DYNAMIC PROPERTIES OF FOUNDATION SUBSOILS AS DETERMINED FROM LABORATORY TESTS

by

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SUMMARY

Elastic and energy dissipation properties of clays are required in the dynamic analysis and design of earthquake resistant structures founded on (or, in the case of dams, constructed of) such materials.

This paper describes a series of dynamic tests on soil samples in which the effects of amplitude and number of repetitions of loading are particularly investigated.

It is found that the elastic modulus decreases as the number of cycles of loading increases. It is also found that the measured value of elastic modulus is markedly dependent on strain amplitude, being greatest at small amplitudes.

The implications of these results, as affecting the dynamic response of structures are discussed.

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INTRODUCTION

As stated by Housner(1) (1954) the main geotechnical problems of earthquakes are: "First, how do the properties of soils affect the local intensities of earthquake ground motion; second, which soils are subject to permanent physical changes during the passage of seismic waves and how will these changes affect superimposed structures; third, how do the properties of the soil affect the behaviour of structures interacting with the ground during an earthquake?"

Other problems include the response of earth structures (such as dams, etc,) to earthquake vibrations.

Before an attempt can be made to solve such problems, fundamental information on the stress-strain properties of soils, under dynamic loading conditions, must be obtained. Most theoretical analyses of these problems assume that the subsoils are elastic. Therefore it is desirable to know how closely this assumption is followed by actual soils, and to determine the appropriate elastic parameters.

One aspect of the subject deals with conditions of failure, under dynamic loads. See, for example, Seed(2). The work described herein, however, is restricted to the determination of soil properties at stresses less than those required to cause failure.

In a previous paper (3) the subject of damping in soils was investigated. In the present paper, extensions of the work on damping are described, and results of investigations on 'elastic' properties of undisturbed soils are given.

THE APPARATUS

For the dynamic tests, the apparatus used was a modification of that described previously (3).

Other experimenters in this field have used either resonance type devices e.g. that described by Wilson and Dietrich(4), or stress-controlled machines, such as that used by Murayama and Shibata(5). In the former method, the sample, free at its upper end, is usually placed on a vertically-vibrating table, the frequency of vibrations being adjusted to resonance. Thus, the frequency at which observations are made is dependent on soil properties and sample size, only.

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(1) The numbers in brackets apply to references listed at the end of the text.

In the latter, the applied dynamic stress is usually generated by counter-rotating unbalanced masses. Thus, unless these masses or their eccentricities are changed, the dynamic force applied is proportional to the square of the frequency.

As it was desired to investigate the range of frequencies encountered in earthquakes, (0-10 cycles/sec.) it appeared desirable that frequency and amplitude should be independently variable, within this range. The strain-controlled apparatus chosen fulfils this requirement without undue complexity.

The soil sample, 4 in. dia. and 8 in. long, sealed in a thin rubber membrane, is enclosed in a triaxial cell to which a constant fluid pressure is supplied. A sinusoidal movement is given to the top platen on the sample by a loading ram, passing through the top of the cell, driven by an eccentric. The throw of this eccentric can be adjusted to give amplitudes of deformation of $\pm$ 0.002 to 0.36 in. (corresponding to strains of $\pm$ 2.5×10^{-4} to 4.5×10^{-2} in./in.). The eccentric is driven by an electric motor through a variable speed drive and a magnetic clutch, to give frequencies within the range 0.08 to 10 cycles/sec. The number of cycles of loading is indicated by a counter on the shaft carrying the eccentric.

As shown in Fig.10, two transducers (utilizing electric resistance strain gauges) are employed. One, mounted on a flat cantilevered spring, measures deformation, while the other, mounted on the loading ram, measures the applied force. As the load transducer is mounted inside the cell, friction in the ram guide is excluded from the measured load. Voltages proportional to load and deformation are applied (respectively) to the vertical and horizontal deflection circuits of a Tectronix (Model 302A) oscilloscope. Thus, hysteresis figures are displayed on the oscilloscope screen. From enlargements of photographs of these figures, measurements of area, maximum stress and maximum strain are made from which elastic and damping parameters can be calculated. Alternatively, load or deformation can be shown, as a function of time, in the conventional way.

In one series of tests, described below, deformation, instead of being sinusoidal, was at constant velocity (but alternating in direction), This was achieved by replacing the eccentric by a cam and follower designed to give the required motion, at a fixed amplitude ($\pm$ 0.125 in.). The deformation-time relationships are shown in Fig.1, for sinusoidal displacement and in Fig.2, displacement at constant velocity.

EXPERIMENTAL INVESTIGATIONS

(a) Soils Tested.

Three soils, described in the table below were tested. Of these, two were undisturbed saturated soils, and the other (No.2) compacted filling from a road embankment.

