The windowing Higuchi's method applied to the analysis of the south of California seismicity

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SUMMARY:
This work examines the seismicity of Southern California since 1981, the aim is to study the events of magnitude greater than 7.0 that occurred in that region with the windowing method of Higuchi to see if it is possible to identify precursor seismicity patterns of great magnitude earthquakes. The methodology consists on isolating segments centered on the events of greater magnitude and to analyze 36 months before and after the earthquake. Several months before the earthquake there is not much variation in the Higuchi's fractal dimension, but closer to the main event, this pattern changes and the fractal dimension decreases. With the Higuchi's method we obtain a straight line whose slope is the fractal dimension, but the intercept has information too, months before the earthquake it has small fluctuations but prior to the earthquake it increments its value, after the earthquake it returns to its normal values.

Keywords: California seismicity, Higuchi's method, Seismic precursor

1. INTRODUCTION

Recent studies have shown that many complex natural systems are characterized by long-range correlations [Peng et al., 1993, 1995], the identification and quantification of these correlations by using spectral analysis fails because the data are not stationary and it is usually not known the origin of these features, instead other methods are used as the detrended fluctuation method (DFA) that is a method that allows the quantification of long-range correlations avoiding spurious detections [Matsoukas et al., 2000; Telesca et al., 2007]. The Higuchi's method [Higuchi, 1988; 1990] is also used to this task and it permits the accurate calculation of the fractal dimension of the time series.

Complex systems as the seismic zones generate time series showing the combination of fractal and periodic components. Since two decades ago the so-called Higuchi`s method to calculate the fractal dimension of complex time series has been used to investigate correlations and non linear dynamic properties embedded in nonstationary time series. For example, this method has been used to analyze electroseismic time series [Guzmán-Vargas et al., 2009; Ramírez Rojas et al., 2007]. Recently, the Higuchi's method has been used to detect periodic components mixed with fractal signals [Peralta et al., 2006; Muñoz-Diosdado et al. 2008, 2009, 2010].

In this work we study the seismicity of Southern California since 1981, this region is highly instrumented, and the catalogues of seismicity in the region can be considered complete since a magnitude of 1.5, so there are a lot of detected events, therefore the results obtained may be statistically significant. The idea is to apply the method of windowing with the Higuchi's method to study if there is some pattern that could be identified as a possible precursor to events of great magnitude. We present here the results of the windowing, which suggest that months before the earthquake there is little variation in the Higuchi's fractal dimension, but closer to the main event this pattern changes and the fractal dimension decreases.

2. HIGUCHI'S METHOD

A time series can be expressed by $x(i)$ $i=1,\ldots, N$, where each datum is taken at equally spaced time intervals, with a uniform time denoted by δ. Usually thought to be $\delta = 1$ because in principle this parameter does not alter the data analysis. The following describes how to apply the Higuchi's method [Higuchi, 1988; 1990] to a time series.

a) From the time series $x(i)$ the new series $x_k^m(i)$ are obtained

$$
\begin{aligned}
&x_k^m;\, x(m),\, x(m+k),\, x(m+2k),\, x(m+3k),\ldots \\
&,x\left(m+\left[\frac{N-m}{k}\right]k\right), \quad (m=1,2,3,\ldots,k)
\end{aligned}
\tag{2.1}
$$

Where k and m are integer numbers, m and k represents the initial time interval width and [ ] denotes the integer part.

b) The length of the series x_k^m (i) is defined as:

$$
L_m(k) = \left\{ \left(\sum_{i=1}^{\left[\frac{N-m}{k}\right]} \left| x(m+ik) - x(m+(i-1)k) \right| \right) \frac{N-1}{\left[\frac{N-m}{k}\right]k} \right\} \Big/ k \tag{2.2}
$$

The term $(N-1)/[(N-m)/k]k$ represents the normalization factor for the length of the subset.

c) The length of the series $L(k)$ for $x(i)$ is obtained by averaging all the subseries lengths $L_m(k)$ that have been obtained for a given k value.

d) If $L(k) \propto k^{-D}$, that is, if it behaves as a power law, we find that the exponent D is the fractal dimension of the series.

