# EARTHQUAKE RESISTANT DESIGN, CONSTRUCTION AND REGULATIONS

## ( SESSION IV )

*The questions not answered in a written form by the author do not appear on the pages of discussion.*

# ON EARTHQUAKE RESISTANT DESIGN OF FLAT SLABS AND CONCRETE SHELL STRUCTURES

By

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## INTRODUCTION

In the design of usual buildings, rigid frames constituted mainly of columns and beams are considered the resisting systems against external forces. Some parts of slabs and walls are very often counted as effective to cooperate with the frames. When flat slabs and shells are connected to column tops, the cooperation of these plate elements with linear members is more important than in usual framed structures. However, difficulties in dealing with these two or three dimensional elements, especially when they are submitted to partial loading, have kept a considerable part of this problem left to be solved in the future.

This paper is a description of experimental studies made to find rational design methods of earthquake resistant flat slabs (including flat plates) and concrete cylindrical shells. Scope was restricted to statics.

## PART I  EARTHQUAKE RESISTANT DESIGN OF FLAT SLABS

### 1.1  POSSIBILITIES OF EARTHQUAKE RESISTANCE OF FLAT SLABS

When the great earthquake attacked Tokyo-Yokohama area in 1923, there were already several flat slab structures in service as office buildings, warehouses and factories in these cities. We can obtain information on the degree of earthquake resistance of flat slab structures from the damage investigation reports made after the earthquake. Damage sustained by 4 flat slab buildings has been reported in detail. The flat slab buildings examined in these reports were two office buildings, a warehouse and a factory building, each having a different amount of structural walls.

Both of the two office buildings reported (five and six storied, respectively) suffered fire after the earthquake. In these buildings some cracks were found on the surface of slabs, columns and walls, and it is hard to determine whether these cracks were due to earthquake shocks or fire, but, in either case, the cracks were not fatal to the structures.

The warehouse also suffered conflagration. Reports clearly indicate that the structural damage caused by shocks was not serious and that the structure had been carrying loads on their slabs soundly until inflammable contents caught fire. According to the reports, heat generated by combustion of inflammable contents of the buildings badly weakened the concrete and reinforcement of the slabs, causing them to hang down and consequently fail.

The factory building in the Kawasaki area is the only example that did not meet fire. This three storied building had essentially no walls to bear

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horizontal forces. Condition of its damage is shown in Fig. 1. It should be noticed that vital failure occured at column tops and not in the slab. This failure could have been avoided if the column tops had been disigned with better reinforcement.

As was seen in this example, experience points out the possibilities of earthquake resistance of flat slab structures, that is, well designed flat slab structures may prove much stronger than supposed by inference from the concept of beam and column structures.

## 1.2    STUDIES IN THE PAST

In the earthquake resistant design of flat slabs, as in the case of usual rigid frame structures, slabs are so designed that they bear the effect of moments produced at column tops by earthquake force in addition to the effect produced by stationary loads.

The problem of elastic plates subjected to moments acting at column tops was first considered by Dr. Tanabashi[1] in 1929. Dr. Tanabashi applied Dr. Lewe's double series method[2] to this problem and examined the theoretical results through steel model tests. The author (Tsuboi) and Mr. Miyazaki[3] applied later the single series method to the same problem. Dr. Gyoten[4] also treated this problem by using the finite difference method to take rigid zone effects into account.

Above mentioned studies were confined to elastic analyses. Problems encountered in designing concrete flat slabs were treated by Messrs. Elstner and Hognestad[5] in their experimental studies on the shearing strength of reinforced concrete slabs subjected to concentrated loads. An investigation on similar problems was recently reported by Mr. Rosenthal[6].

## 1.3    EXPERIMENTAL STUDIES ON FLAT SLABS SUBJECT TO COLUMN TOP MOMENT

### 1.3.1.  Objects and Outlines of Studies

Studies reported herein were made to observe stresses and deformations in mortar slabs in elastic state to examine the approximate solutions mentioned above, and to observe general behaviors of the slabs in elasto-plastic state under monotonously increasing and reciprocally repeated loads. Both loadings were statical. Infinitely continuing flat slabs with square bays were the objects of our present studies.        Specimens have beams along opposite edges, $x = \pm a$ (Fig. 3), which correspond to inflection lines of equally spanned infinitely spreading flat slabs subjected to column top bendings.                      This was to realize the true boundary conditions ($w = 0$, $\nabla^2 w = 0$) with tournament system supporting along these edges. Specimens were free along $y = \pm a$, the boundary conditions being different from true ones that yield $\frac{\partial w}{\partial y} = 0, \frac{\partial^2 w}{\partial y^2} = 0$, but the effect of this difference is considered secondary. Specimens were made of mortar, strength of which being nearly equal for all slabs. Loading mechanism used is shown in Fig. 2. Moment was statically applied through the column stub while reactions were made to occur at four equally spaced points along the inflection lines through tournament systems.