No.	Location	Description	Liquid Limit	Plastic Limit	Moisture Content %
1	Parakai	Soft, sensitive silt	77	35	90
2	Papakura	Compacted sandy clay	62	25	51
3	Ardmore	Silty clay	82	34	46

(b) Tests on 'Virgin' Samples.

Tests at constant frequency and amplitude were made on samples 1 and 2, which had not previously been cyclically loaded. A photographic record of force (against time) was made, during each test, from which the 'absolute' modulus was computed for each cycle. The symbol E is used throughout this paper for the 'absolute' modulus, i.e. the ratio of maximum dynamic stress to maximum strain. The variation of E with number of cycles of loading (N) is shown in Fig.3.

In the remainder of the tests described below, the sample, (No.3), initially undisturbed, had been cyclically loaded for at least 1,000 cycles, so that steady state conditions had been reached.

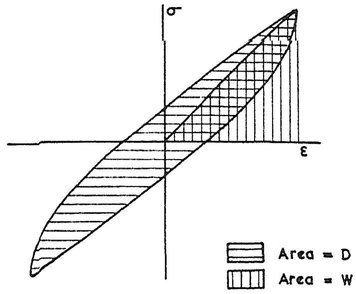
(c) Variation of Strain Amplitude.

A series of tests was made at varying strains, ranging from $\pm 3 \times 10^{-4}$ to $\pm 3 \times 10^{-2}$ in./in. Photographs of the hysteresis loops enabled the absolute modulus, E, to be calculated, and, by measurement of the area of each loop, the specific damping energy, D (in.-lb./cub.in./cycle) i.e. the energy dissipated per cycle, was found.

The damping factor, η , is defined by the relationship

$$\eta = \frac{D}{2 \pi W}$$

where W is the strain energy at maximum strain during the cycle (as shown in the accompanying diagram). The observed variation of E with strain amplitude is shown in Fig.9, drawn with natural scales. The same information is presented in Fig.6 with logarithmic scales, while Figs.7 and 8 show the variations in D and η with amplitude. In Figure 5 the points of maximum stress and maximum strain as obtained from the series of dynamic tests are plotted. For comparison, the stress-strain diagrams for unconfined compression tests on the sample after the dynamic tests and also after complete remoulding, are shown. It can be seen that the dynamic stress-strain curve lies between the curves for the two static tests.



(d) Ultra-sonic Test.

In order to obtain an upper limit to the value of E, wave propagation velocity was measured in an ultra-sonic test. (The method is normally used in the non-destructive testing of concrete). This gives amplitudes of strain much smaller than in most other methods (but of course, at a much higher frequency). The value for E obtained on sample 3 was 450,000 lb/sq.in. compared with the maximum of 1,800 lb/sq.in. obtained mechanically.

(e) 'Constant Velocity' Tests.

As described above, the apparatus was modified, in one series of tests to give constant velocity. A comparison of the observed hysteresis loops for the two types of motion at the same frequency (o.33 cycles/sec.) and strain amplitude (0.016 in./in.).

(f) Variation of Frequency.

Increasing the frequency of vibration, over the range 0.08 to 10 cycles/sec. was found to decrease the observed value of E by about 17%. As the frequencies employed were remote from the computed resonant frequency, and inertia effects can be shown to be negligible, the observed decrease in E must be attributed to properties of the soil.

(g) Effect of Cell Pressure.

Changes in cell pressure, from 5-90 lb./sq.in. appeared to have little effect on E, or D.

INITIAL FALL-OFF OF MODULUS OF ELASTICITY

Several observers have noted a reduction in strength with increasing number of cycles of dynamic loading. Murayama and Shibata(5) for example, found that the strength(of undisturbed clays) bore a linear relationship to log N.

In the present tests, strength was not measured. The modulus, E, however, was found to decrease in a similar manner. After a comparatively small number of cycles, the rate of fall-off decreased, and for sample 1 the modulus became nearly constant, having fallen to about 54% of its initial value, after 40 cycles.

The effect on the compacted sample, (No.2) although still appreciable, was much less.

It is suggested that some re-orientation of the soil particles, equivalent to partial remoulding, occurs under dynamic stress conditions. Just as there is a lower limit below which the strength of a soil, at a certain moisture content, cannot fall, (i.e. the fully-remoulded strength) so there would be expected to be a lower limit to the modulus, E.

Other possible causes of the reduction in modulus include rise of temperature, and rise of pore pressure. Calculations show that the energy dissipation, if entirely converted to heat, would result in negligible temperature rise, for the number of cycles of loading used in the tests.

