call it the "generalized scaling law" in dynamic centrifuge modelling. Tobita et al. (2011) investigated its applicability with a flat saturated sand bed. They conducted a series of centrifuge model tests to verify and find issues on the generalized scaling law under the scheme of the modelling of models technique. In a series of dynamic tests, four different centrifugal accelerations from 5 g to 70 g are applied to the scaled models for which the prototype is uniquely given. With the scaling law, the prototype is scaled down to 1/100. The models are exposed to sinusoidal input accelerations with 0.65 Hz and amplitudes of 2.1 m/s$^2$ and 3.1 m/s$^2$ in prototype scale. For response during shaking, nearly identical accelerations and excess pore-water pressure buildups are recorded for all the cases in the prototype scale. Discrepancies are found on surface settlements and duration time for dissipation of excess pore-water pressure. The major cause of the discrepancy of the latter "duration time" may be associated with low confining stress in model ground under low centrifugal acceleration. The cause of discrepancy of surface settlement has been yet to be identified. Therefore, in this study, the applicability of the generalized scaling law, in particular, the scaling law of displacement is investigated in detail.

### 1.1. Brief review of the generalized scaling law

Scaling factors for physical model tests can be introduced in general forms by choosing a set of basic physical properties to be independent and deriving the scaling factors for other properties via governing equations of the analysed system. In the concept of the generalized scaling law, a model on a shaking table in a geotechnical centrifuge is considered to be a small-scale representation of a 1-g shaking-table test. Figure 1 visualizes this concept by introducing a virtual 1-g model to which the prototype is scaled down via a similitude for 1-g shaking-table tests (Iai, 1989). The virtual 1-g model is subsequently scaled down by applying a similitude for centrifuge tests to the actual physical model. In this way, the geometric scaling factors applied in 1-g tests ($\mu$) [row (1) of Table 1] can be multiplied with those for centrifuge tests ($\eta$) [row (2) of Table 1], resulting in much larger overall scaling factors $\lambda=\mu\eta$ [row (3) of Table 1].

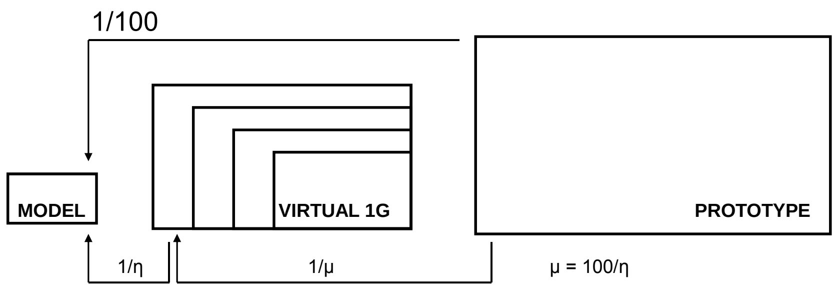



**Figure 1.** Relationship among prototype, virtual 1G model and centrifuge model for the case of scaling factor of 1/100 ($\lambda=\mu\eta=100$)

**Table 1.1.** Scaling factors in physical model testing (Iai, 1989, Iai, et al. 2005)

| | (1) Scaling factors for 1g test | (2) Scaling factors for centrifuge test | (3) Generalized scaling factors |
|---|---|---|---|
| Length | $\mu$ | $\eta$ | $\mu\eta$ |
| Density | 1 | 1 | 1 |
| Time | $\mu^{0.75}$ | $\eta$ | $\mu^{0.75}\eta$ |
| Frequency | $\mu^{-0.75}$ | $1/\eta$ | $\mu^{-0.75}/\eta$ |
| Acceleration | 1 | $1/\eta$ | $1/\eta$ |
| Velocity | $\mu^{0.75}$ | 1 | $\mu^{0.75}$ |
| Displacement | $\mu^{1.5}$ | $\eta$ | $\mu^{1.5}\eta$ |
| Stress | $\mu$ | 1 | $\mu$ |
| Strain | $\mu^{0.5}$ | 1 | $\mu^{0.5}$ |
| Stiffness | $\mu^{0.5}$ | 1 | $\mu^{0.5}$ |
| Permeability | $\mu^{0.75}$ | $\eta$ | $\mu^{0.75}\eta$ |
| Pore pressure | $\mu$ | 1 | $\mu$ |
| Fluid Pressure | $\mu$ | 1 | $\mu$ |

