EARTHQUAKE SPECTRUM PREDICTION FOR THE VALLEY OF MEXICO

By I. Herrera, E. Rosenblueth, and O.A. Rascón[0]

Abstract

Paper reports field and laboratory tests to determine the dynamic properties of Mexico City clay. The data are used in conjunction with a linear, one-dimensional theory of multiple wave reflection in stratified media and the results are treated in accordance with an approximate theory, which permits computing the probability distributions of spectral responses for various degrees of damping. Expected spectra are compared with those obtained from earthquake records. Missing information for deep strata is found by trial and error and velocities measured for the upper layers are adjusted on reasonable bases. The comparison is deemed good.

Notation

A = spectral acceleration

a = maximum ground acceleration

B = parameter in expected pseudovelocity

L = specimen length

c = maximum ground velocity

x = maximum ground displacement

s = earthquake duration

T = natural period

t = time

V = spectral pseudovelocity, defined in such a manner that ωV is the maximum absolute acceleration of the system

v = shear-wave velocity

α = expected ordinate in pseudovelocity spectrum of white noise

ζ = percentage of critical damping in the structure

η = percentage of critical damping in the soil

ω = circular frequency

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Introduction

The one-dimensional problem of multiple wave reflection in linear stratified soil has received much attention in earthquake engineering(1-4). The treatment has been mostly confined to deterministic spectra for zero structural damping. Developments in the probabilistic approach to earthquake-resistant design(5,6) have recently made it possible to predict the probability of spectral responses on soft ground even for finite damping(7). Assumptions concerning one-dimensional vibrations of the ground and of its linear behavior are retained in the probabilistic theory.

The situation over large portions of the Valley of Mexico is such that linear behavior of the soil may be expected to yield reasonable approximations up to earthquakes of substantial intensities, and the phenomena of multiple wave reflection have such importance that they cannot be neglected in any approach to earthquake-resistant design in the valley. Indeed, the leading conclusions derived from the effects of the earthquake of 28 July 1957 were the need for a seismic zoning of Mexico City and the marked prevalence of certain ground periods(8), and subsequent quakes have borne out these statements.

Accordingly it has seemed justified to apply the theory developed in Ref. 7 to the Valley of Mexico. In this manner, undamped and damped expected spectra have been computed for the site where a strong-motion accelerograph has recorded several earthquakes.

The dynamic properties to incorporate in the theory have been derived from field and laboratory measurements(9-11). The expected spectra of the earthquakes arriving at the rock-soil interface from below are also required. These have been borrowed from an empirical study(12 and 13). The theoretical expected spectra obtained from the ground surface are compared with those derived from records of two moderate and two mild earthquakes.

Soil Conditions

The site selected for determination of soil properties lies in the neighborhood of the accelerograph through which records were obtained for calculation of the spectra used here for comparison. It lies on Mexico City's Alameda Central Park. Fig. 1 shows a soil profile through the city and the location of this site.

Static properties and long-time behavior of the soil down to a depth of about 70 m have been extensively studied(14). Little information is available on the deeper formations. The deepest well in the city has penetrated approximately 400 m without meeting rock(15). From an interpolation between points distant about 70 km from each other it is estimated that bedrock lies at a depth of 500 m at the Alameda and it is believed to be an andesite(15). Between 70 and presumably 500 m of depth the material is coarse and noncohesive, save for a few clayey and silty lenses near the top.

Field Measurements

Attempts were made at determining shear-wave velocities by means of conventional seismic exploration techniques, with surface charges. The velocities of P waves were successfully determined in this manner down to a depth of 60 m, but S waves could not be clearly detected in the records due to interference with longitudinal waves. The difficulty was overcome through the use of depth charges. A detailed description of these tests is given elsewhere(11). Fig. 2 shows a typical record obtained in this manner. In this particular record the ratio of P and S wave velocities is 15; this should be compared with 1.73 for a Poisson's ratio of 0.25, which is not grossly in error for most rocks(16). The very high ratio of P- to S-wave velocities which implies a Poisson's ratio of 0.498, is no doubt associated with the very high water content of the saturated clay.