With regard to pore pressure, Lo(7) observed that, for 'static' tests over three cycles, there is a unique relationship between pore pressure change, and resultant deformation. If this observation were applicable to dynamic stress conditions, then, after any number of complete cycles, as the resultant deformation is zero, the pore pressure should be unaltered. Although pore pressures were not fully investigated, there is evidence to suggest that this does not apply under dynamic loading conditions, and that pore pressure, after a number of cycles of loading, does increase. This is compatible with the concept of particle reorientation.

The total reduction in modulus is no doubt related to the <u>sensitivity</u> of the soil i.e. the ratio of compressive strengths in the undisturbed and remoulded states. The rate at which the fall-off occurs is probably also related to the <u>susceptibility</u> of the soil, a property noted by Newland and Allely(8). Thus a more 'susceptible' soil requires less remoulding energy to reduce its strength than does another, with the same sensitivity, but lower susceptibility.

ENERGY DISSIPATION

Typical hysteresis loops are shown in the photographs Figs.11 and 12. The shape is very different from the ellipse which would be produced by purely viscous damping. There is some resemblance (particularly in Fig.11) to the parallelogram which would result from parallel friction (Coulomb) damping. The upward curvature of the upper part of the loop, however, suggests a non-linear elastic element (e.g. a 'hardening' spring). Such properties are not generally found in static tests on soils, where the curvature is in the reverse direction.

It was considered that the upward curvature might be due to the sinusoidal nature of the displacement, coupled with a time-dependent increase in modulus occurring during cyclic loading. This thixotropic regain(9) might explain the increasing gradient as displacement increased towards its maximum value, where the rate of strain decreased to zero. The observed fact that this curvature was greater at lower frequencies tended to support this opinion.

Accordingly, the 'constant velocity' tests were made, so that, with constant rate of strain throughout each half-cycle, any velocity-dependent effect would remain constant. As shown in the comparative hysteresis diagrams (Fig.4) the curvature was not greatly altered. Apart from 'peaks' at the ends of the diagram for the constant velocity test, caused by high accelerations at these points, the general shapes are much the same. Thus it appears that thixotropic effects are not greatly significant, at the frequency employed, (0.33 cycles/sec).

In the study of damping properties of metals, plastics etc., the following empirical relationship is found to be approximately true over certain ranges of the variables:

$$D = J\,\sigma^{n} \qquad \text{.................... (1)}$$

where D is the specific damping energy

σ is the maximum stress and
J and n are constants.
If ε is the maximum strain and
E is the elastic modulus, then the relationship may be written

$$D = JE^{n}\,\varepsilon^{n}$$

$$= K\,\varepsilon^{n}$$

For a material for which E is constant, the product JE^{n} is also constant. In the tests described, however, E was found to vary considerably. Under these conditions, it was found that the conventional relationship (equation 1, above) did not apply, i.e. the parameters J and n did not have constant values, whereas the relationship

$$D = K \varepsilon^{n'} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

was found to apply over certain ranges. (The exponent is now written n' to distinguish it from the more conventional parameter in terms of stress). Thus the relationship (equation 2) in terms of strain is found, empirically, to be more widely applicable than that in terms of stress.

In Fig.7 the gradient of the line is equal to the value of the exponent n'. For strains up to 1.5×10^{-3} in./in. n' remains constant at 1.82. The graph is no longer straight, in the range of strain 1.5×10^{-3} to 7×10^{-3} in./in., but at higher strains becomes more nearly linear with a gradient of 2.15.

(For purely viscous damping, n = 2 and η is independent of amplitude while for ferrous metals values of n between 2 and 3 are observed for moderate stress levels and η increases somewhat with amplitude).

Variations in damping factor (Fig.8) also suggest that different damping mechanisms operate for strains below 1.5×10^{-3} in./in. and above 7×10^{-3} in./in., while the intermediate range forms a transition zone. For low amplitudes some characteristics of viscous damping are found (including a more nearly elliptical hysteresis diagram) while at high amplitudes damping may be described as 'plasto-elastic'.

It should be noted that the damping factor, η, is twice the ratio $\frac{C}{C_c}$ used in theoretical analyses involving viscous damping, where C is the 'damping constant' and C_c its value for critical damping. Thus the observed values are between 16% and 36% of equivalent critical viscous damping.

VARIATION OF ELASTIC MODULUS WITH AMPLITUDE

It has been realised for some time that the elastic modulus, for a soil, is dependent on the method of measurement. Different methods (seismic velocity, resonance, static tests, etc.) differ (a) in the rate of strain and (b) in the strain amplitude applied. Here, the latter variable is investigated.

The tests show (Fig.9) a low, but reasonably constant value of E for strains over 0.01 in./in. (The tests on 'virgin' samples, described above, were both in this range). For lower amplitudes, however, E increases markedly as the amplitude decreases. It was found that these observations were repeatable, and results were the same, whether amplitude was being increased or decreased. This dependence of E on strain amplitude may account, to a great extent, for the differences in values obtained by various methods.