Applying the above relation implies the proper choice of a maximum value of k for which the relationship $L(k) \propto k^{-D}$ is approximately linear (Figure 1).

In the case of self-affine curves, this fractal dimension relates to the exponent β (by means of $\beta = 5 - 2D$, where if D is in the range $1<D<2$ then $1<\beta<3$. Higuchi showed that this method provides an accurate estimate of the fractal dimension of even a small number of data. Higuchi developed his method as an alternative to spectral analysis because although there is a relationship between D and β, the standard deviation of the fractal dimension obtained by using the fast Fourier transform (FFT) is greater than the standard deviation which is obtained by calculating the fractal dimension with this method. As the FFT method requires calculating averages of power spectra to obtain a stable spectrum, this will require many of these averages to obtain precise and stable values as those afforded by this technique. Also the Higuchi's method has allowed to define clearly the two or more regions in which the graph of $\log L_m(k)$ vs $\log k$ is divided in case it has crossovers, i.e. the points that divide the different scaling regions with different values of the fractal dimension D [Higuchi, 1988; 1990].

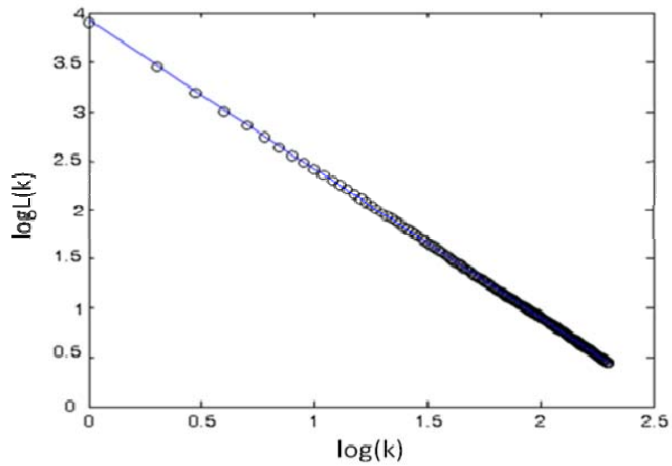

Figure 1.Evaluation of the fractal dimension of a Brownian noise with the Higuchi's method. In this case the slope is approximately 1.5, then $\beta = 5 - 2D = 2$ that is the β value that corresponds to Brownian noise.

3. METHODOLOGY

We used the catalog with data of the Southern California Seismic Network (SCSN), a project led by Caltech. The catalog is complete from a magnitude of 1.5. So that the used catalog has all the data since 1981 until 2011 (between 32^o and 37^o of North latitude and 114^o and 122^o West longitude), event of magnitude less than 1.5 were not considered in the analysis and however there were still thousands of events to make the calculations. It is really important to work with a catalogue that is complete from a small magnitude because the statistical results will be more significant given that there are many events in the catalogue. If we consider the same period in Mexico, our catalogues hardly would be complete from a magnitude of 4.3, so the number of data would be very limited. When applying the Higuchi's method we always found long-range correlations because the obtained D values obtained oscillate around 2.0, by which the spectral exponent β is around 1.0 which corresponds to $1/f$ noise, i.e. long-range correlations. As we wished to analyze the seismicity around the main events that have taken place in that region, the first with magnitude 7.3, occurred in 1992, 9 km to the N of Yucca Valley, CA, the second of magnitude 7.1 occurred in 1999, 51 km to the N of Joshua Tree, CA and the third of magnitude 7.2 occurred in 2010, 54 km to the SSE of Calexico, CA. We made a windowing with the Higuchi's method around all the three events in the way we describe below.