Field measurements were supplemented with laboratory tests on undisturbed samples from a continuous boring drilled at the site. The samples were subjected to standard classification tests and to free torsional vibrations of unconfined cylindrical specimens (Fig. 3). The tests are described elsewhere(9,10). The main results concern the variation of internal damping η and natural period in specimens having various lengths. These results appear in Fig. 4. Tests of statistical significance lead to the conclusion that the percentage of damping not depend significantly on the frequency of vibration within the range of these tests. If this behavior were to hold for all frequencies, its interpretation in terms of the linear theory of viscoelasticity would imply a relaxation function proportional to $1/t$ for all t. This is inadmissible because such a relaxation function leads to the condition in which a deformation impressed at $t = 0$ produces stress for negative t, that is, before the deformation is applied.

One type of hysteretic damping would lead to the same independence of the wave velocity and percentage of damping from the frequency and amplitude of vibration(17). It is probable, therefore, that this type of hysteretic behavior approaches closely the actual behavior of the material tested. In fact, the small degree of nonlinearity detected in the tests favors the assumption of a type of hysteretic damping only slightly different from the one treated in Ref. 17(10).

Shear-wave velocities, v, derived both from the field determinations and from laboratory tests, are shown in Fig. 5. Also shown is a stepped variation of v with depth, for which the average values have been made to coincide with the averages derived from the field determination, but the elevations at which the wave velocity is assumed to change have been derived from the laboratory soil classification and dynamic tests.

The large spread in the velocities thqt were determined from the laboratory determinations and the fact that they are systematically smaller than the field-measured values should be attributed to the presence of fissures in the samples. Indeed, many specimens had visible haircracks and yet their stiffnesses were not significantly different from those of specimens apparently exempt from cracks. The same type of spread and of discrepancy is found when comparing shear strengths determined in the laboratory from unconfined compressive tests with field determinations from wave tests(14).

No direct laboratory information is available on wave velocities or internal damping for material below a depth of 54 m. The velocities can be estimated from correlations between natural water contents and dynamic shear moduli obtained from forced vibration tests in torsion on specimens from other borings in Mexico City(18).

Comparison with Spectra of Actual Earthquakes

After several trials the velocity and density profile in Fig. 6 was chosen. Values down to a depth of 54 m are derived directly from field measurements of shear velocities and laboratory determination of unit weight. Properties below 500 m are based on values reported in the literature(19) for similar rocks. Between 54 and 70 m, values are extrapolated from measurements at higher elevations using available data on water contents down to 70 m. Between 70 and 500 m no more can be claimed than that the values seem reasonable in the light of experimental information on other noncohesive materials subjected to comparable confining intergranular pressures(20).

Once the soil and rock properties have been decided upon it is possible to calculate the distribution of responses of a simple linear system that rests on the ground surface, to earthquakes defined by their magnitude and focal distance. Actually only the expectations of these responses need be computed for any specific instance since, if the ground motion is idealized as a Gaussian process, the distribution of responses normalized in terms of their expectations can be taken equal to that of responses to white noise without excessive error(6).

In what follows the expected spectral ordinates predicted from the experimentally determined soil properties are compared with the spectra that correspond to four earthquakes recorded at the Alameda Park. Both horizontal components of each ground motion are used for the comparison. The method used for obtaining these spectra has been published by J.I. Bustamante(21). Other calculations of the spectra for the two strongest earthquakes have also been published(22,23). The characteristics of these quakes are given in Table 1.

Magnitudes are those reported by Berkeley(24), Pasadena(24), and J. Figueroa(25). Intensities are in the modified Mercalli Scale. Focal distances are based on focal coordinates reported by J. Figueroa(25). a, c, and x are, respectively, the expected maximum acceleration, velocity, and displacement at the surface of bedrock, computed by the empirical relations in Refs. 12 and 13.

The expected pseudovelocity spectra $V(\omega,0) = \omega^{-1} A(\omega,0) = 2V_0(\omega,0)$ at the surface of bedrock, if there were no soft ground, was calculated with the criterion given in Refs. 12 and 13 (Fig. 7).