Three of the nine specimens were made of plain mortar (FO-1, FO-2 &

FO-3). The rest were reinforced in three different ways, that is, reinforcement in the direction of the x - axis (the direction included in the plane in which the column stub moment is applied) was apportioned more to the column strip than to the intermediate strip for Fl-1 & Fl-2, uniformly distributed for F2-1 & F2-2 and apportioned less to the column strip than to the intermediate strip for F3-1 & F3-2 (Fig. 3). One of the two specimens having identical reinforcement distribution was subjected to monotonously increasing moment, while the other was subjected to reciprocally repeated moment. In the latter case, loading was returned at three stages; first little outside the elastic region, then in elasto-plastic state and finally very near the ultimate strength. Strains at various locations on slab surfaces and the angle of rotation of column stubs were measured.

## 1.3.2. Test Results in Elastic State

### a. Stress Distribution:

As strains were measured in a quarter part of each test slab, symmetry of loading regarding the x - axis is essential to have the results in this part represent the whole slab.
This was checked by the strains at the points of the opposite portion with respect to the x - axis.          In the test of FO-2 (plain mortar) the best symmetry was achieved (difference of strains of two opposite points was less than 7 per cent), so strain measurements of FO-2 were used for comparison with theoretical approximations.          Similar strain distributions were recorded for the other slabs.

Calculations were made by the two approximations for a slab identical in dimension with those tested; one of these approximations was single series method in which loads distributed over the column top areas to represent applied external moments were expanded in Fourier's series, and the other was finite difference method in which perfectly rigid zones were assumed over the column top areas.

Distributions of $\overline{M}_x = -D\frac{\partial^2 w}{\partial x^2}$ obtained by these two methods are shown in Fig. 4.          Comparison with the test results shows that either method is practically sufficient to express stress distributions. Ratios of longitudinal moments apportioned to column and intermediate strips are as shown in Fig. 4.

### b. Stiffness——Effective Width:

When interaction of columns and slabs is to be considered in flat slab structures, the conception of effective width —— width of an imaginary beam having the same depth, span and stiffness against rotation of the column as those of the slab under consideration —— is introduced.

As stated above, introduction of rigid zones around the column tops is not necessarily essential to obtain stresses in the slabs with practically sufficient accuracy. However, it can be easily supposed that the effect of rigid zones is important for the estimation of effective stiffnesses. This effect was examined as follows.

In single series method, if we have the longitudinal tangent of the de-

flection surface at the column top center represent the angle of rotation of the column top,

$$\theta = \frac{\partial w}{\partial x}\bigg|_{\substack{x=0 \\ y=0}} = 0.0625 \frac{M}{\pi^5 R^4 D} \ . \tag{1}$$

This gives the following effective width for an imaginary beam with no rigid zone for poisson's ratio $\nu = 1/6$ .

$$2 b_e' = \frac{M}{E h^3 \theta} (2a) = 0.672 \, (2a) \ . \tag{2}$$

When translated for a beam, for comparison, with a perfectly rigid zone for $-C \leq X \leq C$ , this gives

$$2 b_e = 2 b_e' \left(\frac{a-c}{a}\right)^3 = 0.345 \, (2a) \ . \tag{3}$$

On the other hand, finite difference method in which perfectly rigid zone is assumed over the column top area gives, for $\nu = 1/6$ ,

$$2 b_e = 0.620 \, (2a) \tag{4}$$

Test results are as shown below. In calculating effective width from observed angle of rotation of column stubs, effect of reinforcement was taken into consideration for the cross section as a whole, and the difference in distributions of reinforcement was not considered. Results show that the latter effect is not appreciable within the elastic region. Effective width thus obtained by the tests is

$$2 b_e = 0.58 \sim 0.61 \, (2a) \ , \tag{5}$$

showing a good agreement with that obtained by finite difference method assuming perfectly rigid zone.