Jones(10) determined the resonant frequency of an oscillator placed on a soil, and reported reduced values (corresponding to lower values of E) when increased power was supplied to the oscillator, i.e. when strain amplitude was increased. Research by others, also, has shown a decrease in resonant frequency of machine foundations with increase in excitation force.

CONCLUSIONS

While it is realised that the observations made apply strictly only to the type of soil and conditions of test employed in the experiments, it is felt that the following conclusions are valid in general, for cohesive soils:-

1. The modulus of elasticity of an undisturbed soil falls rapidly during the first 100 cycles of loading, for cyclic loading over a certain amplitude.

2. Energy dissipation in soils approximates, in some respects, to 'viscous' damping, at low amplitudes (though less dependent on frequency) while at higher amplitudes 'elasto-plastic' characteristics predominate. The specific damping energy, D, increases with amplitude.

3. The elastic modulus is strongly dependent on strain amplitude, decreasing with increase in amplitude. For this reason, different values are obtained by different experimental techniques. In order of decreasing modulus obtained, dynamic tests may be listed:

(i) Ultra-sonic method
(ii) Seismic (wave-velocity) methods
(iii) Laboratory vibration methods, at low strains
(iv) " " " , at high strains

IMPLICATIONS

Theories are available for the dynamic analysis of structures, which include soil-structure interaction, one of the most comprehensive being that presented by Thomson(11).

Such theories assume that the elastic modulus has a single constant value, and material damping in the soil is usually neglected. It appears that in fact the elastic modulus is dependent on number of cycles of loading, and on amplitude. Thus, appropriate values of modulus will be dependent on intensity of vibration, position with respect to the foundation, and number of cycles of loading. It would be impracticable to incorporate all these variables. A more satisfactory approach may be to assess an 'effective average modulus' for use in such theories, to obtain approximate results for certain conditions.

In order to assess the amplitude range which may be appropriate beneath a foundation in a major earthquake, let us consider a typical building on a raft foundation sited on the Ardmore clay tested. Let the static bearing pressure be 10 lb/sq.in. and suppose earthquake loading to alter this value by ± 50%. This provides a factor of safety of about 4½ for static conditions and 3 for earthquake loading (based on static strength) which are typical values used in practice. From Figs. 5 and 9 it appears that, after a large number of cycles, dynamic stresses of ± 5 lb/sq.in. will be in the range where the dynamic modulus is low. These high values of stress(and hence strain) will apply only to the region immediately adjacent to the dynamically stressed foundations. At greater distances the modulus will increase rapidly. Also, in the region where dynamic stress is high, damping is high. Thus there may be a semi-plastic zone of limited extent formed beneath the structure during an earthquake of high intensity, in which energy dissipation may occur.

In comparison, when structures are mechanically vibrated to determine their resonant frequencies, the dynamic stresses beneath the foundations are small and the elastic modulus will be high.

It appears possible, therefore, that ground-structure interaction may be a more important factor (reducing the resonant frequencies of the various modes and increasing energy dissipation) during a high intensity earthquake than in smaller tremors, or with artificially induced vibrations.

ACKNOWLEDGEMENTS

This work forms part of a general research project on earthquake resistant design being pursued at the University of Auckland. The advice and encouragement of Professor N.A. Mowbray, and other members of the Civil Engineering Department is acknowledged. The work has been greatly assisted by a University Research Grant.

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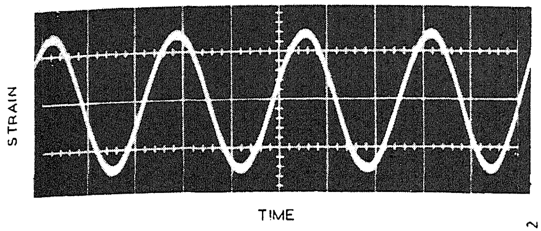
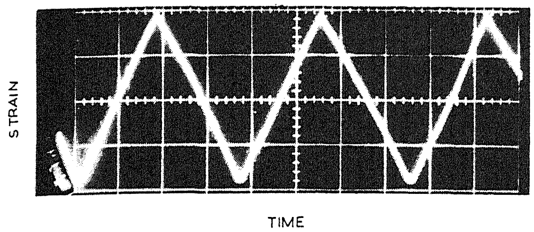
FIGURE 1