The expected pseudovelocity spectra at the ground surface were found from the following expression(7)

$$E\left[V_1(\omega_0, s)\right] = \left[\frac{\int_{-\infty}^{\infty} |F_v|^2 |F_1|^2 (E\left[V_0(\omega, 0)\right])^2 \, d\omega}{\int_{-\infty}^{\infty} |F_v|^2 \alpha^2 \, d\omega} \right]^{1/2} E\left[\bar{V}_\alpha(\omega_0, s)\right]$$

where V_1 is the pseudovelocity spectrum at the ground surface, F_v is the Fourier transform of the system's basic solution (its response to a Dirac delta acceleration pulse), F_1 is the Fourier transform of the soil transfer function, and $\bar{V}_\alpha$ is the pseudovelocity spectrum of a white-noise disturbance with (constant) expected undamped pseudovelocity spectrum α and duration s.

Owing to previous experience in the Valley of Mexico it is justified to take a value for the expected spectral ordinates considerably greater than can be deduced from the empirical relations of Refs. 12 and 13, which were established from average conditions. This adjustment should consist in multiplying all expected ordinates by a constant factor, as the modification follows exclusively from the greater facility with which waves are transmitted to one locality than to others. It is also conceivable that the empirical relations used do not apply at very large focal distances. A magnification factor of 2 was found to furnish satisfactory results for the earthquakes originating near Acapulco; no magnification factor was assumed for the motion of 1961, which originated in a very different location and at shorter focal distance.

The computed expected velocity spectra at the ground surface, assuming for the internal damping of soil a value of 5.36 percent(10) are compared in Figs. 8 and 9 with the actual spectra of these earthquakes. Agreement as to the general shape of the spectra is good with both horizontal components of both earthquakes of May 1962, save for the horizontal scale of these curves. It is poor indeed with the spectra of the two milder earthquakes. The discrepancy may be due to various causes. It is likely that the initial portion of the two weaker earthquakes had been lost before the beginning of the records. It is also conceivable that the semiempirical formulas systematically do not apply to low-magnitude earthquakes originating several hundred kilometers from Mexico City. Even if this were the case, the difference with predicted spectra would not be especially significant since, from the viewpoint of earthquake engineering, the smaller motions are relatively uninteresting. Attention will be confined therefore to the two earthquakes of May 1962.

The main difference between the actual and predicted spectra lies in that the most pronounced peak in the theoretical curves is shifted to the right. In order to have the prevailing periods coincide, the velocities determined through geophysical exploration and laboratory tests must be increased 50 percent; no change in the assumed properties of the lower formations will achieve the desired result. The corrected $|F_1|$ function is shown in Fig. 10. The corrected expected spectra for internally damped soil are compared again with those of the May earthquakes in Figs. 11 and 12. Pseudovelocity spectra for damped linear systems are included in the figures.

This shift may be due to several reasons. Probably the most important are the following two.

The number of homogeneous layers used in the analysis. This concept has an important influence on the predicted prevailing periods. Consider for example an analysis based on a single 500-m layer lying on rock. The layer's fundamental period of vibration will give the prevailing ground period for this case. This period would normally be computed, then by preserving the sum of travel times for the various layers and would equal

$$4\left(\frac{11}{51} + \frac{23.5}{90} + \frac{11.5}{105} + \frac{24}{134} + \frac{430}{1050}\right) = 4.7 \text{ sec}$$

which is nearly twice the actual prevailing ground period. Conceivably, therefore, consideration of a large number of homogeneous strata would have given a more realistic estimate of the periods.

Horizontal heterogeneity of individual strata. The soil underlying the Alameda Park is practically virgin. The park is surrounded by heavily consolidated formations, and toward the east important consolidation began six centuries ago. A few kilometers west of the park lies the old lake shore, and the soft clay disappears entirely. Consequently, effective stiffness of the soil is no doubt higher than that derived from local properties, so the 50 percent increase is not unwarranted.

Once the above correction is introduced the correlation between the predicted expectations and the actual spectral ordinates is good. The remaining minor discrepancies are not surprising in view of the simplifications introduced in the theoretical model.

Conclusions

From the results of this study it seems likely that the simplified one-dimensional linear treatment in Ref. 7 will give satisfactory results on soft ground, provided the following conditions exist.