## 1.3.3. Test Results in Elasto-Plastic State and Ultimate Strength

As the behavior of plain mortar slabs is of little interest to us from the view-point of earthquake resistance, detailed description is omitted. Notable tendancies are; though failure (bending) was visually quite sudden with no cracks found before it, strain measurements show them spreading from the center towards edges, making slabs fail at about half of ultimate loads obtained through beam tests by assuming the slabs also to be beams. Moment – Strain and Angle of Rotation Curves of FO-2 are shown in Fig. 5.

Six of nine test slabs were reinforced in three different ways, though the total cross sectional area of reinforcement was kept the same for all slabs. All of them were doubly reinforced, upper and lower reinforcings being the same. For test slabs F1-1 and F1-2, 70 % of the total longitudinal reinforcement was distributed within column strips. For F2-1 and F2-2 the longitudinal reinforcement was uniform over the whole section. For F3-1 and F3-2 only 30 % of the longitudinal reinforcement was arranged in column strips, the rest being distributed in intermediate strips.

In the tests of F1-1, F2-1 and F3-1, column moment was monotonously increased up to failure, while in the case of F1-2, F2-2 and F3-2, it was statically repeated three times in different stages as shown by an example for F1-2 in Fig. 8.

Moment-Angle of Rotation (of column stub) curves of F1-1, F2-1 and F3-1 are shown in Fig. 7 by solid lines and virgin curves for F1-2, F2-2

and F3-2 by dotted lines. It should be noticed that in both loadings, distribution of reinforcement has considerable effects on the slab stiffness in elasto-plastic region.

Of six reinforced slabs, only F1-1 which had denser reinforcement in its column strip and was monotonously loaded failed in pure flexure. On the other hand, only F3-2 which had lighter reinforcement in its column strip and was repeatedly loaded, failed in punching. For the remaining slabs, two kinds of failures apparently coexisted. As shown in Fig.9 pattern of punching observed in the test is of a different nature from that experienced in usual flat slab tests made without or with little column moment. Punching took place very gradually and no sudden decrease of load was observed. Crack patterns for a few slabs are shown in Fig. 6.

Ultimate moments of test slabs are indicated in Fig. 7. Ultimate moment of F1-1 which failed in pure flexure is in good agreement with that calculated by the yield line theory, taking into consideration the effect of strain hardening of reinforcement, and confirmed for the present case by beam tests. As to F2-1 & F3-1, and F1-2 & F2-2 in positive loading, yielding was spreading when punching began, and for F3-2, and F1-2 & F2-2 in negative loading, punching strength was reached before yielding began.

Two tendencies are seen with respect to the ultimate strength of test slabs: Distribution of longitudinal reinforcement affects the punching shear resistance around the column; and repeated load reduces the punching shear resistance, especially after the foregoing opposite load reached near the ultimate load.

## PART II   EARTHQUAKE RESISTANT DESIGN OF CONCRETE CYLINDRICAL SHELLS

## 2.1   GENERAL VIEW

As shell roofs are comparatively new types of structures, we do not have sufficient knowledge about how they behave under earthquake shocks. As one of the most popular kinds of shell roofs, concrete cylindrical shells are taken up here.

Cylindrical Shells are often constructed on columns, and it is usual for traverses or stiffners to cross the sections at the locations of columns. Sometimes they are also provided with edge beams in the longitudinal direction.

As we lack the knowledge concerning the earthquake resistance of shell roofs, cylindrical shell roofs are usually so designed that they may receive no moments from the column tops, by making or assuming pin joints at the connections with columns. Consequently, columns must be designed as cantilevers clamped at the bases. However, this is by no means rational nor economical. Design methods by which the cooperation of shell slabs and edge beams can be properly evaluated are required.

When the horizontal component of earthquake force is transversely applied, stresses in a shell due to body force may be of some importance in some cases[5], but, as long as the effect of column top moment is concerned,

a traverse with an effective fraction of the shell does not essentially differ from a beam of the usual frame structure.

On the other hand, when longitudinal moments are applied at column tops, shell slabs seem to lend more effective cooperation to beams, and they will play quite an essential role when no edge beam exists. Thus it is the longitudinal column top moment that particularly draws our attention.

## 2.2. STUDIES IN THE PAST

The above mentioned problem in the elastic region was considered by Mr. Heki[9] who applied Dr. Zerna's approximation,[10] which is essentially identical to the second approximation of ASCE Manual No. 31, by expanding antisymmetrically distributed line loads at column tops in Fourier's series. The same problem was recently treated by Mr. Okamoto.[11]

## 2.3. EXPERIMENTAL STUDIES ON CYLINDRICAL SHELLS SUBJECT TO COLUMN MOMENT

### 2.3.1. Objects of Studies

In the analysis of a cylindrical shell subject to longitudinal earthquake force, the column top moment is first replaced by a line load distributed on the column top, which is then expanded in trigonometric series to give longitudinal boundary conditions to the shell.