For each of the three above-mentioned earthquakes we analyzed periods of 6 years, 3 years before the earthquake and three years after the same (with the exception of the last event, because as it happened in 2010 we only had less than two years after it). If one of those three earthquakes is j-th, then the windows were taken forward and backward. Each window has 1000 data. For example, the first window to the right contained data from the $j + 1$ to the $j + 1001$ data, the second from the $j + 101$ to the $j + 1101$ data, the third from the $j + 201$ to the $j + 1201$ data, and so forth until that was no longer possible to take a complete window. As it can be seen, the windows overlap with 100 data. The backwards windowing is performed in the same way, for example, the first back window comprised from the $j\text{-}1001$ data until the $j\text{-}1$ data. The slope was calculated for each window as described above and the graphs of the different values of the slope are plotted for each of the windows. Also the value of the y-intercept was calculated, because it has been shown that this y-intercept also has important information [Gálvez - Coyt et al., 2012].

4. RESULTS

As already mentioned, the Higuchi´s method was not applied directly to the time series. We selected the three events of greater magnitude in the catalogue, these were events of magnitude greater than 7.0, the first of magnitude 7.2 which occurred on June 28, 1992; the second of magnitude 7.1 which occurred on October 16, 1999 and the third which had its epicenter on the Mexican side and had a magnitude of 7.2 on April 4, 2010. After that, these events were removed from the catalogue and we take subsets of the catalogue with a duration of six years, to these time subseries we applied the Higuchi' method, but taking windows (hence the name of Higuchi's windowing), the windows overlap in order to have an adequate number of data. The Higuchi´s dimension and the y-intercept were calculated before and after the earthquake for each window and we obtained the graphs shown below. For each earthquake we show three figures, the first showing the location of the earthquake in the catalogue in periods of six years (except for the last which has almost five years), the second shows the variation of the fractal dimension in the windows before and after the earthquake and the third shows the y-intercept for each window before and after the earthquake. In general it is observed that for the three events there is a variation of the fractal dimension *D* before and after the quake, but before the earthquake we noted a decrease in the fractal dimension which is evident in all three events. In the graphics of the y-intercept, it is observed that prior to the earthquake there is an increase in its value, which is seen in the three graphs. Indeed in Figure 3 we can see on the right another peak of important variation of the fractal dimension, and it is also observed in Figure 4 as other peak in the y-intercept, but you can see in Figure 2 that this peak correspond to an event of magnitude 6.7.

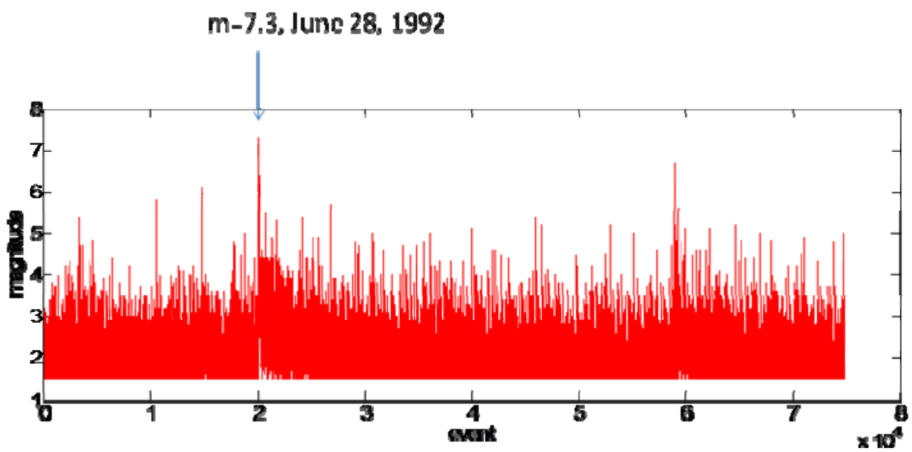

Figure 2. The earthquake of magnitude 7.3 on June 28, 1992, the graphic shows 6 years of events major or equal to 1.5, 3 years before and three years after the earthquake.