1. The density of noncohesive strata and the cohesion in the upper formations are such that soil behavior will be reasonably linear up to the intensity of motions under consideration.
2. The soil is horizontally stratified.
3. Every stratum is essentially homogeneous, it is isotropic in horizontal planes (orthotropic), and it extends over a large distance in both directions, as compared with the total thickness of the soil formations.

When these conditions are fulfilled, the very high magnification factors predicted by the theory of multiple wave reflection are not a matter of conjecture. Factors in excess of 10 are a reality for the undamped spectra of earthquakes whose Modified Mercalli intensity approaches VII in Mexico City.

If condition 3 is not rigorously met, an adjustment in the assumed wave velocities will yield satisfactory results.

The distribution of spectral ordinates, normalized in terms of the corresponding expectations, is nearly the same as for white noise.

The empirical formulas in Refs. 12 and 13, which predict the shape of spectra on hard ground, seem to furnish satisfactory results provided a proportionality factor is introduced, which depends on the locality and on the focal coordinates. For Mexico City the percentage of internal damping determined from laboratory tests(10) on unconfined specimens subjected to free vibrations was adequate for the calculation of expected spectra. This percentage (5.36 percent of critical) turned out independent of the frequency of vibration. This type of behavior may not be extrapolated to the whole spectrum of frequencies, as it leads to spurious states of stress prior to the application of any stipulated state of deformation. Still the frequency-independent type of internal damping may be used in the linear theory of visco-elasticity over a wide range of frequencies to describe certain phenomena, although a more realistic characterization of behavior should center on hysteretic damping.

There are laboratory techniques, for determination of dynamic properties, far more precise than the one used in this study. The better techniques should be resorted to in order to supplement field determinations, while the simpler tests should prove adequate in cohesive material that is not fissured. The fact that adequate results were obtained for the soil's internal damping in this case may be due to a compensation of errors: it is likely that in situ damping is much smaller than that determined from tests on fissured samples, but that the effective damping for earthquake motions is again much greater than the actual internal damping, due to irregularities in reflection surfaces.

Acknowledgments

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TABLE 1

Characteristics of the Earthquakes Studied

Date	Magnitude	Intensity	Focal distance, km	a,cm/sec^2	c, cm/sec	x, cm	s,sec
10 Dec. 1961	5.25	V	82	19.85	1.71	1.70	32
11 May 1962	7.25	VI	330	6.06	1.18	1.29	131
19 May 1962	7.00	VI$\frac{1}{2}$	240	9.37	1.59	1.75	99
30 Nov. 1962	5.50	V	256	2.48	0.31	0.25	86

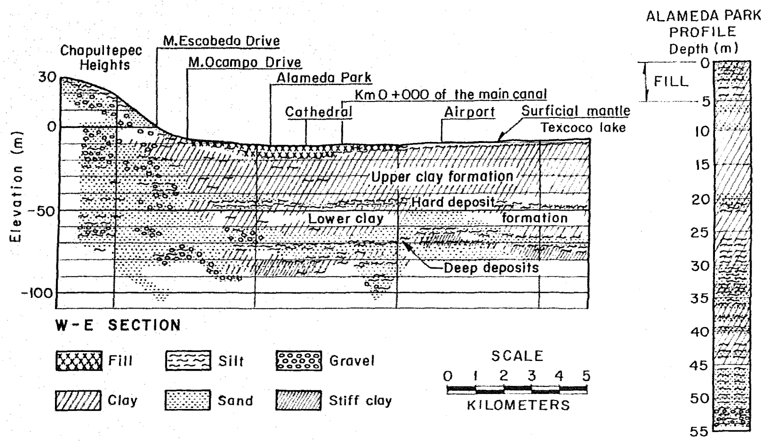

FIG. 1 SUBSOIL PROFILE IN THE VALLEY OF MEXICO (After Marsal and Mazari, 1962)

