

BUILDING CODE PROVISIONS ON TORSIONAL OSCILLATIONS  
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Introduction

Damage from torsional oscillations due to earthquakes has been observed in many buildings. In instances the distribution of damage can be explained only by admitting that the center of rotation lay inside the floor area. (1) This implies actual eccentricities of horizontal shear much in excess of what can be accounted for by static computations.

One may cite several reasons for the discrepancy: (2) uncertainties in calculation of relative rigidities and hence in location of the center of torsion; differences between dynamic and static behavior; uncertainties in load distribution; non-linear behavior; etc.

There are few data and no means at present for a rational evaluation of these factors. Consciousness of their importance has led to adoption of certain provisions in at least three building codes. Thus the Structural Engineers Association of California proposes a minimum eccentricity equal to  $\pm 5$  percent of the plan dimension. (3,4) The Mexican emergency regulations require an eccentricity, additional to that computed statically, equal to  $\pm 5$  percent of the plan dimension for ordinary buildings and  $\pm 7$  percent for warehouses; besides, at no elevation should the torsional eccentricity be less than half of the maximum computed for all stories below that elevation, nor the design torsion less than half of the maximum torque computed for all stories above. (5,6) The proposed requirements for Mexico's new Federal District code tentatively specify an eccentricity equal to  $\pm 5$  percent of the plan dimension of the story, to be added to 1.5 times the torsional eccentricity statically computed for all buildings; static analysis is permitted only when the computed eccentricity does not exceed 10 percent of the plan dimensions. (5)

Dynamic studies of single-story buildings (7) indicate clearly that differences between dynamic and static eccentricity in ideal elastic buildings may be important. The main object of the present paper is to generalize those studies. Conclusions derived therefrom are far from definitive but may aid judgment in the setting up of code requirements. They show the problem has sufficient importance to warrant further investigation.

Assumptions

Three different design acceleration spectra have been used in the present analysis (Fig. 1). Spectrum 1, in which acceleration is inversely proportional to natural period,  $T$ , in the entire range of  $T$ , is the same as assumed by Housner and Outinen in their study of torsion. (7) Spectrum 2 has been proposed in Mexico's new building code for structures on firm ground. (6) In it the acceleration is independent of the  $T$  up to  $T = 0.5$  sec, and inversely proportional to  $T$  thereafter. Spectrum 3 has been pro-

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posed for structures on soft ground in the Federal District; the design acceleration is a linear function of  $T$  up to  $T = 1$  sec, where its value is twice that for  $T = 0$ ; it is independent of  $T$  between 1 and 2.5 sec; and thereafter becomes inversely proportional to  $T$ .

These spectra are intended to correspond to 0.1-0.2 of critical damping. Accordingly the structures analyzed were treated as undamped.

It was assumed that the structures could deform only in torsion and shear and that they had no eccentricity parallel to the motion analyzed. Hence, they were treated as systems with two degrees of freedom per story with no influence from motion perpendicular to the direction considered.

The motion of the structure was found from expansion in terms of the natural modes and frequencies. The coefficients of the expansion are the participation factors. After calculation of all natural modes and periods for a given structure, the contribution of each mode to the motion was found by multiplying the mode in question by the corresponding participation factor and by the spectrum ordinate associated with that natural period. This is known as the method of modal analysis.

Design responses (shears and torques) were obtained as the square root of the sum of squared responses associated with the natural modes. This assumes that earthquakes consist of random arrays of pulses.<sup>(8)</sup> It is the method specified in the proposed code when one chooses to use modal analysis.

Assumptions adopted in this paper lead to results consistent with a proposed building code. They are not intended to afford bases for an attempt at predicting actual behavior. Yet, with qualifications they should permit drawing general conclusions. Spectrum 1 was included with the purpose of gaining some generality.

### Multistory Buildings

Four and eight story buildings have been analyzed using the modal analysis method described above. In all cases considered the plan of every floor is rectangular (in most cases it is square) and the center of mass of all floors lies on the same vertical line. All the floors in a given building have the same mass and, except series 2 and 3, they have the same polar moment of inertia. In all the examples, shear stiffnesses were assigned such a variation with height above ground that the relative displacement between consecutive floors, neglecting eccentricity, would be the same for all stories when the building were analyzed statically under horizontal accelerations proportional to height. Thus in a four-story building, if the stiffness of the ground story is 1.000, those of subsequent stories are 0.900, 0.700, and 0.400. In an eight-story structure they are 1.000, 0.972, 0.916, 0.833, 0.722, 0.583, 0.416, and 0.416. (The stiffness of the 8th story was taken equal to that of the 7th story.)

Torsional eccentricity of horizontal shear was assumed due to an end frame or wall. In that direction the entire stiffness, save for that of

the end frame or wall, was assumed uniformly distributed. In the direction perpendicular to the ground motion the entire stiffness was uniformly distributed within the floor area and had a magnitude such that at every story stiffnesses in both directions were proportional to the side lengths; they were equal for square floors.

In many of the cases analyzed there was static eccentricity (distance from the center of torsion to the statically computed horizontal shear) in only one story, while in all the stories the center of torsion lay on the same vertical line as the centers of mass of all floors. In other cases two or more stories had static eccentricity. The ratio  $e/a$  selected for each story was in most cases 0, 0.1, 0.25, or 0.5, and either positive or negative.

The ratio of mass to shear stiffness was chosen so that the fundamental period of the structure was equal to 0.5 sec for series A and to 2.5 sec for series B. Each series was analyzed in conjunction with the three design spectra described above.

The problems were dealt with by setting up a matrix formulation of the free vibration problem and using modal analysis. Computations were carried out in an electronic digital computer.

Table 1 gives a summary of the geometric characteristics of most of the buildings studied. They are divided in eight groups that conform with minor variations to the characteristics described above. All buildings, except those in group 5, are square in plan. The distinctive characteristics of the group are:

Group	Characteristics
1	Uniformly distributed mass in all floors: $j = a^2m/6$ .
2	In floor immediately above story having static eccentricity, $J = a^2m/3$ ; in all other floors, $J = a^2m/6$ .
3	In floor immediately above story having static eccentricity, $J = a^2m/12$ ; in all other floors, $J = a^2m/6$ .
4	4-story buildings; shear stiffnesses from the bottom up are $k$ , $0.972k$ , $0.916k$ , and $0.833k$ .
5	Rectangular floors with uniformly distributed masses: $J = (a^2 + b^2)m/12$ ; shear stiffness proportional to side length.
6	Unequal shear stiffnesses in x and y directions.
7	$J = 2a^2m/3$ in all floors.
8	$J = a^2m/24$ in all floors.

The effect of presumably important variables was sought through study of these groups. Dynamic eccentricity and shear were computed in every story. Results were compared with those corresponding to simpler dynamic structures and with results of static computations, as follows.

1. Ratio of computed dynamic shear to that in a similar building in which static eccentricity is neglected. Figs. 3-6 depict the ratio  $V/V_0$  for each story, series B, spectrum 2.