## 2. DYNAMIC CENTRIFUGE TESTS ON FLAT, LOOSE, DRY SAND DEPOSIT

To investigate the applicability of the generalized scaling law described above, a series of dynamic tests was conducted following the principle of "modelling of models." This technique was introduced by Schofield (1980) to assess the behaviour of a prototype through repetition of the test at different scales and comparison of the results in prototype scale. In the present study, without changing the actual size of the physical model but varying the virtual 1-g dimension, the overall scaling factor ($\lambda=\mu\eta=100$) is kept constant (Fig. 1). Here, it is set to a fixed value comprising different combinations of the scaling factors for 1-g model testing, $\mu$, and centrifuge testing, $\eta$. Table 2 lists the applied geometric scaling factors as well as frequencies and amplitudes of the input motions employed in the study. As shown in Table 2, the scaling factors of displacement are relatively larger than the other physical quantities (200 in 25 g and 141.42 in 50g). This fact demands precise measurements in displacement. In total 5 tests [3 tests in 25 g (25 g_1, 25g_2, and 25g_3) and 2 tests in 50 g (50g_1 and 50g_2)] are conducted. In what follows, units are in prototype unless otherwise specified.

**Table 1.2.** Test cases and scaling factors used in the present study

| | Case 1 | 25 G | | Case 2 | 50 G | |
|---|---|---|---|---|---|---|
| Quantity | scaling factor 1g test | scaling factor centrifuge test | generalized scaling factors | scaling factor 1g test | scaling factor centrifuge test | generalized scaling factors |
| Length | 4.00 | 25.00 | 100.00 | 2.00 | 50.00 | 100.00 |
| Density | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| Time | 2.83 | 25.00 | 70.71 | 1.68 | 50.00 | 84.09 |
| Frequency | 0.35 | 0.04 | 0.01 | 0.59 | 0.02 | 0.01 |
| Acceleration | 1.00 | 0.04 | 0.04 | 1.00 | 0.02 | 0.02 |
| Velocity | 2.83 | 1.00 | 2.83 | 1.68 | 1.00 | 1.68 |
| Displacement | 8.00 | 25.00 | 200.00 | 2.83 | 50.00 | 141.42 |
| Stress | 4.00 | 1.00 | 4.00 | 2.00 | 1.00 | 2.00 |
| Strain | 2.00 | 1.00 | 2.00 | 1.41 | 1.00 | 1.41 |
| Stiffness | 2.00 | 1.00 | 2.00 | 1.41 | 1.00 | 1.41 |
| Permeability | 2.83 | 25.00 | 70.71 | 1.68 | 50.00 | 84.09 |
| Pore pressure | 4.00 | 1.00 | 4.00 | 2.00 | 1.00 | 2.00 |
| Fluid Pressure | 4.00 | 1.00 | 4.00 | 2.00 | 1.00 | 2.00 |

### 2.1. Test setup

A series of dynamic tests under two different centrifugal accelerations of 25 g and 50 g are conducted with the geotechnical centrifuge (arm length=5.0 m) at the IFSTTAR (Institut français des sciences et technologies des transports, de l'aménagement et des réseaux), Nantes, France. The model ground constitutes of a flat dry sand layer, which is constructed with air-pluviation method (pluviation height=0.6 m, slot width=4 mm) to form the relative density of 50% of the Fontainebleau sand NE34 ($e_{min}$=0.545, $e_{max}$=0.866). With the scaling law, a prototype ground is scaled down to 1/100. The flexible ESB (equivalent shear beam) box whose inside dimension is 800 (W) x 400 (H) x 340 (D) (mm) in model scale is employed (Fig. 2). A sinusoidal input acceleration of frequency 1.0 Hz, maximum amplitude 0.5 g, and duration 14 sec is applied to the model ground. Each model is exposed to the identical input motion sequentially 10 times in order to increase the number of measurements.