From the standpoint of design, however, this approach has its faults that the convergence of series is not rapid, especially when the ratio of the longitudinal width of column top to the longitudinal span is small.

In the case of flat slabs, the problem is of course similar, but column width – span ratio is comparatively large. On the contrary, in cylindrical shells, large column width is usually considered unnecessary as long as stationary loads are concerned, because internal forces of the shells are transfered to columns chiefly through traverses. Thus the above ratio often falls on a value less than 1/10. Moreover, calculation involved is much more laborious in this case than in flat slabs.

Objects of our studies were to examine the validity of elastic approximation, to find simpler and more practical ways to obtain stresses and the effective stiffness of the shell, and to observe the general behavior of concrete shells (mortar models were used here) in its elasto-plastic state.

### 2.3.2. Parallel Shells

a. Celluloid model test:

To reproduce the phenomenon which would take place when parallel continuous shells are subjected to moments at equally spanned column tops, a celluloid specimen shown in Fig. 10 was made and moment was applied through the centrally located column.

This model has only two spans in the transverse direction. As it is expected that stresses and deformations rapidly decrease with the distance

from the column, this kind of model may be used to represent infinite parallel shells. The longitudinal edges of the model were supported by celluloid diaphragms, and, as in case of flat slab tests, inflection lines were assumed there. Longitudinal span was varied as shown in Fig. 10. Strains at various points, deflection of the longitudinal column line and rotation of the column stub were measured.
Results of the tests were as follows.

Stresses:

Transverse bending moment $M_\varphi$ and longitudinal stress $\sigma_x$ obtained from strains observed are indicated in Fig. 11 for the case where L = 17 cm. Results of the series method based on Zerna's approximation are shown in the same figure for comparison. First 6 terms were taken in the above calculation. Convergence of $\sigma_x$ was not very good yet, but the both stresses are seen in fairly good agreement with those observed. For L = 43.8 cm, distributions of $\sigma_x$ on a few transverse measuring lines are shown in Fig. 12 compared with those calculated by the beam method. It can be seen that observed $\sigma_x$ approaches that obtained by the beam method when the distance from the column increases. It was also observed that considerable changes in longitudinal span produces little changes in stresses around the column.

Effective Stiffness:

Rotation of the column stub measured by the mirror method is plotted in Fig. 13 for various longitudinal spans. It can be seen that the straight line applied to the observed angles of rotation of comparatively long specimens ( $L \geqslant r/2$ ) is parallel to the line obtained by the beam method.

The test results mentioned above can be reduced to the following statement concerning the nature of comparatively long parallel shells subject to column top moments: Deformations of the shells are chiefly governed by beam action, but disturbance takes place around the columns. This disturbance diminishes rapidly with increase of the distance from the column, and it seems that stress distributions around the column have little sensitivity to the changes in longitudinal span in the case of comparatively long shells. As it will be compared later with that of single shells, this is a nature peculiar to parallel shells. This characteristic helps us make an engineering approach to the present problem. As stated above, convergence of series is generally very slow in long shells. In such cases, shorter shells may be considered instead and the results can be modified by the beam method. This nature also simplifies the way of thinking when we try to find effective stiffnesses of parallel shells. If we try to assume the effective width in shells as in flat slabs, we encounter the difficulty that there are too many factors which affect the results.
In the case of flat slabs these factors are the shape and size of the bay and the column top section. When both the bay and the column top section are square, the only variable is the column top width – span ratio. For cylindrical shells, two variables must be added to the general case of flat slabs, that is, radius of the section and thickness of the shell slab. Thus, at least four variables, the ratios of longitudinal and transverse spans, shell slab thicknesses and longitudinal column top widths, to radius of the section, for instance, have to be involved in the expression of an effective width. On the other hand, taking advantage of the characteristic

of parallel shells, we can separate the longitudinal span from the above variables for comparatively long shells. Then column stub rotation is considered a sum of that due to beam deformation and that due to partial deformation around the column, the latter being independent of the longitudinal span.