STRAIN – TIME

SINUSOIDAL STRAIN

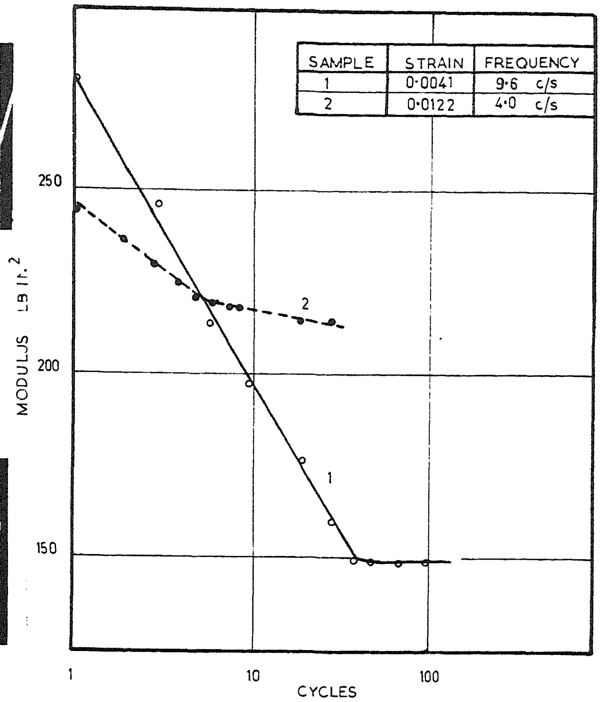
FIGURE 2



STRAIN – TIME

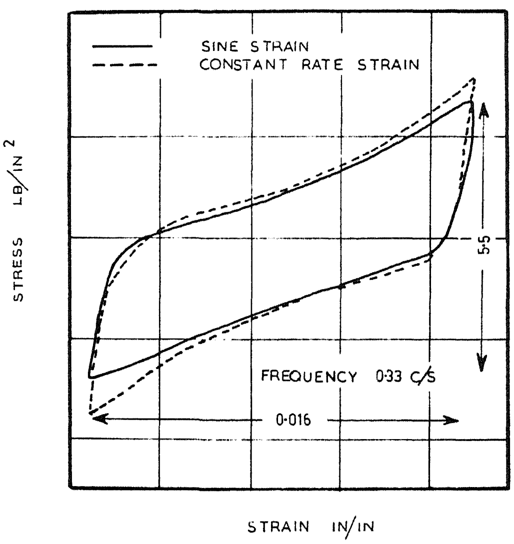
CONSTANT RATE STRAIN

FIGURE 3



MODULUS E – NUMBER OF CYCLES - N

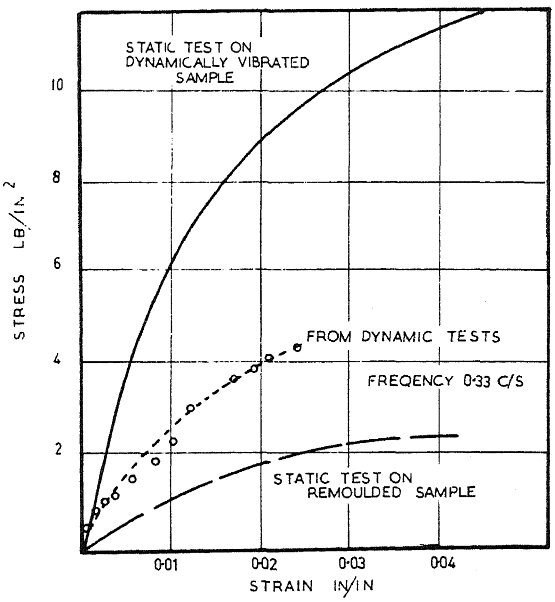
FIGURE 4



HYSTERESIS LOOPS

SINE AND CONSTANT RATE STRAIN