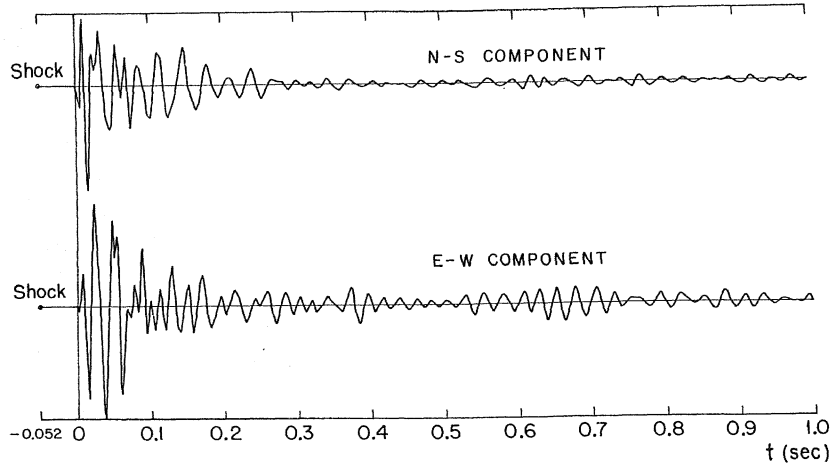

FIG. 2 TYPICAL RECORD FROM TESTS AT THE ALAMEDA-PARK SITE

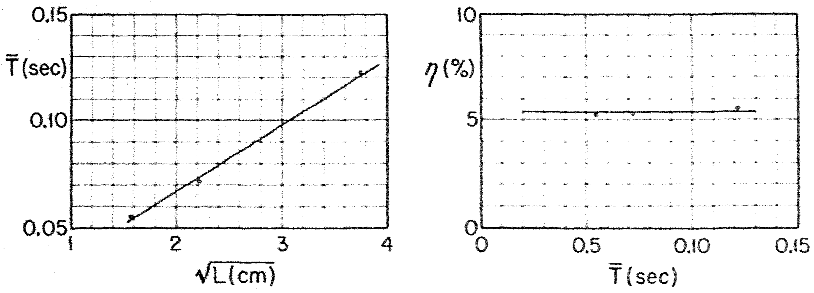

FIG. 4 VARIATIONS OF NATURAL PERIOD WITH SPECIMEN LENGTH AND OF INTERNAL DAMPING WITH THE NATURAL PERIOD

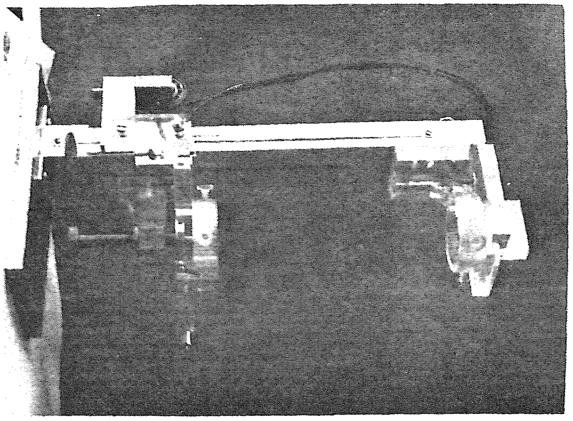

FIG. 3 APPARATUS FOR TORSIONAL VIBRATIONS

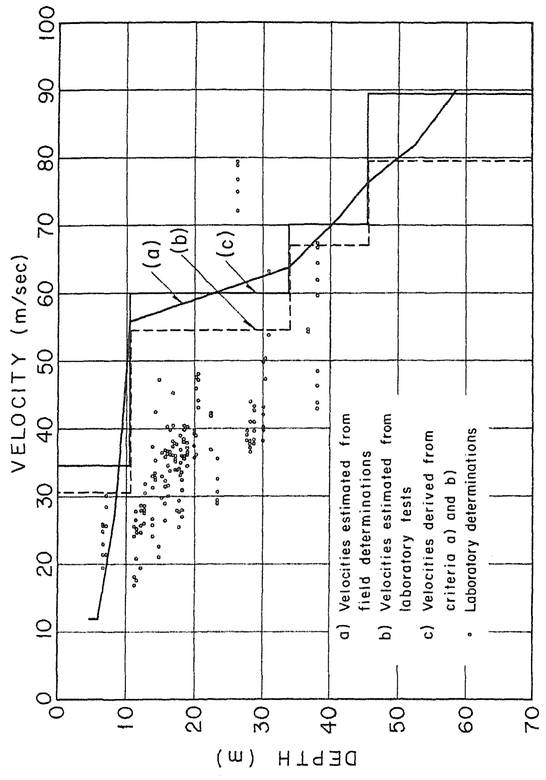