In total nine accelerometers are installed in the model (Fig. 2). Surface settlements are measured by laser displacement transducers. Settlements at three different depths (300, 200 and 50 mm – model scale - from the surface) are measured by settlement gauges (Fig. 3), which are made of a plate, rod connected to potentiometers.

As shown in Fig. 4(a), settlement gauges are carefully placed at the specified depth (Fig. 2) with fishing strings. The PVC plates without attaching the potentiometers are installed for comparison purposes. Three potentiometers are mounted after completing model ground

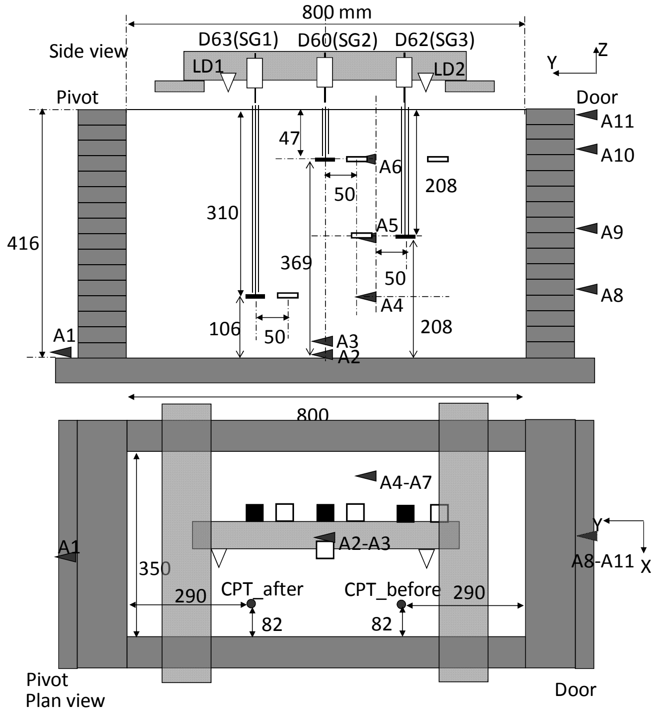



**Figure 2.** Schematic view and sensor location of the model

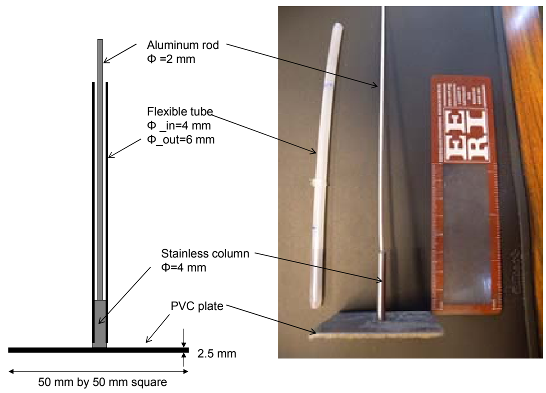
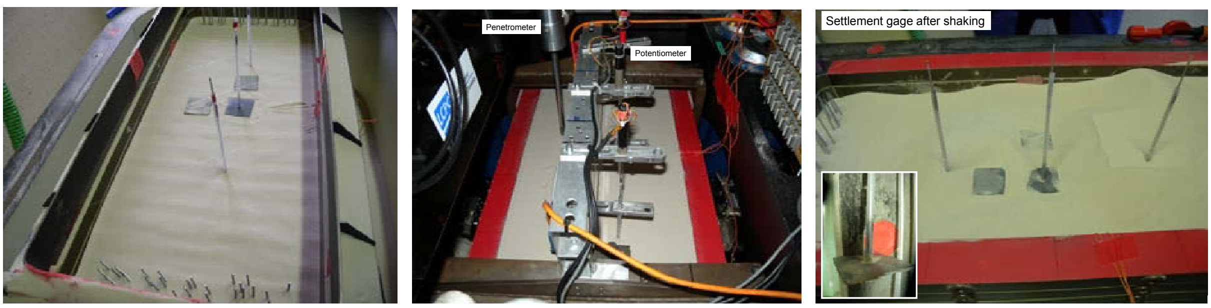