FIGURE 5



STRESS – STRAIN

FIGURE 6

ELASTIC MODULUS
E

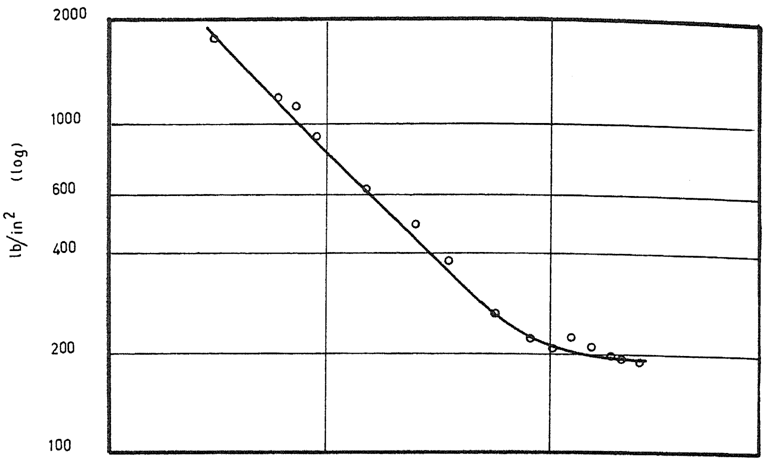


FIGURE 7

DAMPING ENERGY

D

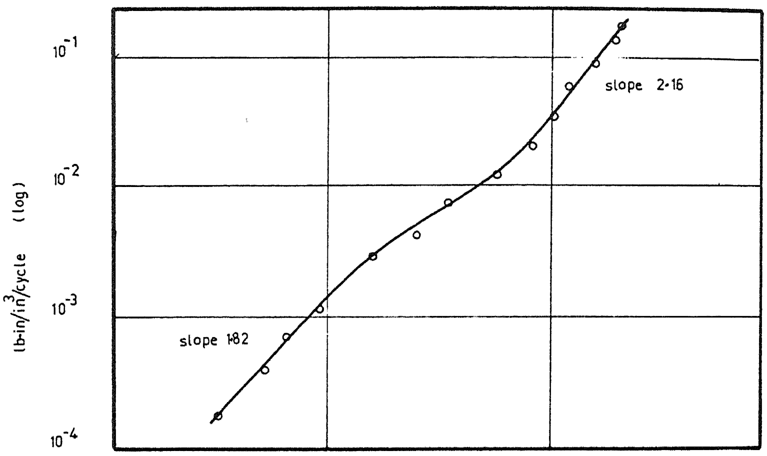


FIGURE 8

DAMPING FACTOR

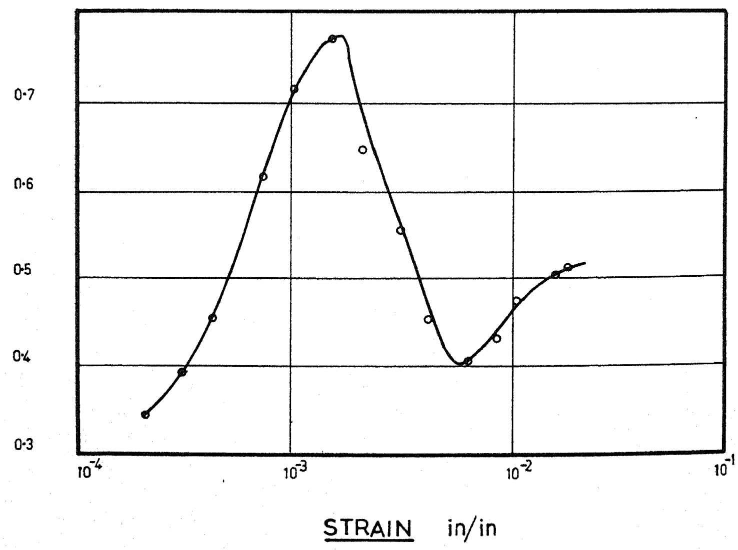
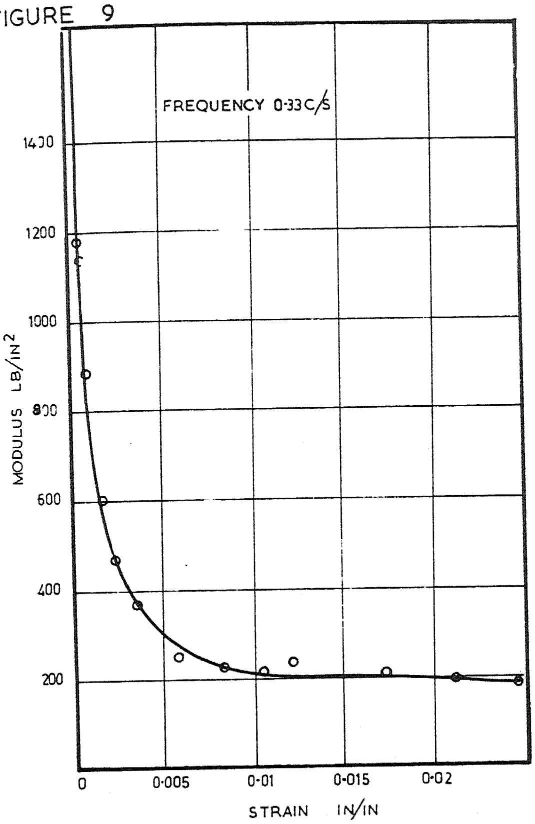
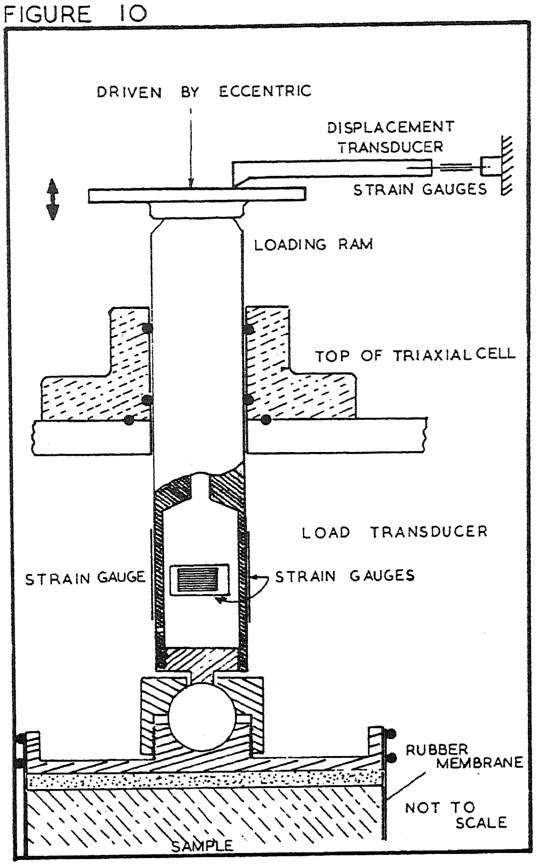