FIG. 5 SHEAR WAVE VELOCITIES

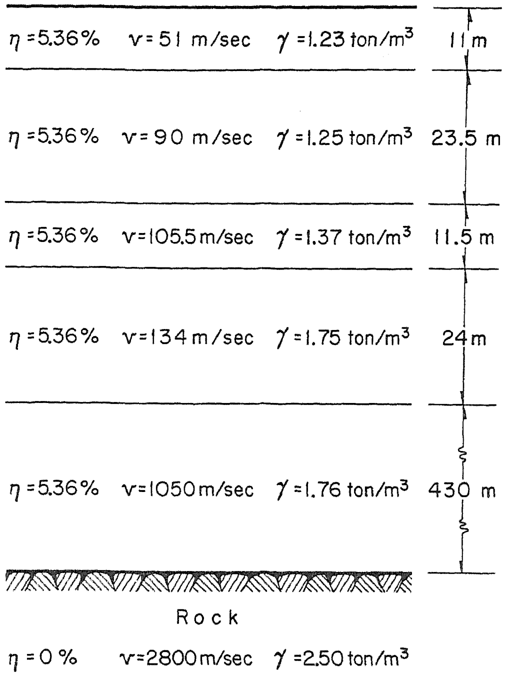

FIG. 6 VELOCITY AND DENSITY PROFILE

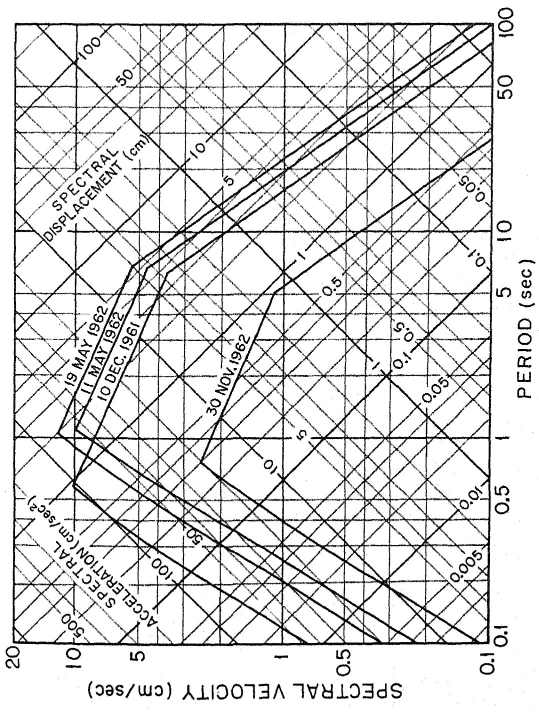

FIG. 7 EXPECTED UNDAMPED SPECTRA CORRESPONDING TO THE SURFACE OF BEDROCK

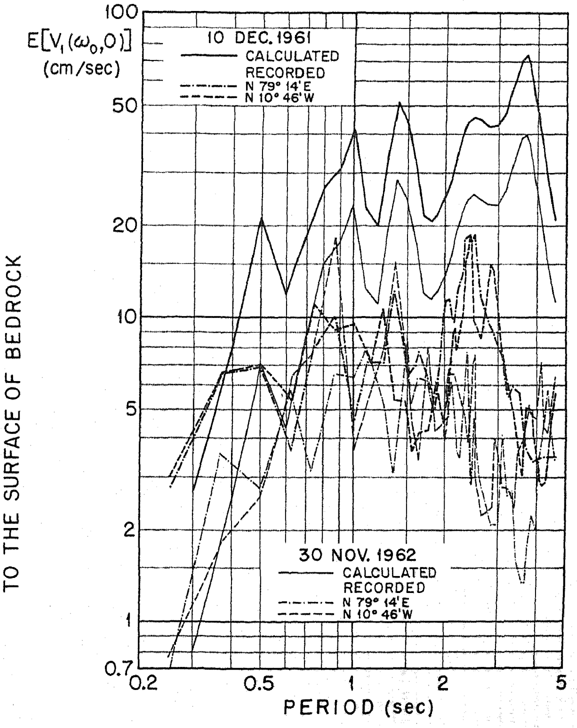

FIG. 8 COMPARISON OF THEORETICAL AND ACTUAL SPECTRA EARTHQUAKES OF 10 DEC. 1961 AND 30 NOV. 1962