**Figure 3.** Detail of a settlement gauge




**(a)** **(b)** **(c)**

**Figure 4.** Model setup (a) Installation of settlement gauge, (b) Setup of penetrometer and potentiometers, and (c) settlement gauges and PVC plates after the test

## 3. RESPONSE OF THE MODEL GROUND

### 3.1. Input and ground acceleration

As shown in Fig. 5, nearly identical input accelerations are given to the model ground. Figure 6 summarises intensity of input motion in the form of Arias intensity (Arias 1970) for all the cases employed in the present study. As shown in Fig. 6, in model scale, intensities of cases in 25 g are quite different from those of cases in 50 g. In 25 g, the value of the intensity is about 180 m/s, while in 50 g, it is about 600 m/s. In prototype scale [Fig. 6(b)], the intensity becomes about 20 m/s for both centrifugal accelerations. As shown in Fig. 6(a), for the first 5 shakes in the case of 50g, the intensity varies between 450 m/s to 700 m/s in model scale. As shown in Fig. 6(b), the intensity of case 50g_2 is closer to the intensity of cases in 25 g than case of 50g_1. This variation may be due to the instability of shake table control. As explained later, this small variation exerts influence on the ground settlements.

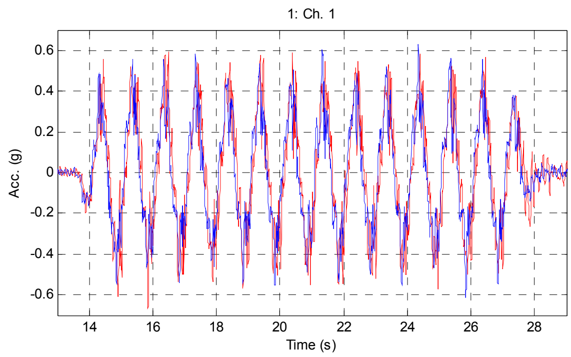



**Figure 5.** Time histories of input acceleration in prototype scale (red 25g_1, blue 50g_1)

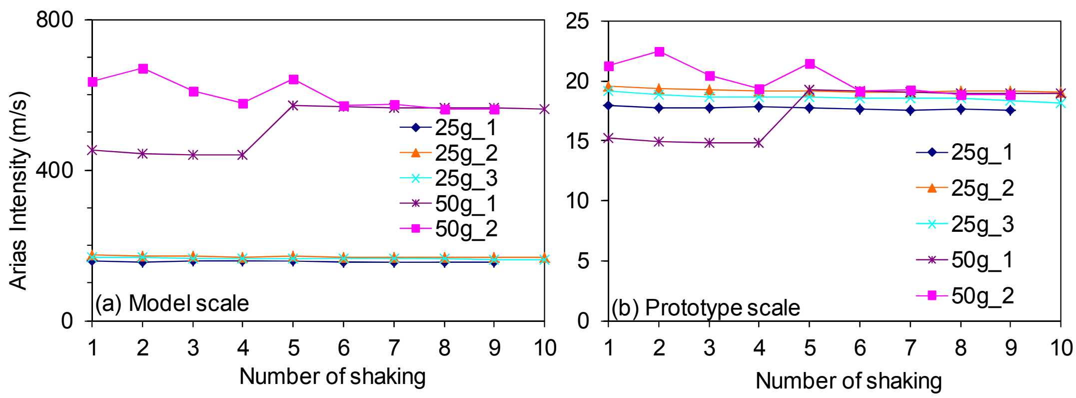



**Figure 6.** Arias intensity of the recorded input acceleration: (a) model scale and (b) prototype scale

### 3.2. Penetration resistance

Before the initial shaking and after the 10th shaking, resistance of the model ground was measured by the miniature penetrometer shown in Fig. 4(b). As shown in Fig. 7(a), penetration resistance in depth under 50 g in model scale is, as it is expected, larger than that of 25 g. While in prototype scale [Fig. 7(b)], they approach each other and the curve of 25 g becomes slightly larger. This clearly shows that the generalized scaling law works correctly for the scaling of penetration resistance. The same can be said for the penetration resistance after the 10th shaking (Fig. 8). However, degree of coincidence becomes lower especially in deeper depth.