2. Ratio of dynamic eccentricity in each story to the maximum static eccentricity in all floors,  $e_j/e_{st \max}$ . Dynamic eccentricities were also compared with the floor dimension  $a$  parallel thereto. Both groups of ratios appear in right and left axes of Figs. 7-10 for various maximum static eccentricities, series B, spectrum 2.
3. Ratio between shear ratios in multi- and single-story structures. By shear ratio is meant the quantity defined under comparison 1. Single-story buildings chosen for comparison have the same polar moment of inertia as the floor above the story with maximum static eccentricity in the corresponding multistory building; static eccentricity and torsional rigidity are the same as for that story in the multistory building; and the ratio of shear stiffness to mass in the one-story structure is the same as for the first story of the multistory building. Ordinates in Figs. 11-14 are ratios of shear ratios for the story with maximum static eccentricity, spectra 1, 2, and 3, series A and B. For buildings eccentric in more than two stories, only the maximum and minimum ratios found are shown; this holds for comparison 4 also.
4. Ratio of dynamic eccentricity in the multistory buildings to that in one-story structures having the foregoing characteristics. Ordinates in Figs. 15-18 give these ratios for the story with maximum static eccentricity, spectra 1, 2, and 3, series A and B.
5. Comparison between maximum static eccentricity and the dynamic eccentricities in all stories of some 4-story buildings. Fig. 20-22 show results for series B, spectrum 2, and Fig. 19 for series A, same spectrum. In Fig. 19,  $J = a^2m/6$ ; in 20,  $J = a^2m/6$ ; in 21,  $J = 2a^2m/3$ ; and in 22,  $J = a^2m/24$ .

These results justify the following conclusions.

Effects of static eccentricity in a given story are essentially independent of eccentricities in other stories if these are not numerically greater than the eccentricity in the story considered (Figs. 3-10). Effects are slightly reduced when eccentricities alternate signs in consecutive stories (Figs. 7 and 8).

Dynamic eccentricity in a story with static eccentricity is roughly the same as for a single story building with the same static eccentricity (Figs. 15-18).

Dynamic eccentricity in stories other than the ones in which there is appreciable static eccentricity are roughly equal to  $0.4a$  and nearly independent of static eccentricity when  $J = a^2m/6$ . In every case they are an appreciable fraction of the maximum dynamic eccentricity in the building (Figs. 7-10).

Horizontal shears are somewhat reduced due to appreciable static eccentricity. The reduction is comparable to that in single-story buildings.

An outstanding feature in these results is the fact that for  $J = a^2m/6$  dynamic eccentricity seems to remain finite even when the static eccentricity is made ever smaller. For other  $J$ 's this no longer holds, although there is always some dynamic magnification. The situation is dilucidated in the Appendix for single-story buildings. It is shown that for a critical  $J$ , which is  $a^2m/6$  for the type of structure considered here, the two differential equations of motion tend to become identical when the static eccentricity approaches zero. The condition leads to two natural modes having equal frequencies. An analogous situation can be anticipated in structures having several stories.

Actually the torques associated with the pair of equal natural periods have opposite signs and tend to become equal as the static eccentricity approaches zero. The true combined dynamic eccentricity also tends to zero, but the sum of squared torques (or of their absolute values) does not. Hence the apparent discontinuity at  $e = 0$  when  $J$  is critical

Accordingly the anomaly should be ascribed to the method of analysis. In this range neither the criterion of the root sum square nor that of the sum of absolute values holds in combining natural modes. Both assume that the natural periods of the modes to be combined are sufficiently different from each other that oscillation in one mode may be regarded as a process independent of oscillation in the other modes. The same difficulty is not to be expected under large static eccentricities or other polar moments of inertia.

#### Concluding Remarks

Since modal analysis does not apply when two or more natural periods are very close to each other compared to the expected earthquake duration, at least the range of small static eccentricities coupled with near-critical polar moments of inertia should be further explored.

The following items also deserve further study. Eccentricity along a line different from the principal axes of the building; the presence of unequal masses in the various stories; centers of mass not on the same vertical line; combined effects of ground motion in more than one direction; non-linear behavior and pronounced damping; and torsional components of ground motions.

Nonetheless some general conclusions are warranted:

1. Dynamic eccentricity may exceed statically computed values. The excess is particularly important when the polar moments of inertia are close to their critical values; these are attained within the range of usual characteristics of buildings.
2. A rough estimate of torsional dynamic effects in multistory buildings can be obtained from the response of single-story structures with similar characteristics.
3. Excessive dynamic eccentricity due to closeness to the critical polar moments of inertia may be greatly reduced by changing the

critical values. This may be accomplished by increasing in the stiffness of perimetral frames.

References

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Appendix. Single-Story Buildings

Consider a structure having a single story (Fig. 2). D'Alembert's equations of equilibrium in free oscillation lead to the pair of equations

$$\left. \begin{aligned} (k - mp^2) y - ek\theta &= 0 \\ -eky + (R - Jp^2) \theta &= 0 \end{aligned} \right\} \quad (1)$$

where  $p$  denotes natural circular frequency. Their solution gives

$$\begin{aligned} p_i^2 &= \lambda_i^2 \frac{k}{m}, \quad i = 1, 2 \\ \lambda_i^2 &= \frac{\rho + 1}{2} \pm \sqrt{\left(\frac{\rho - 1}{2}\right)^2 + \frac{c^2}{J}} \end{aligned} \quad (2)$$

$$V_i = \frac{c^2 m \dot{y}_i}{c^2 + j(\lambda_i^2 - 1)^2}, \quad M_i = \frac{R' V_i (\lambda_i^2 - 1)}{ack \lambda_i^2} \quad (3)$$

The dynamic eccentricity is

$$e_d = \left( \frac{M_1^2 + M_2^2}{V_1^2 + V_2^2} \right)^{1/2} \quad (4)$$

Values of  $c \ll 1$  are particularly interesting. Consider first the case  $\rho = 1 + 2\alpha c$ , where  $\alpha$  is any arbitrary constant. Then, from Eq. 2,

$$\lambda_i^2 = 1 + \left( \alpha \mp \sqrt{\alpha^2 + \frac{1}{j}} \right) c$$

Equations 3

$$\frac{M_i}{V_i} = \frac{R' \left( \alpha \mp \sqrt{\alpha^2 + \frac{1}{j}} \right)}{\alpha k \lambda_i^2}$$

This ratio is finite for all  $\alpha$  even when  $c = 0$ . Hence the dynamic eccentricity (Eq. 4) is also finite even when the static eccentricity is nil.

Now, if  $\rho - 1$  does not tend to zero with  $c$ ,

$$\lambda_1^2 \doteq 1 - \frac{c^2}{(\rho-1)j}, \quad \lambda_2^2 \doteq \rho + \frac{c^2}{(\rho-1)j}$$

$$e_d \doteq \frac{cR'}{\alpha j k (\rho-1)} \sqrt{\left( \frac{\ddot{y}_2}{\ddot{y}_1} \right)^2 \frac{1}{\rho^2} + 1}$$

which is of the order of the static eccentricity.