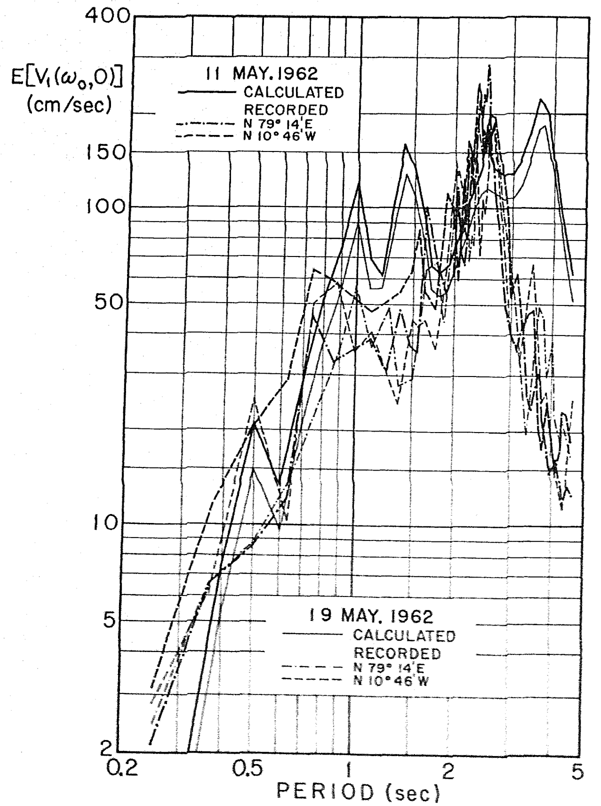

FIG. 9 COMPARISON OF THEORETICAL AND ACTUAL SPECTRA EARTHQUAKE OF 11 AND 19 MAY 1962

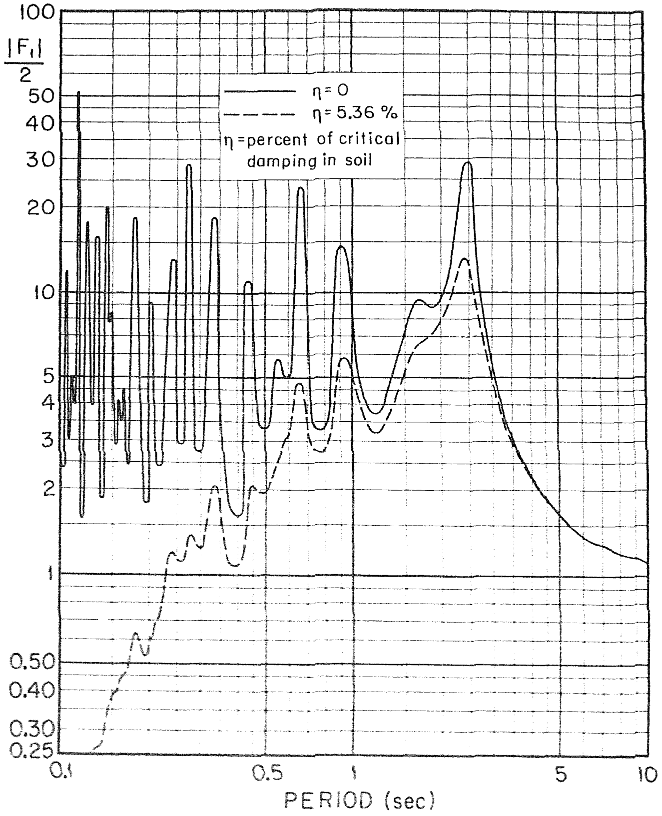

FIG. 10 FOURIER TRANSFORM OF THE SOIL TRANSFER FUNCTION

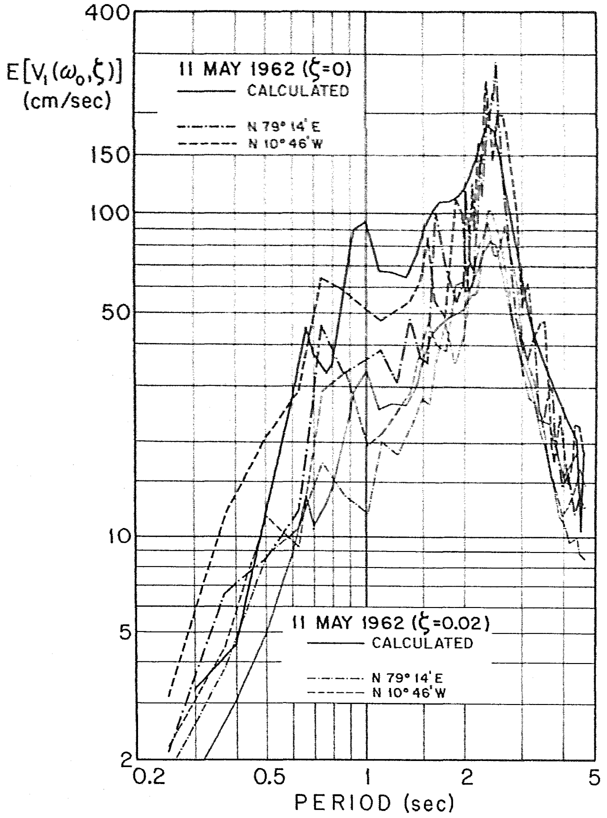

FIG. 11 COMPARISON OF ADJUSTED THEORETICAL SPECTRUM WITH SPECTRA OF THE EARTHQUAKE OF 11 MAY 1962