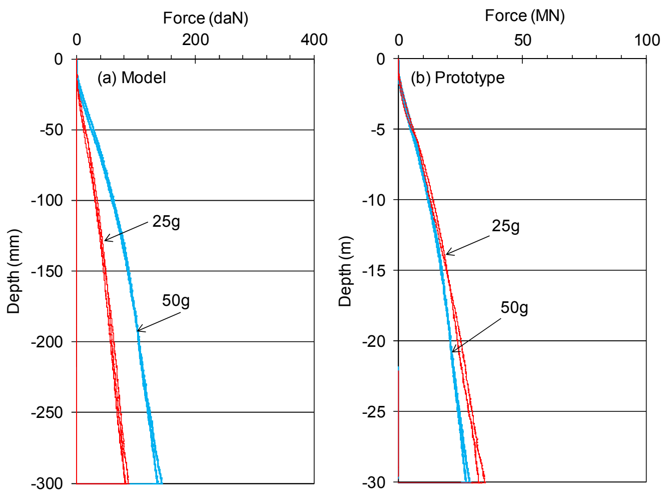



**Figure 7.** Penetrometer resistance before the initial shaking in model scale (a) and prototype scale (b).

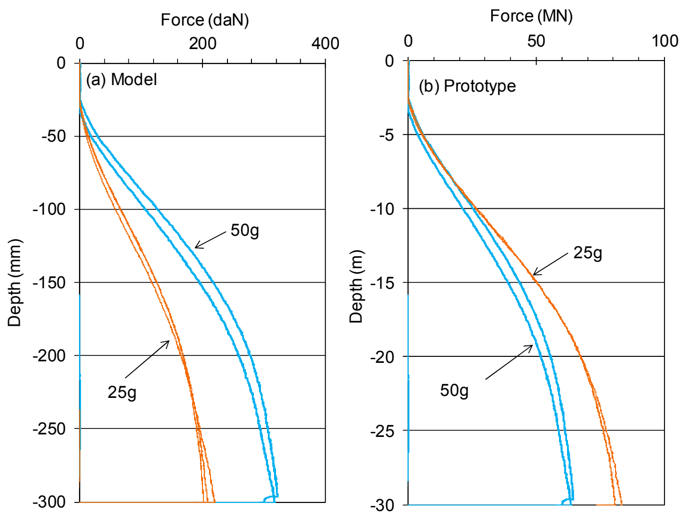



**Figure 8.** Penetrometer resistance after the 10th shaking in model scale (a) and prototype scale (b).

### 3.3. Ground settlements

Ground settlements at the ground surface are measured by laser displacement transducers, and those in the ground are by settlement gauges (Fig. 3). Before shaking, the ground is consolidated by applying the specified centrifugal accelerations consecutively 3 times to stabilize the ground. This process is manifested in Fig. 9 (model scale). For example, as shown in Fig. 9(a), as centrifugal acceleration increases, the ground surface settles about 1.1 mm (D122) to 1.3 mm (D116). After the centrifugal acceleration of 25 g is kept about 5 min, centrifuge is stopped (1 g). At this moment, the ground settlements are reduced to about 1 mm. Then again the centrifugal acceleration is applied, and as shown in Fig. 9(a), the surface settlements further increases up to about 1.5 to 1.7 mm. This process is repeated 3 times. Common trend shown in Fig. 9 is that the ground settles as centrifugal acceleration increases and in every application of centrifugal acceleration, the amount of ground settlements gradually increases. However, the rate of settlement is decreasing, i.e., the amount of settlement asymptotically approaches to some maximum values.