Accordingly, in the single-story building, there is always a critical value of  $\rho$ , and hence a critical polar moment of inertia, for which the dynamic eccentricity tends to a finite value as the static eccentricity tends to zero. For all other polar moments of inertia  $e_d$  remains of the same order as  $e$  when the latter tends to zero.

As a special case consider a building square in plan view, of side  $a$ . Let the shear stiffness  $k$  come from a number of resisting elements sufficiently close to each other and sufficiently similar to each other plus an end element that is more rigid than the rest, and let the shear stiffness perpendicular to the direction of motion be equal also to  $k$ . Then,

$$R = \frac{a^2 k}{6} (1+2c), \quad \rho = \frac{1+2c}{6j}$$

The critical polar moment inertia obtains when  $j = 1/6$ , that is, for example when the mass is uniformly distributed in the slab. Correspondingly, with  $c \ll 1$ , if  $\ddot{y}_1 = \ddot{y}_2 = \ddot{y}$ ,

$$V = 0.756 m \ddot{y}, \quad e_d \doteq 0.349 a (1+c)$$

If  $\ddot{y}_1/\ddot{y}_2 = \lambda_1/\lambda_2$

$$V \doteq 0.756 m \ddot{y}_1 (1+0.45c), \quad e_d \doteq 0.349 a (1+2.60c)$$

On the other hand, if  $j \neq 1/6$ , and  $\bar{y}_1 = \bar{y}_2$ ,

$$\frac{e_d}{e_{st}} = \frac{\sqrt{1+(6j)^2}}{1-6j}$$

while if  $\bar{y}_1/\bar{y}_2 = \lambda_1/\lambda_2$ ,

$$\frac{e_d}{e_{st}} = \frac{\sqrt{1+6j}}{1-6j}$$

Figures 2a and b show results computed for this type of square building (without the simplifying assumption that  $c \ll 1$ ). They correspond to spectrum 2, series A and B. The latter have been computed for structures with fundamental periods equal to 0.5 and 2.5 sec respectively. These results are in accordance with the above derivations for small static eccentricities. Results for spectrum 3 and  $T_1 = 2.5$  sec are similar to those in Fig. 2a.

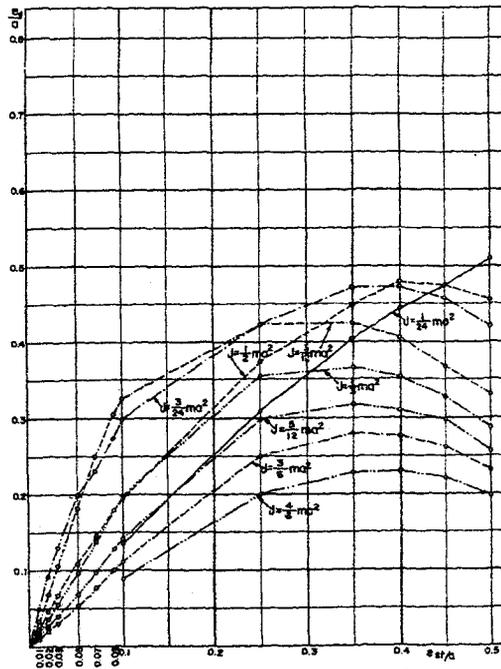
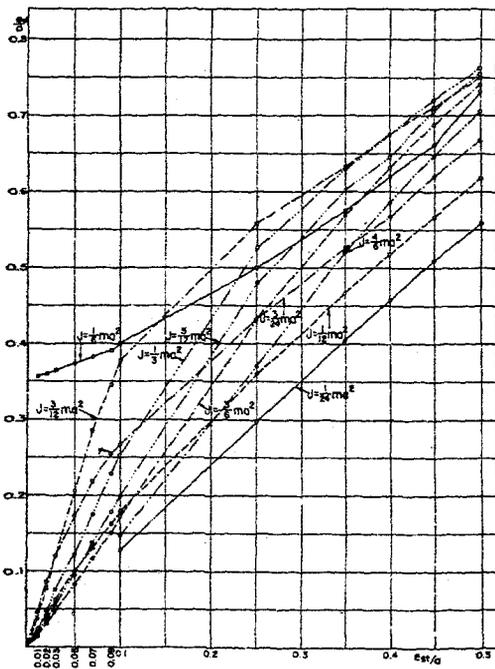
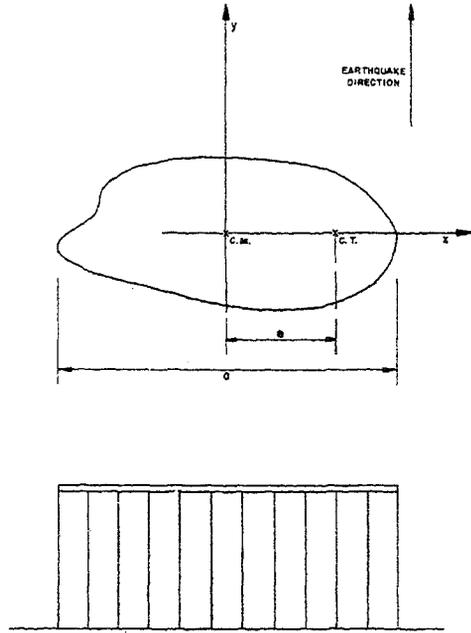
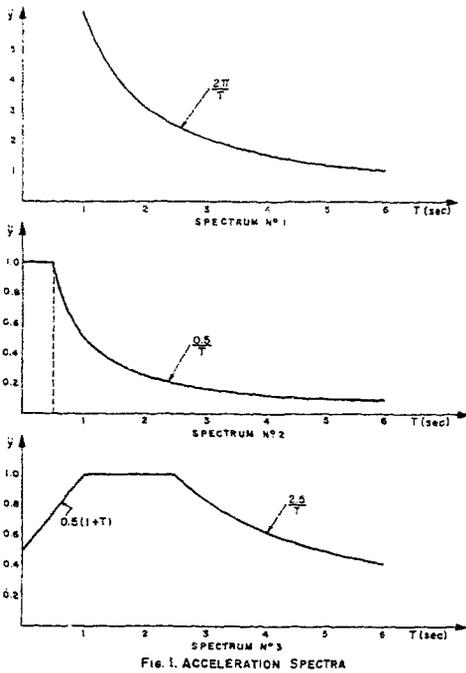
#### Notation