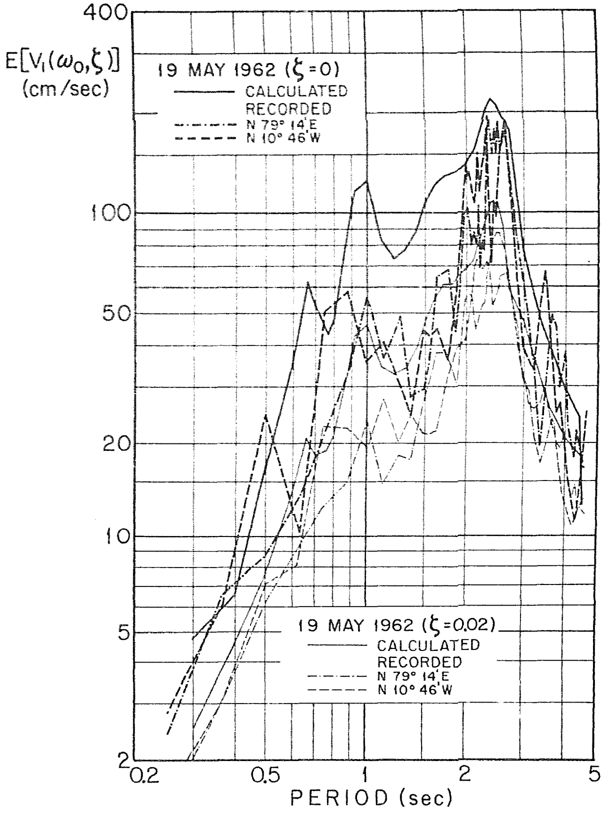

FIG.12 COMPARISON OF ADJUSTED THEORETICAL SPECTRUM WITH SPECTRA OF THE EARTHQUAKE OF 19 MAY 1962

ERRATA

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PAGE 63	Para 2, Line 8: "that the percentage of damping does not depend ...."
PAGE 68	Reference 13; add: (Unpublished Manuscript Instituto de Ingenieria, UNAM, Mexico, D.F., 1965)