Thus the problem can be simplified to some extent and a more rational approach to the practical solution seems to be possible by this method than by the conception of effective width.

b. Mortar model tests

Eight models shown in Fig. 14 were made of reinforced mortar and subjected to repeated column top moments. They have various rises ( $f/b = 0$, 0.05, 0.10, 0.20 ) and one of the two specimens which have the same rise has, and the other does not have, an edge beam. The loading system used in flat slab tests was again employed without any change in principle. The test results were as follows.

Stresses:

As it was difficult to fabricate specimens with constant and uniform thicknesses, and as values of strains were small, we could not expect sufficient accuracy in observed strains. However, the following information was obtained: Prominence of beam action is common for all specimens. Edge beam restrains partial deformation around the column and diminution of local stresses with increase of the distance from columns was seen as well as in specimens with no edge beams. Increases in rise also decrease stresses in shell slabs, but the influence of this change in rise was not very large.

Stiffness:

Observed angle of rotation of column stub is plotted in Fig. 15 with curves obtained by the beam method shown for comparison. When $f = 0$ (flat), as stated in the description of flat slab tests, rigid region of column top area restrains the rotation of column stub, and observed rotation of both (with and without edge beam) specimens were much less than expected by the beam method. With increase in rise, this effect becomes less, and conversely, the effect of local deformation becomes prominent. Fig. 15 also suggests that a change in rise does not have a large effect on local deformation.

Cracks and Failure:

All specimens were subjected to reciprocally repeated loads. The method of loading was the same as that used for flat slab tests. Two flat specimens failed in bending, and, as they were sufficiently long, behaved completely like beams. In shell specimens, initial cracking took place at around 20 % of ultmate load. In the case of specimens without edge beam, first cracks appeared on the upper surface along the longitudinal column line. This is apparently due to $M_\varphi$ which shows high concentration around the column in the theoretical solution, too. However, this crack did not extend long, nor was it fatal. In the case of those with edge beams, initial cracking occured at the point of contact of column and beam.

For all of the shell specimens, most prominent and vital were shear cracks which spread from the corner of columns at approximately 45 degrees to the generatrix. All of the shells failed by local failure around the columns.

In Fig. 16, applied ultimate moments of shell specimens are plotted with calculated maximum moments of the section (single span) for comparison. It can be seen from the figure that S1-1 and S1-2 would develop the maximum strength of their sections in actual structures. Examples of crack patterns are shown in Fig. 17.

## 2.3.3. Single Shells

Three single shell specimens as shown in Fig. 18 were made of reinforced mortar. All of them have edge beams. The feet of the specimens were so pinned that horizontal reactions at these four points will be equal, and a reciprocally repeated horizontal load was statically applied at the level of column tops. Deflections of shell surfaces and strains at various locations were measured.
Test results were as follows.

Deflections and Stresses:

Deflection surfaces of the three specimens are shown in Fig. 18 for the same load. There is a great difference between the deflections of flat and shell specimens, both in magnitude and in character. But there is no appreciable difference between those of shell specimens with different rises ($f = 0.10b$ & $0.20b$). The same thing can be observed regarding the stresses.

Concerning the stress distributions, another characteristic of single shells is that the beam method can not be applied. Distribution of longitudinal direct stresses at a section near the column is characterized by Fig. 19. Effective stiffness is much less than expected by the beam method ($1/8 \sim 1/10$).

Therefore, the concept of beam deformation plus local deformation is not suitable for single shells. For these, the conception of effective width is preferable for the estimation of effective stiffnesses.

Cracks and Failure:

Crack pattern is different from that of parallel shells. As transverse edges largely deflect over the longitudinal span, cracks appear along these edges, but do not spread much toward the top of the shells. Crack patterns of shell specimens are shown in Fig. 20. Failure occured at the connection of shells and columns. Mode of failure was complicated.

## SUMMARY

Preliminary studies for rational design methods of earthquake resistant flat slabs and concrete cylindrical shells were described in this report.

Attempts were made to grasp the general nature of their behaviors under column top moments. For both structures, series methods can be applied to obtain stresses, but for the estimation of effective stiffnesses, either some modification of the methods or application of other methods by which the effect of rigid zone can be introduced is required. In the practical design of parallel long shells, slow convergence of series can be avoided by making use of the prominence of beam action in these structures. Concept of effective width is suitable for the practical design of flat slabs and single shells, but in the case of parallel shells, introduction of beam action will give simpler and more rational design methods. In the placement of reinforcement of flat slabs, not only stress distribution in elastic region, but also the influence of reinforcement on stiffness in elasto-plastic region and the ultimate strength should be considered. Further studies are required to learn the nature of punching of flat slabs under column top moment. In parallel shells, shear strength of shell slabs is important, while the most prominent stresses in elastic solutions are longitudinal direct stresses and transverse moments. In single shells deformation is very large, and shells can not be expected to stiffen edge beams to a great extent. Cracks appear over a wide range. Strength of connection is essential.