FIGURE 9



MODULUS E - STRAIN ε

FIGURE 10



STRAIN and STRESS TRANSDUCERS

FIGURE 11

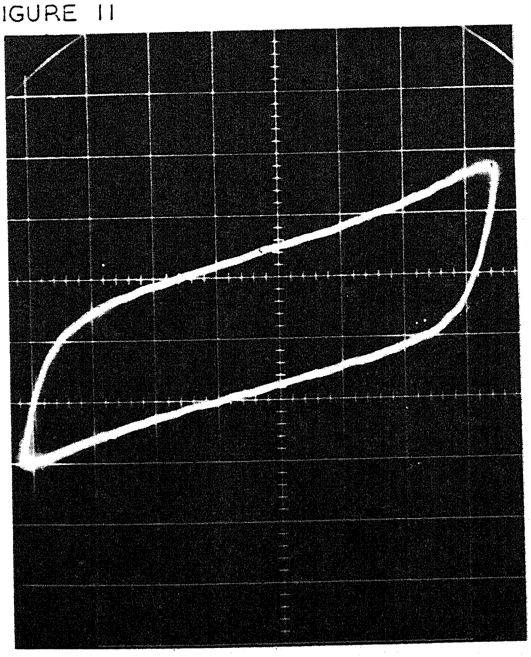
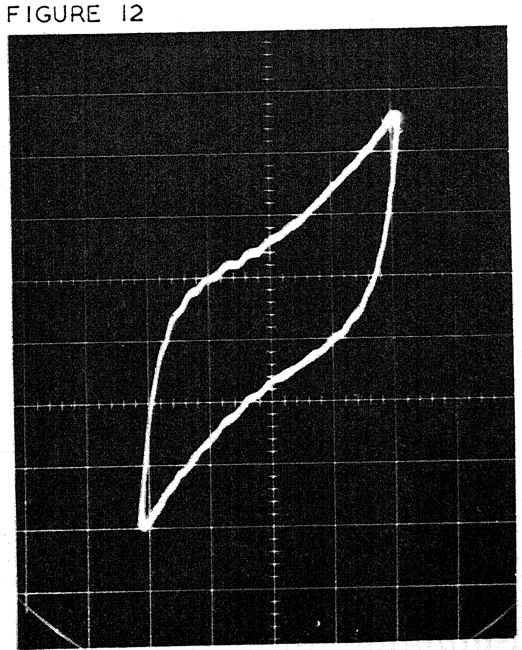


FIGURE 12



TYPICAL HYSTERESIS LOOPS

DYNAMIC PROPERTIES OF SAND SUBJECTED TO DYNAMIC LOAD BY SHALLOW FOOTINGS

BY P.W. TAYLOR AND J.M.O. HUGHES

QUESTION BY: E. ROSENBLUETH - MEXICO

As pointed out by the authors, the values of the damping factor shown in Fig. 8 must be halved to obtain a more conventional form of equivalent viscous damping ratio. When this is done, ratios of 16 to 36 per cent are obtained. Yet D.E. Hudson has shown that the maximum possible equivalent damping ratio in a hysteretic one-degree-of-freedom system is 15.9 per cent if the equivalence is based on a criterion that leads to quite accurate computation of the amplitude of steady-state harmonic oscillations. He has also shown that for most simple systems this ratio is smaller than the upper limit quoted and that, if the criterion be based on maximum deformation response to earthquakes, the equivalent damping ratio is usually smaller than 4 per cent. The apparent discrepancy with the values reported in the present paper occurs from a difference in definition. If one adopts Hudson's criterion it will be found that the ratios of 16 to 36 per cent of critical damping become much smaller, giving values consistent with the upper limit and with the typical ratios contained in Hudson's paper.