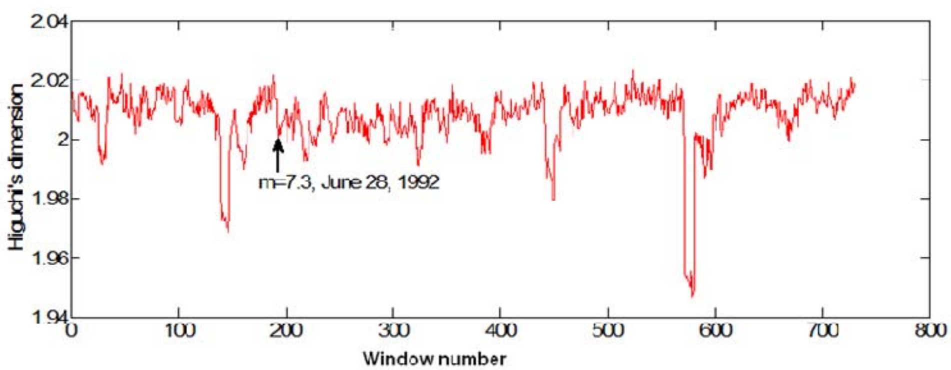

Figure 3. The Higuchi's windowing method implemented over a period of six years around the event on June 28, 1992. With EQ it is shown when the aforementioned earthquake occurred. Note that there is much variation in the Higuchi's fractal dimension and a decrease before the earthquake.

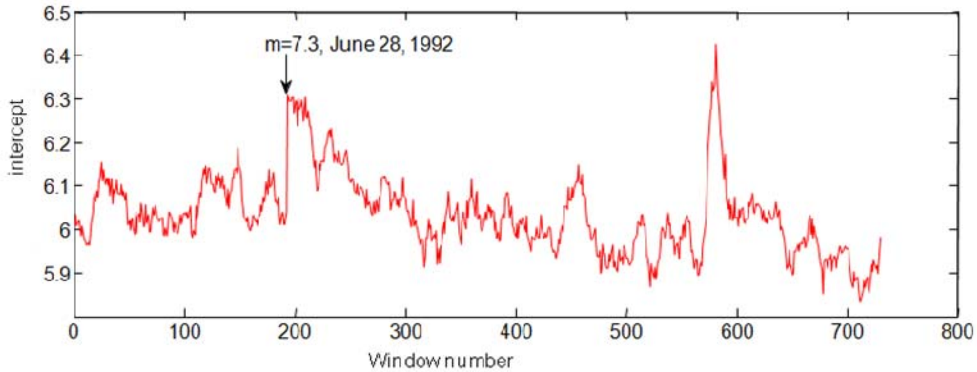

Figure 4. The *y*-intercept for each one of the windows of the windowing Higuchi's method implemented over a period of six years around the event on June 28, 1992. Note that the calculated maximum values of the *y*-intercept happen prior to the earthquake.

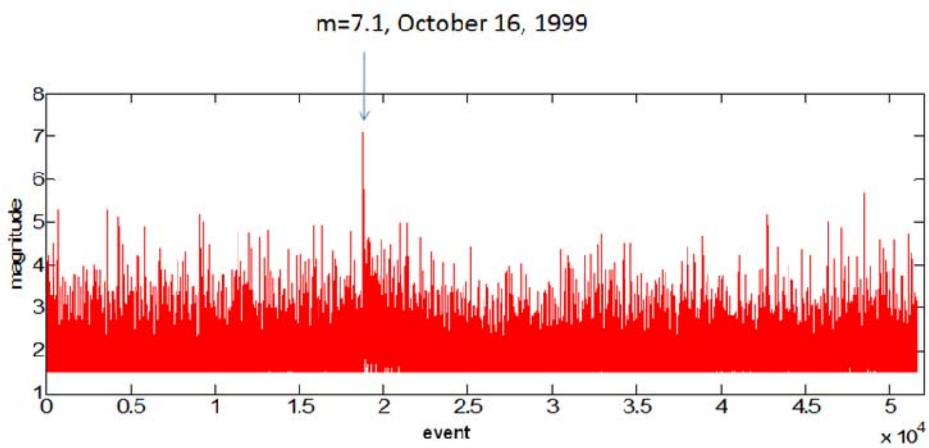

Figure 5. The earthquake of magnitude 7.1, on October 16, 1999, the graphic shows 6 years of events major or equal to 1.5, 3 years before and three years after the earthquake.