- a, b = floor dimensions, respectively perpendicular and parallel to earthquake motion;
- c = e/a;
- $e = e_{st.max}$  = maximum static eccentricity;
- $e_d$  = dynamic eccentricity;
- $e_j$  = dynamic eccentricity in jth story;
- $e_{jE}$  = dynamic eccentricity in statically eccentric story;
- $(e_E)_1$  = dynamic eccentricity in a one-story structure similar to a multistory building;
- $I_{ky}$  =  $a^2 k_{yj} (1 + 4c)/12$  = moment of inertia of the shear stiffnesses in y direction with respect to the y axis through the center of mass;
- $I_{kx}$  =  $ab k_{yj}/12$  = do. x direction, x axis;
- J = centroidal polar moment of inertia of mass;
- $j = \frac{J}{a^2 m}$ ;
- $k_y$  = total shear stiffness in the y direction;
- $k_x$  = do. x direction;
- $M_i$  = torsional moment in the ith mode;

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- M** = torsional design moment;  
**n** = number of degrees of freedom of the building;  
**P<sub>i</sub>** = natural circular frequency of the *i*th mode;  $p_i^2 = k_{yi}\lambda_i^2/m_i$ ;  
**R** =  $I_{kx} + I_{ky}$  = torsional stiffness with respect to the center of mass;  
**R'** =  $R - e_x^2 k_y$  = torsional stiffness with respect to the center of torsion;  
**r** =  $a/b$ ;  
**T<sub>i</sub>** = natural period of the *i*th mode;  
**V<sub>i</sub>** = shear force in the *i*th natural mode of a statically eccentric structure.  
**V** = design shear force;  
**V<sub>0</sub>** = design shear force when neglecting static eccentricity;  
**(V/V<sub>0</sub>)<sub>1</sub>** = ratio of shear forces in single-story structure similar to a multistory building;  $(V/V_0)_m$  ratio of shear forces on the multistory building;  
**y<sub>i</sub>** = ordinate of the acceleration spectrum, corresponding to the *i*th natural period;  
**y<sub>i</sub>** = free-vibration displacement of the center of mass in the *i*th natural mode;  
**β** = stiffness ratio =  $I_{kx}/I_{ky}$ ;  
**θ<sub>i</sub>** = free-vibration rotation in the *i*th natural mode;  
**λ<sub>i</sub><sup>2</sup>** = *i*th eigenvalue;  
**ρ** =  $R/(a^2k_j)$ .

Subscript *j* refers to *j*th floor or the story immediately below. Usually the *j* subscripts are dropped in single-story buildings.



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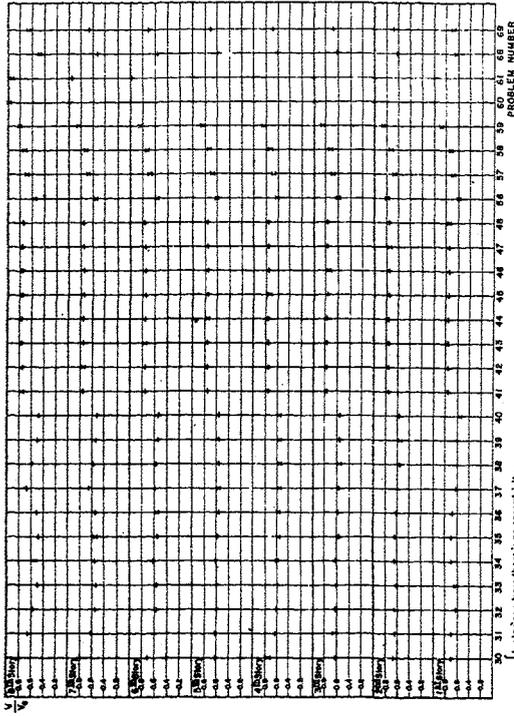