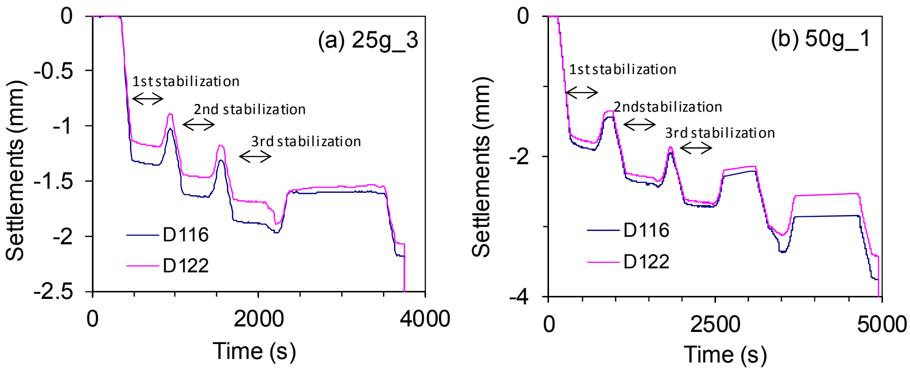



**Figure 9.** Time histories of ground settlements due to consolidation before shaking (model scale): (a) 25g_1 and (b) 50g_1

Figure 10 shows time histories of the ground settlements during shaking. The curves indicate step wise increase of settlements at each shaking. In Fig. 10(b), the curve of D63 is increasing, i.e., settlement gauge is uplifted. This may be because of mal-functioning of the sensor or sands near the PVC plate were flowing under the plate.

Amounts of settlements after each shaking are summarized in Fig. 11. If the generalized scaling law works correctly, those curves in prototype scale [Fig. 11 (b)] are identical. Results show, for example, after 10th shaking, the ground settlement is about 2,800 mm (50 g) and 3,400 mm (25 g). As number of shaking increases, the difference seems to be increasing. However, at the initial shaking, the amount of settlements is about 900 to 1,100 mm, variation of difference in settlement between cases of 25 g and 50 g is much smaller than those after 2nd shaking. Thus, in what follows, settlements after the 1st shaking are investigated in detail.