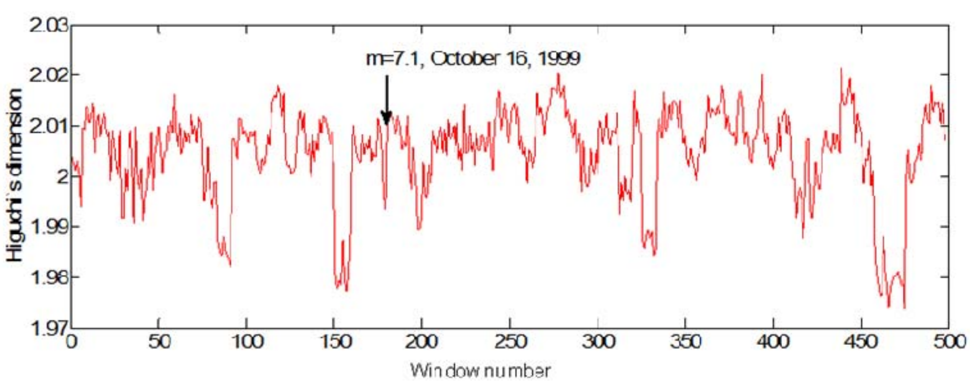

Figure 6. The windowing Higuchi's method implemented over a period of six years around the event on October 16, 1999. Qualitatively it is observed a similar behavior to the 1992 event shown in Fig. 3

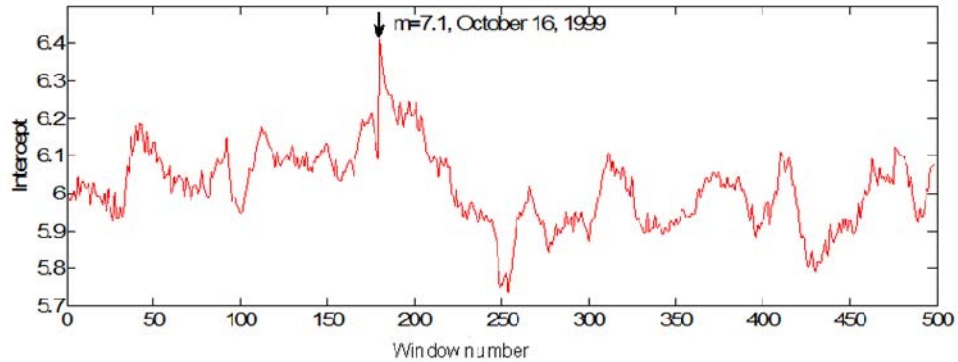

Figure 7. The *y*-intercept for each one of the windows of the Higuchi's windowing method implemented over a period of six years around the event on October 16, 1999. Note the structure of maximum values that occurs before the earthquake is also repeated in this case.