Fig. 4. DYNAMIC SHEAR RATIOS, 6-STORY BLDGS, SPECTRUM 2, SERIES B

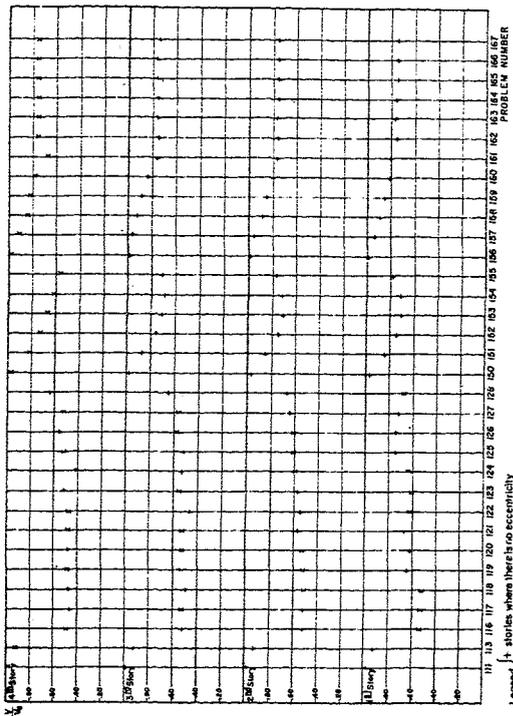


Fig. 6. DYNAMIC SHEAR RATIOS, 4-STORY BLDGS, SPECTRUM 2, SERIES B

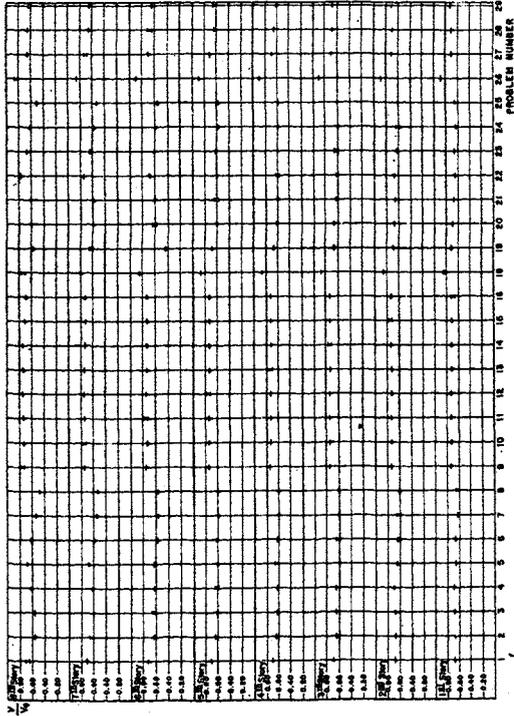


Fig. 3. DYNAMIC SHEAR RATIOS, 6-STORY BLDGS, SPECTRUM 2, SERIES B

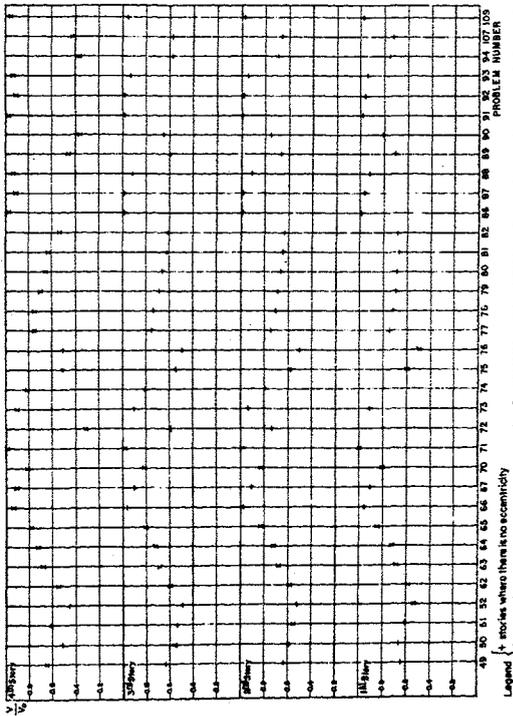
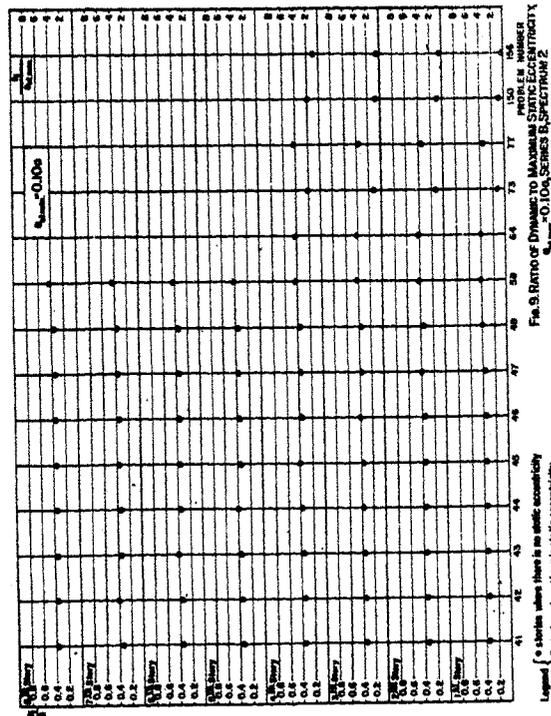
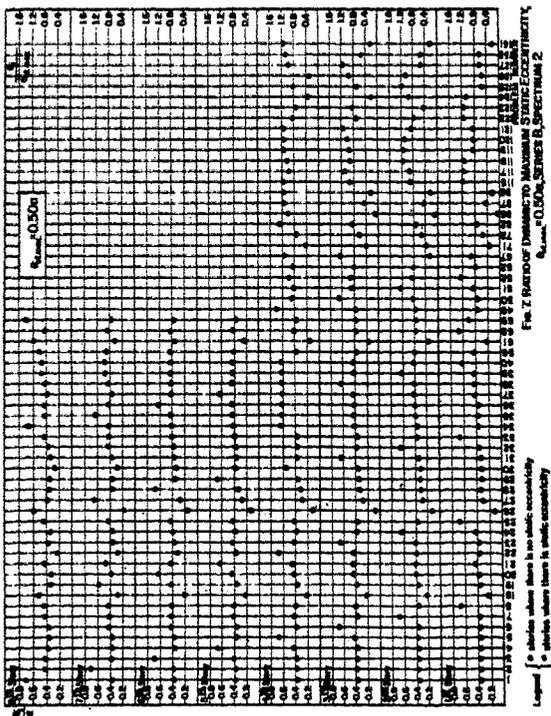
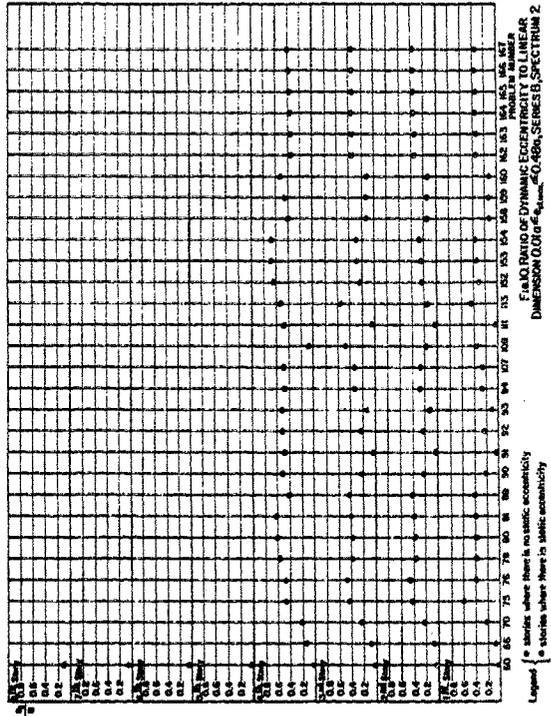
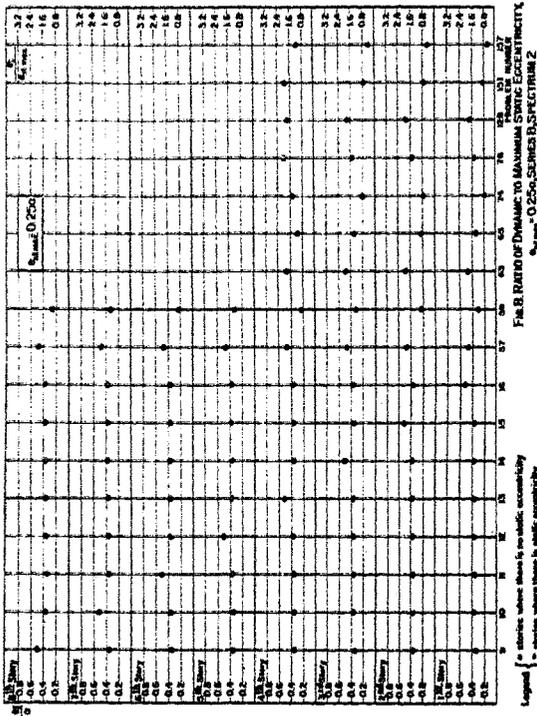
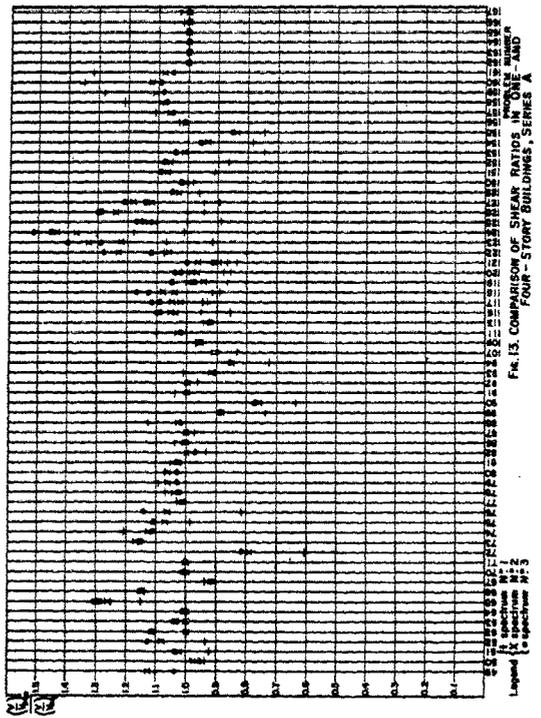
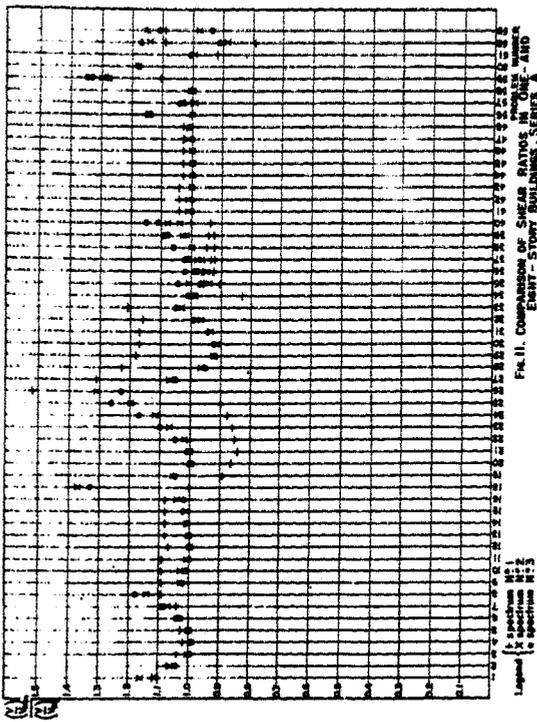
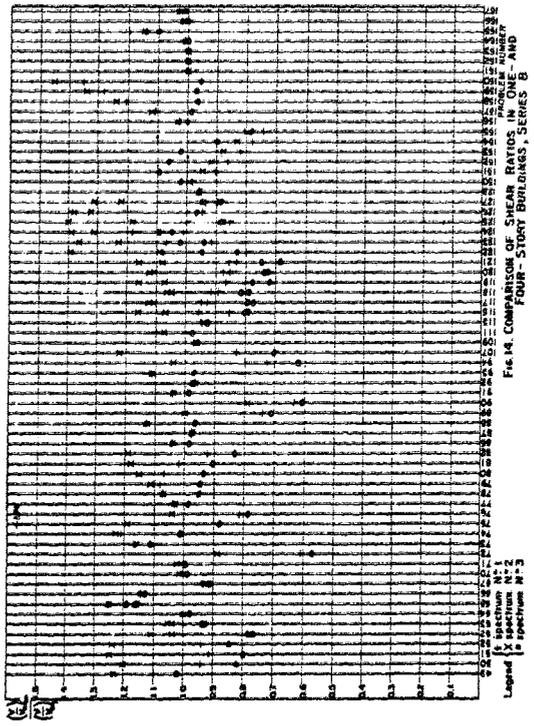
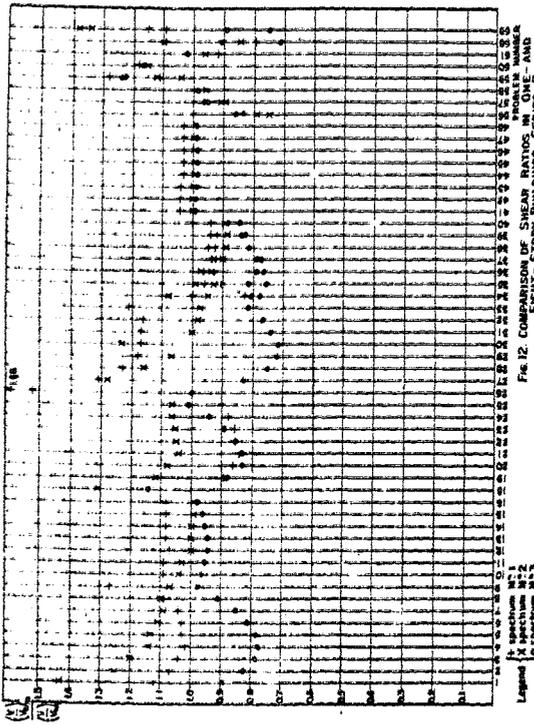