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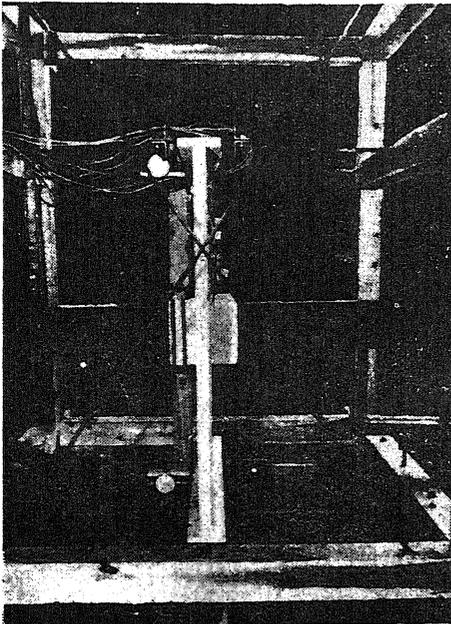
Fig. 1
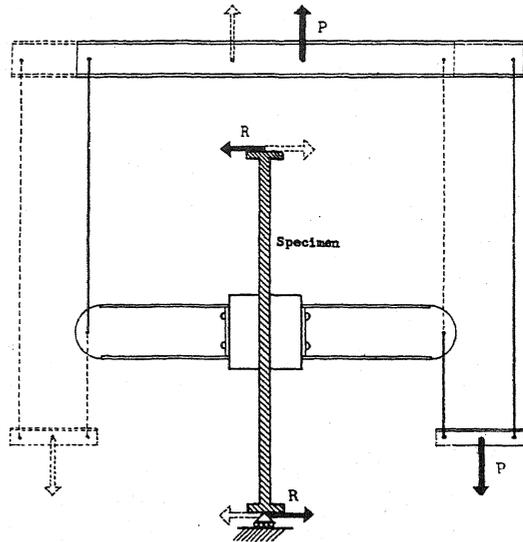
Example of
Damage