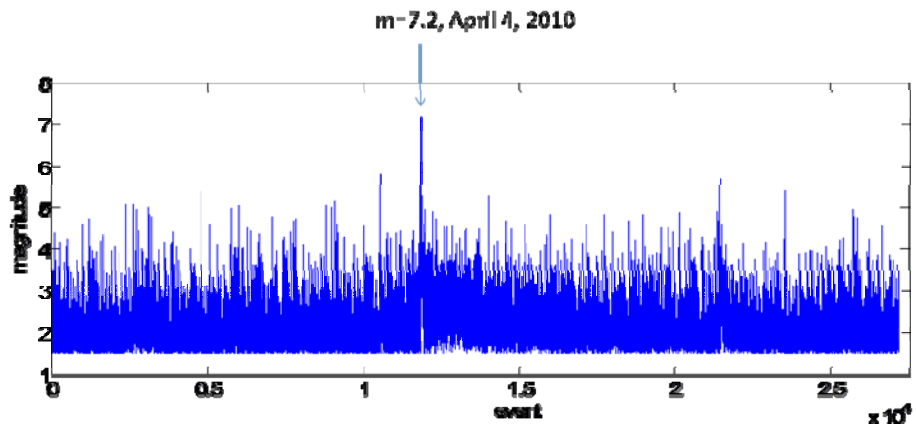

Figure 8. The earthquake of magnitude 7.2, on April 4, 2010, the graphic shows almost 5 years of events major or equal to 1.5, 3 years before and almost two years after the earthquake.

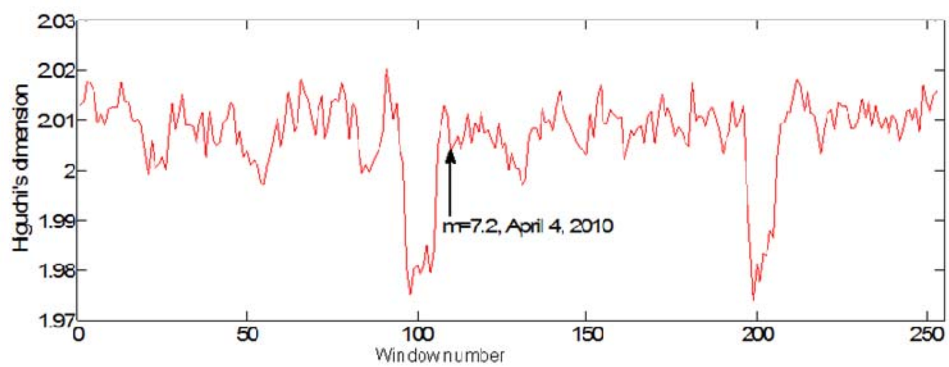

Figure 9. The Higuchi's windowing method implemented over a period of almost five years around the event on April 4, 2010. Note again the decrease of the fractal dimension prior to the earthquake.

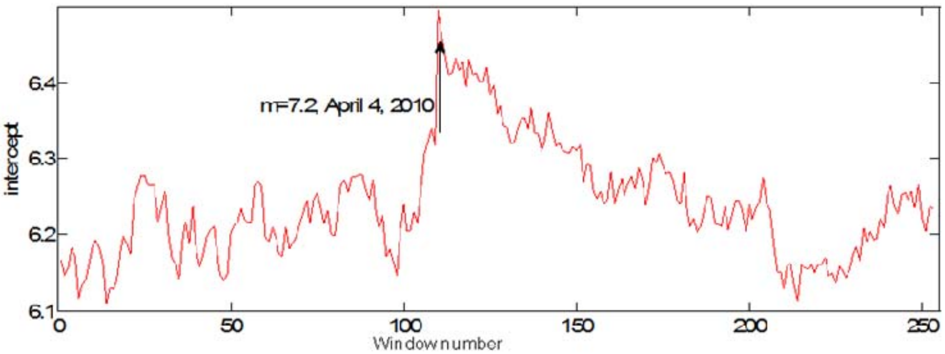

Figure 10. Note how it is clear once again that the earthquake occurs after the uprising in the values of the *y*-intercept.

5. CONCLUSIONS

We have found very interesting results that show anomalous behavior in the fractal dimension that possibly indicate the imminence of an earthquake of great magnitude, these results were found using the windowing Higuchi's method and calculating the fractal dimension and the value y-intercept in each window, what we see is a pattern of decrease of the fractal dimension prior to the 3 events studied earthquakes of magnitude greater than 7.0. Also we observe an increase in the value of the *y*-intercept prior to the three earthquakes. We have to perform more calculations under different conditions, but the two observed patterns for the three earthquakes suggests the possibility of a possible precursor via the results of the Higuchi's windowing method.

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