Fig. 5. DYNAMIC SHEAR RATIOS, 4-STORY BLDGS, SPECTRUM 2, SERIES B



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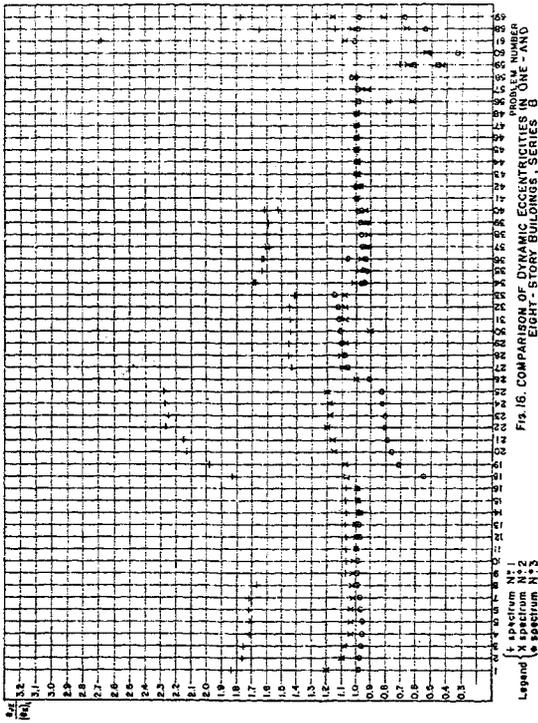