REPLY BY: P.W. TAYLOR

The authors thank Dr. Rosenblueth for explaining the apparent contradiction between the paper under discussion and Hudson's, in the damping factors quoted. The statement, in the paper, that the damping factor used must be halved to obtain the more conventional form $\left(\frac{C}{Cc}\right)$ should be qualified. This relationship is strictly true only for a system in resonance.

Because the terminology used to describe material damping (particularly in non-linear systems and those in which damping is not truly viscous) tends to be confusing, the authors think it preferable to describe material damping properties in terms of damping energy, per unit volume (D).

REPLY BY: J.M.O. HUGHES

Further to the work contained in this paper the annexed results are presented to give a more quantitive estimate of the effects of the results.

IMPLICATIONS:-

Because of the rapid decrease in absolute modulus with increase in the stress or strain, it appears that under reasonably large dynamic loads, as in a major earthquake, the modulus of the ground reaches a low value.

To obtain a more quantitative estimate of the rotation that would occur under a footing subject to an applied moment, a simple stress distribution was considered (see accompanying figures). The angle of spread ϕ was determined such that the overall effect of this stress distribution would be approximately the same as that obtained theoretically for a moment applied on the surface of a linear elastic half-space. This spread is not true in the case of a non-linear soil, however, it is a first approximation.

From the results of the Paper it was shown that for strain below 1% the absolute modulus and damping energy are functions of strain to a power, two-thirds and 1.82 respectively. For the purpose of computer analysis the soil is then devided into a series of strips with the above stress distribution uniformly through each. The strain that would occur at the point of maximum stress and the total damping energy dissipated within each later can be calculated. Summing up the effect of these strips the total energy dissipated per cycle and the rotation can be found.

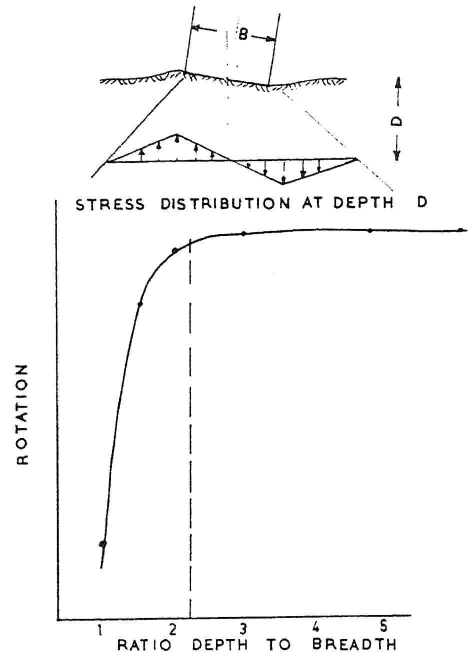
From this three points emerge:

1. Practically all the rotation occurs within the region of limited depth, near the base. For the Ardmore soil considered in the paper, it can be seen from the accompanying graph that the soil below $2\frac{1}{4}$ times the footing breadth has little effect.

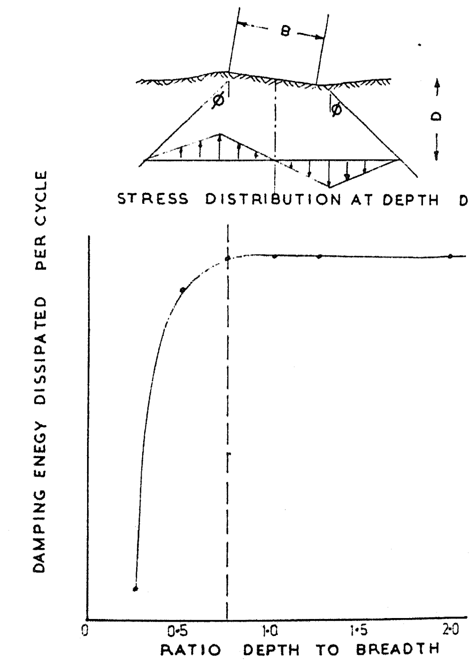
2. The damping is even more confined to the surface. For the above soil the graph indicates that the soil below 0.75 times the footing breadth has little effect.

3. The mass taking part in the motion - i.e. the concentrated mass moving with the footing - is small.

It appears possible, therefore, that ground structure interaction may be a more important factor in reducing the resonance frequency and increasing the energy dissipated during a high intensity earthquake than in smaller tremors, or with artificially induced vibrations.



ROTATION OF THE FOOTING UNDER CONSTANT MOMENT WITH DEPTH OF SOIL CONSIDERED



DAMPING ENERGY DISSIPATED UNDER CONSTANT MOMENT WITH DEPTH OF SOIL CONSIDERED