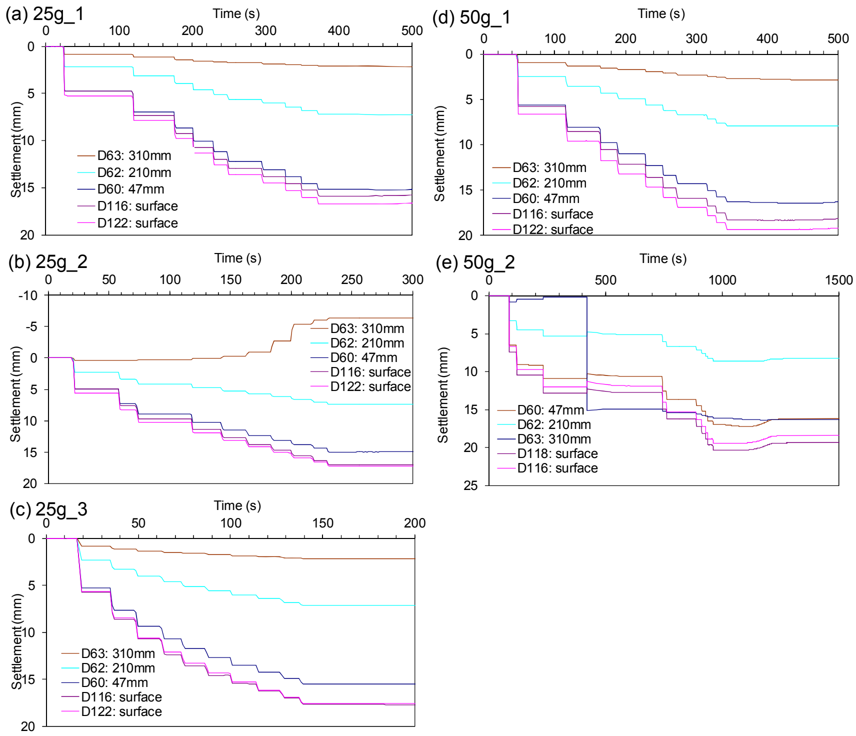



**Figure 10.** Time histories of ground settlements due to shaking (model scale)

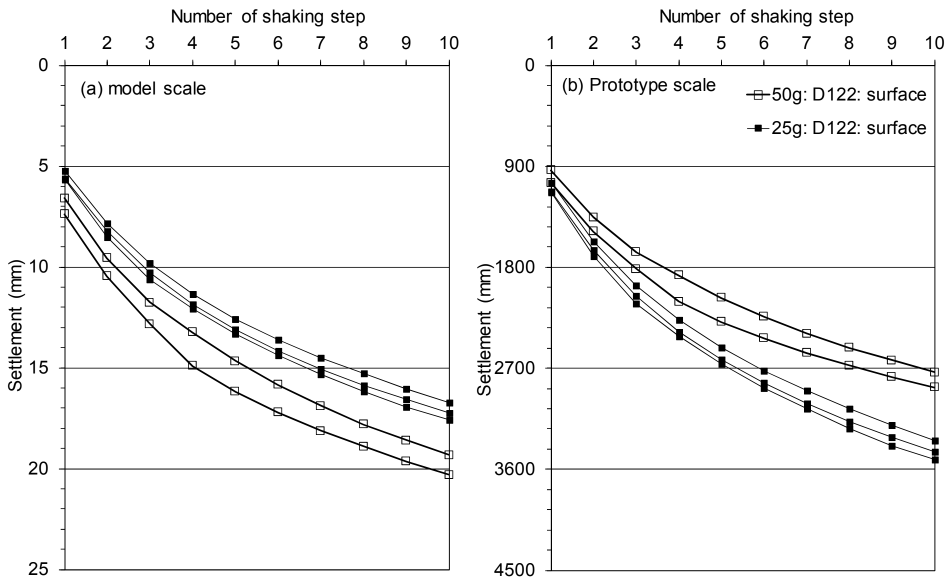



**Figure 11.** Cumulative settlements after each shaking: (a) model scale, (b) prototype scale

Figure 12 shows the averaged amount of settlements after the 1st shaking in model [Fig. 12(a)], and prototype scale [Fig. 12(b)]. As it is shown in Fig. 12(b), the perfect match was not obtained between 25 g and 50 g. Near the surface, the difference of settlement between 25 g and 50 g is about 100 mm. The amount of settlement in 50 g is systematically underestimated compared with the ones of 25 g. To have better match, settlements in 50 g has to be much larger in model scale, or in the experiments.

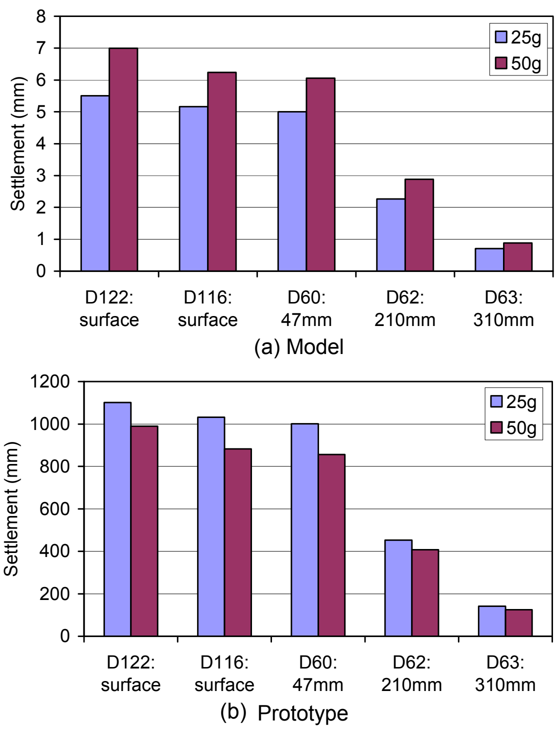



**Figure 12.** Comparison of averaged settlements measured in 25 g and 50 g: (a) model scale, and (b) prototype scale

Figure 13 shows settlements observed after the 1st shaking for all cases. Even though special care had been taken for model construction processes, there are considerable amount of variations in the amount of measured settlements. In each case, it is possible to have minor variation in constructing the model ground, in sensor setups, and in the input accelerations. As mentioned earlier, scaling factor of settlements (displacement) is as large as 200 for 25 g and 141 for 50 g. Those minor variations may cause large difference in results. Another possible cause of the difference is, as shown in Fig. 6, that due to the variation of input energy shown by the Arias intensity, case 50g_1 had lower and case 50g_2 had slightly larger intensity of shaking. This trend is found in the amount settlement shown in Fig. 13(c). Considering that the intensity of case 50g_2 is close to the ones in 25 g, and if we compare results of 50g_2 with the ones in 25 g [see dotted horizontal lines in Fig. 13 (b) and (d)], the amount of settlement in prototype scale seems to be matching well.