Fig. 16. COMPARISON OF DYNAMIC ECCENTRICITIES IN ONE - AND FOUR - STORY BUILDINGS, SERIES B

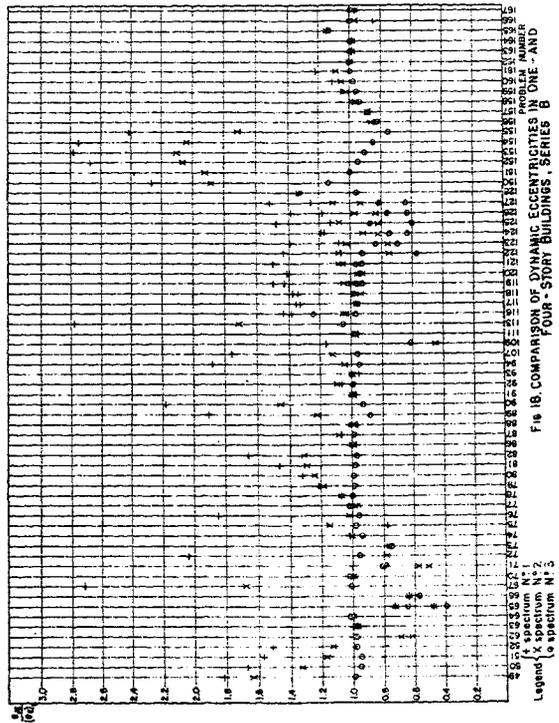


Fig. 17. COMPARISON OF DYNAMIC ECCENTRICITIES IN ONE - AND FOUR - STORY BUILDINGS, SERIES A

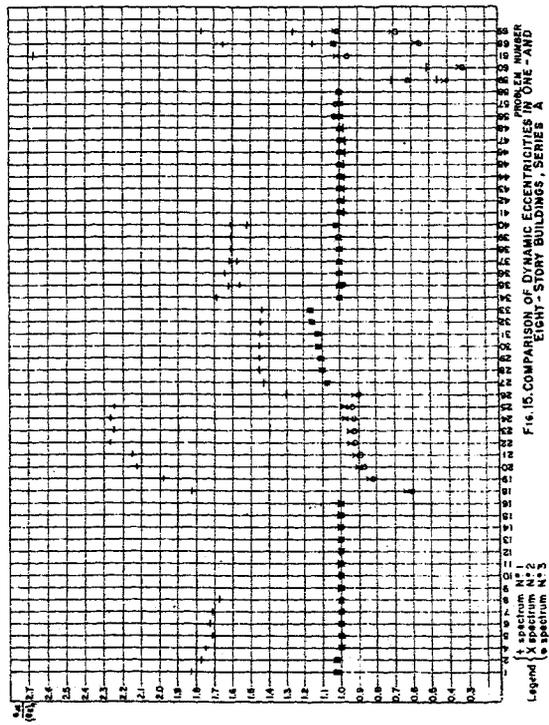


Fig. 16. COMPARISON OF DYNAMIC ECCENTRICITIES IN ONE - AND FOUR - STORY BUILDINGS, SERIES B

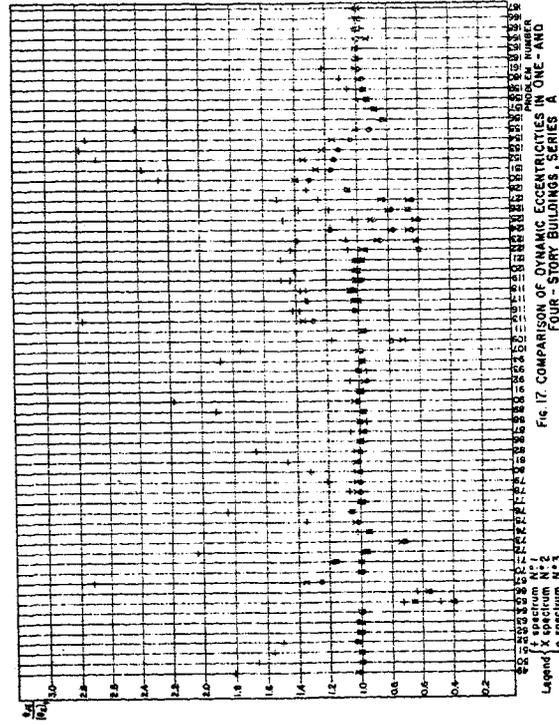


Fig. 17. COMPARISON OF DYNAMIC ECCENTRICITIES IN ONE - AND FOUR - STORY BUILDINGS, SERIES A

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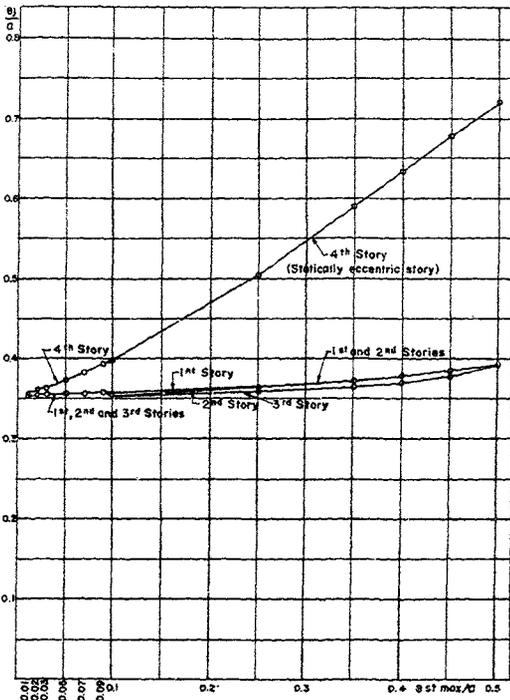