Fig. 2-1

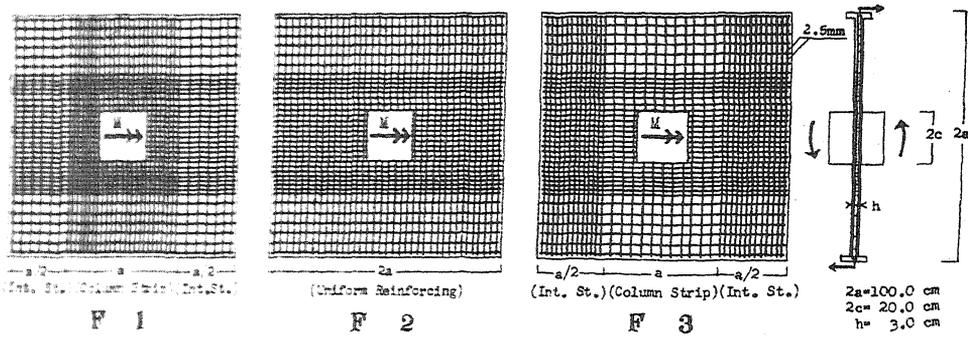

Fig. 2-2



Fig. 2  Loading System
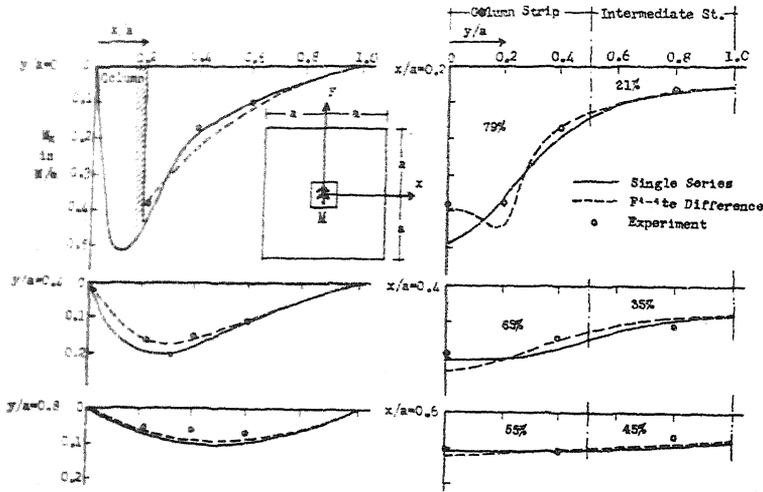
Fig. 3  Flat Slab Specimens
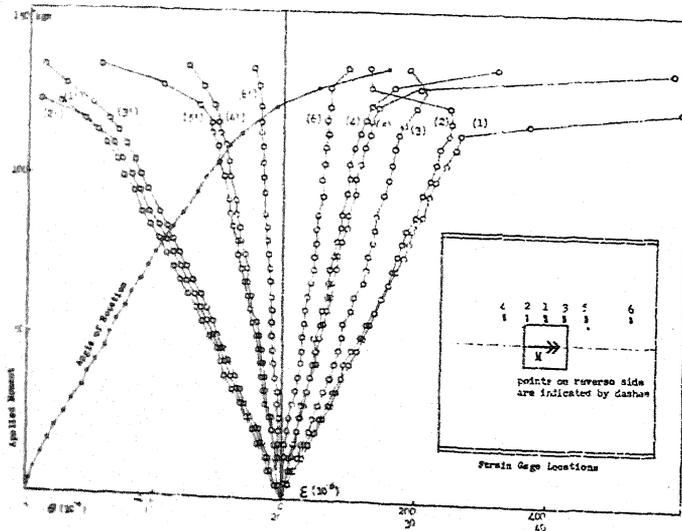


Fig. 4

Distribution
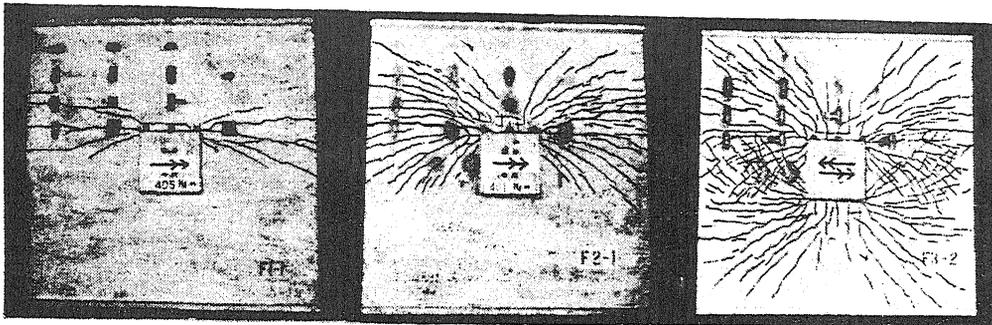
of $\bar{M}_x$



Fig. 5

M-ε }
M-θ } Curves

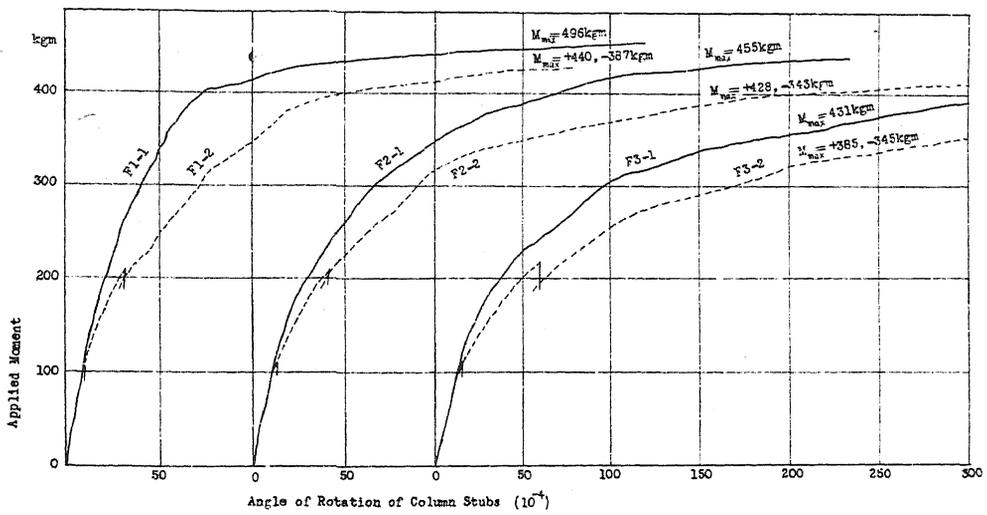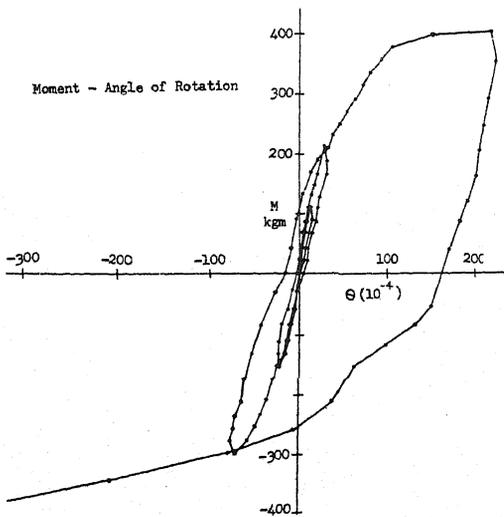Fig. 6  Crack Patterns at Failure