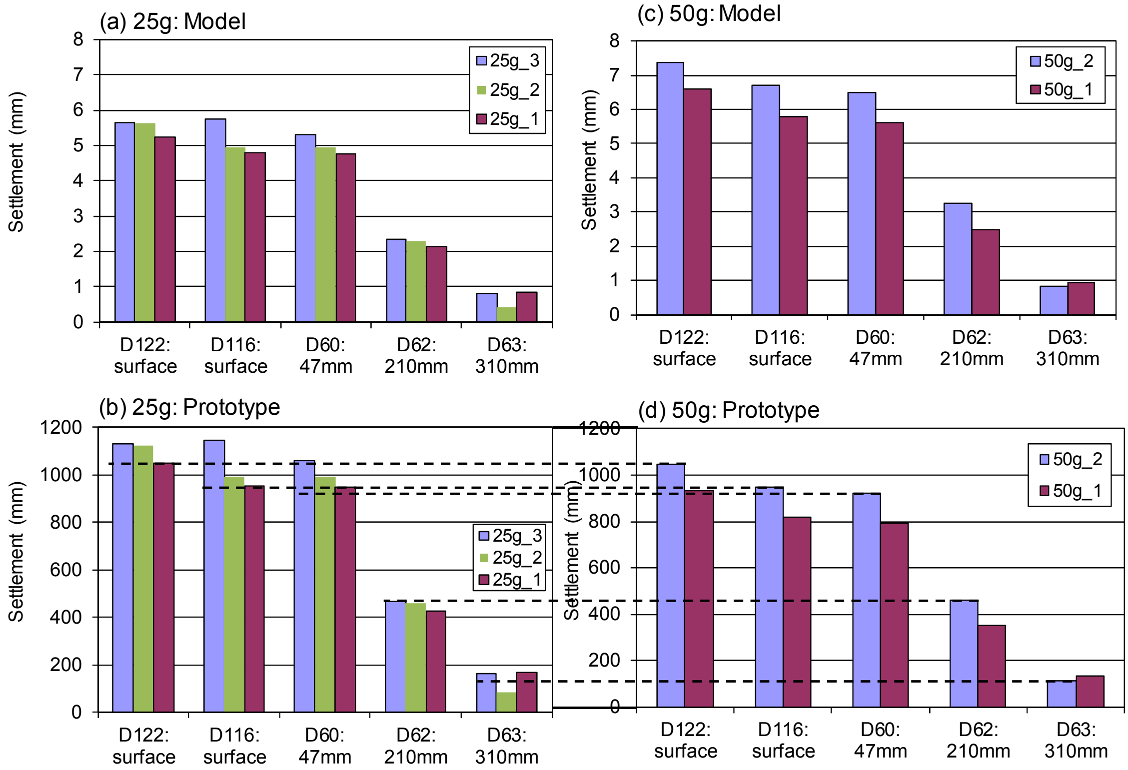



**Figure 13.** Summary of settlements measured in all the test cases: (a) 25 g in model scale, (b) 25 g in prototype scale, (c) 50 g in model scale, (d) 50 g in prototype scale

## 4. CONCLUSIONS

To examine the applicability of the generalized scaling law, a series of dynamic tests under two different centrifugal accelerations of 25 g and 50 g were conducted. The model ground constituted of a flat dry sand layer. A prototype ground and input accelerations were scaled down to 1/100 according with the scaling law. A sinusoidal input acceleration of frequency 1.0 Hz, maximum amplitude 0.5 g, and duration 14 sec in prototype scale was applied to the model ground. Each model was exposed to the identical input motion sequentially 10 times. Measured settlements after the initial shake in prototype scale showed agreements between the two models when the intensity of shaking was nearly identical.

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