Fig. 19. DYNAMIC VS. STATIC ECCENTRICITIES, FOUR-STORY BUILDINGS, SERIES A, SPECTRUM 2,  $J = \frac{1}{4} m/s^2$

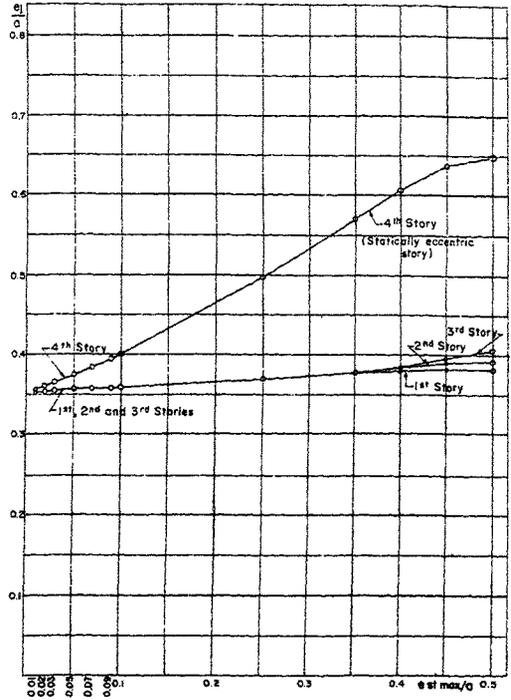


Fig. 20. DYNAMIC VS. MAXIMUM STATIC ECCENTRICITIES, FOUR-STORY BUILDINGS, SERIES B, SPECTRUM 2,  $J = \frac{1}{8} m/s^2$

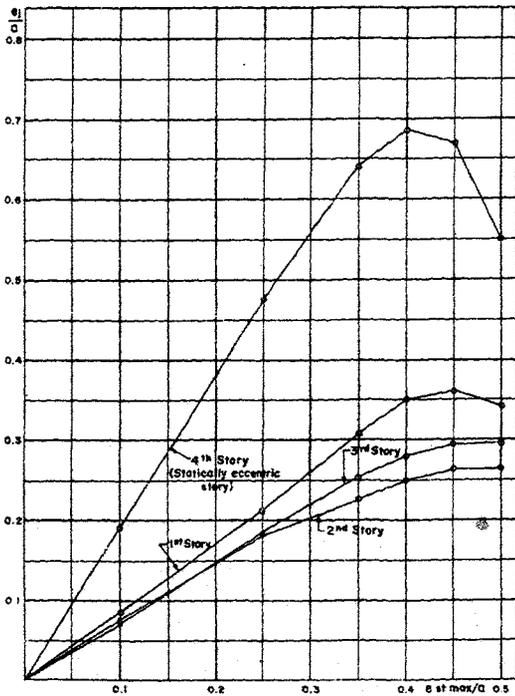


Fig. 21. DYNAMIC VS. MAXIMUM STATIC ECCENTRICITIES, FOUR-STORY BUILDINGS, SERIES B, SPECTRUM 2,  $J = \frac{1}{4} m/s^2$

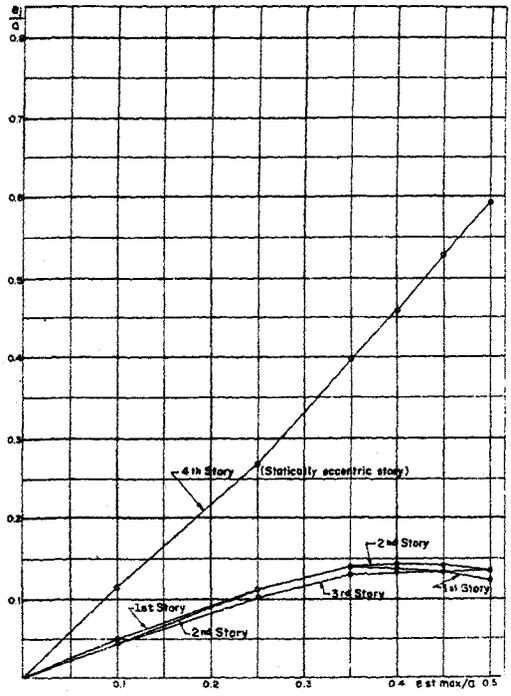


Fig. 22. DYNAMIC VS. MAXIMUM STATIC ECCENTRICITIES, FOUR-STORY BUILDINGS, SERIES B, SPECTRUM 2,  $J = \frac{1}{16} m/s^2$

TABLE I, 2<sup>ND</sup> PART

BUILDING NUMBER	GROUP NUMBER	NUMBER OF STORIES	STORIES IN WHICH THERE IS ECCENTRICITY	$C = \frac{e}{a}$	$r = \frac{a}{b}$	EXPLANATORY NOTES
49 TO 52	4	4	SEE NOTES	0.50	1.00	ECCENTRICITY IN ONE STORY. FOR BLDG 49 THE ECC. WAS IN THE 4 <sup>TH</sup> STORY; FOR BLDG 50, IN THE 3 <sup>RD</sup> ; ETC
56 TO 58	1	8	ALL STORIES	SEE NOTES	1.00	FOR BLDG 56, C=0.50; FOR 57, C=0.25; FOR 58, C=0.10
59	1	8	ALL STORIES	±0.25	1.00	THE ECCENTRICITIES ALTERNATE. AT THE 8 <sup>TH</sup> STORY C=0.25; AT THE 7 <sup>TH</sup> C=0.20; ETC
60	5	8	8	0.05	10.00	
61	5	8	8	0.50	0.100	
62 TO 64	1	4	ALL STORIES	SEE NOTES	1.00	FOR BLDG 62, C=0.50; FOR 63, C=0.25; FOR 64, C=0.10
65	1	4	ALL STORIES	±0.25	1.00	THE ECCENTRICITIES ALTERNATE. AT THE 4 <sup>TH</sup> STORY C=0.25; AT THE 3 <sup>RD</sup> , C=0.20; ETC
66	5	4	4	0.05	10.00	
67	5	4	4	0.50	0.100	
68	1	8	1 AND 8	0.50 0.10	1.00	AT THE 1 <sup>ST</sup> STORY C=0.50; AT THE 8 <sup>TH</sup> , C=0.10
69	1	8	1 AND 8	0.25 0.50	1.00	AT THE 1 <sup>ST</sup> STORY C=0.25; AT THE 8 <sup>TH</sup> , C=0.50

TABLE I, 4<sup>TH</sup> PART

BUILDING NUMBER	GROUP NUMBER	NUMBER OF STORIES	STORIES IN WHICH THERE IS ECCENTRICITY	$C = \frac{e}{a}$	$r = \frac{a}{b}$	EXPLANATORY NOTES
94	5	4	4	0.48	10.00	
107, 109	5, 6	4	4	0.45	SEE NOTES	FOR BLDG 107, $\frac{e}{a} = 10$ ; FOR BLDG 109, $\frac{e}{a} = 0.10$ ; FOR BOTH AT 4 <sup>TH</sup> STORY $\beta = 0$
111, 113	5, 6	4	4	0.45	SEE NOTES	FOR BLDG 111, $\frac{e}{a} = 10$ ; FOR BLDG 113, $\frac{e}{a} = 0.10$ ; FOR BOTH AT 4 <sup>TH</sup> STORY $\beta = 3.87$
116 TO 118	1	4	SEE NOTES	0.50	1.00	ECCENTRICITY IN 1 <sup>ST</sup> STORY AND IN ONE OF THE OTHER THREE; FOR BLDG 116 IN STORIES 1 AND 4; FOR BLDG 117 IN STORIES 1 AND 3; FOR BLDG 118 IN STORIES 1 AND 2
119, 120	1	4	SEE NOTES	0.50	1.00	ECCENTRICITY IN TWO STORIES; FOR BLDG 119 IN STORIES 2 AND 4; FOR BLDG 120 IN STORIES 2 AND 3
121	1	4	SEE NOTES	0.50	1.00	ECCENTRICITY IN STORIES 3 AND 4
122 TO 124	1	4	SEE NOTES	±0.50	1.00	ECCENTRICITY IN 1 <sup>ST</sup> STORY C=0.50 AND IN ONE OF THE OTHER THREE C=0.50 FOR BLDG 122 IN STORIES 1 AND 3; FOR BLDG 124 IN STORIES 1 AND 2
125, 126	1	4	SEE NOTES	±0.50	1.00	ECCENTRICITY IN 2 <sup>ND</sup> STORY C=0.50 AND IN ONE OF THE TWO UPPER STORIES C=0.25; FOR BLDG 125 IN STORIES 2 AND 4; FOR BLDG 126 IN STORIES 2 AND 3
127	1	4	SEE NOTES	±0.50	1.00	IN STORY 3, C=0.50; IN STORY 4, C=0.50
128	6	4	1	0.45	1.00	FOR 1 <sup>ST</sup> STORY $\beta = 0$
150 TO 155	7	4	4	SEE NOTES	1.00	FOR BLDG 150, C=0.10; FOR 151, C=0.25; FOR 152, C=0.35; FOR 153, C=0.40; FOR 154, C=0.45; FOR 155, C=0.50
156 TO 161	8	4	4	SEE NOTES	1.00	FOR BLDG 156, C=0.10; FOR 157, C=0.25; FOR 158, C=0.35; FOR 159, C=0.40; FOR 160, C=0.45; FOR 161, C=0.50
162 TO 167	1	4	4	SEE NOTES	1.00	FOR BLDG 162, C=0.1; FOR 163, C=0.2; FOR 164, C=0.3; FOR 165, C=0.5; FOR 166, C=0.7; FOR 167, C=0.9

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