Fig. 7  M-θ Curves



Fig. 8  M-θ Curve under
Repeated Load



Fig. 9  Punching Pattern

Fig. 10  Celluloid Specimens

M.L.=Measuring Line of Strains

r = 30.0 cm
b = 23.1 cm
h = 0.15 cm
% = 22.6°
f(rise) = b/10 = 2.31 cm

Fig. 12   $\bar{\sigma}_x$

Fig. 11   $M_\varphi$ & $\bar{\sigma}_x$





Fig. 13  Rotation-Span

Specimens

| f/b | no edge beam | with edge beam |
|-----|--------------|----------------|
| 0 | S 0 - 1 | S 0 - 2 |
| 0.05 | S 1 - 1 | S 1 - 2 |
| 0.10 | S 2 - 1 | S 2 - 2 |
| 0.20 | S 3 - 1 | S 3 - 2 |

Fig. 14   Parallel Shell Specimens
(Mortar)

Parallel Shells without Edge Beam

——— Beam Method
- - -o- - - Experiment

Parallel Shells with Edge Beams

Fig. 15   Angle of Rotation

——— Calculated Ultimate Moments
of the Shell Cross Sections

●, O   Positive & Negative Maximum
Moments of the Specimens
with Edge Beams

▼, △   Those of the Specimens
without Edge Beam

Fig. 16   Ultimate Load

Fig. 17   Crack Patterns
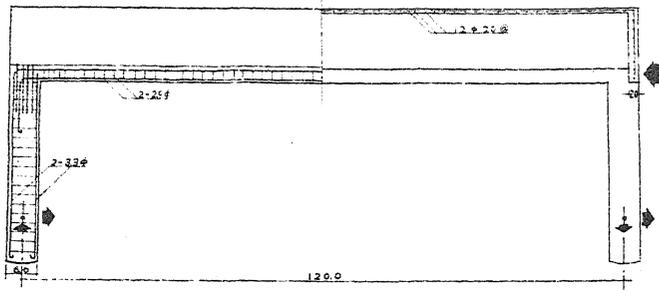
Fig. 18-1



Fig. 18-2

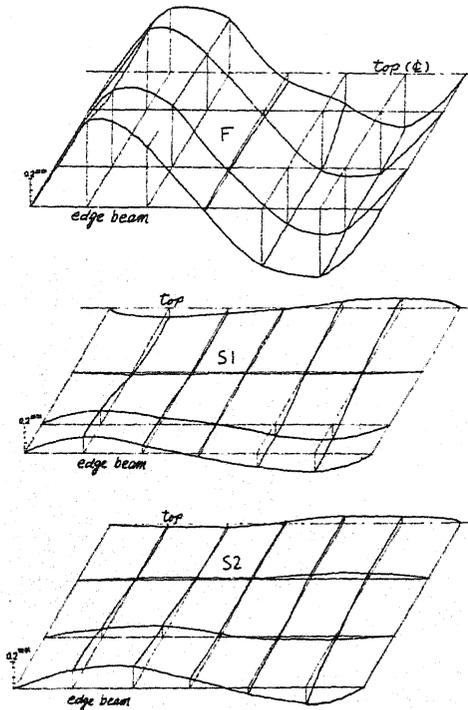Fig. 18  Single Shell Specimens

Fig. 19  Deflections



Fig. 20  Longitudinal Stress



Fig. 21  Crack Patterns