DISPLACEMENT SPECTRUM IN THE DYNAMIC RESPONSE OF INELASTIC STRUCTURES, FOR DESIGN PURPOSES

By

JULIO IBANEZ

ABSTRACT

Facts, obviously important in the damaging capacity of eartquakes, are ignored in some acceleration spectrums for earthquake-proof buildings Codes. Among those facts, the mechanical characteristics of the structural materials, and the duration of the seisms are prominent examples. A procedure is presented here, devoted to take these facts into account.

ACCELERATION SPECTRUM FOR PERFECT SYNCHRONISM

Fig 1, curve "A", shows the ground acceleration spectrum, derived from the corresponding accelerogram Fig 2, of the N-S component of the El Centro earthquake, California, May 18, 1940 (1). Ordinates are the maximum cumulated accelerations, in a mass of a one degree of freedom vibrating system, as that of Fig 3-a. Abscissas are the corresponding self vibration periods of the system.

The curve "A" contains a number of peaks at various points. They must be attributed to the persistent vibrations of the ground, synchronized with the period of the vibrating system and, consequently, generating cumulative displacements and accelerations in the said peak (Appendix A).

Various heterogeneous stratas of soft soil, laying on the rocky bed, forming a multy-mode vibrating system, are likely to be responsible for these persistent vibrations (2).

Let us examine one of the peaks: that of 0.18 seconds period, for instance, whose maximum cumulated acceleration is q = 5.19 g. Since the base accelerogram Fig 2, does not exhibit greater accelerations than 0.32 g, the persistent ground vibration of 0.18 seconds period does not have individual waves of greater acceleration. Furthermore, the duration of the significant part of the seism was, approximately, 30 seconds, as shown in Fig 2.

As seen in Appendix A, n synchronic, succesive impulses cause displacements and accelerations n times greater than those of the first impulse. So, for building up the acceleration 5.19 g of the considered peak, in the 167 impulses that go into the 30 seconds of duration of the seism, the mean acceleration of the persistent ground vibration must be 5.19/167 = 0.031 g = 30.4 cm/sec^2. It is in all probability that the actual maximum value of the acceleration in any particular wave differs from the maximum in the mean wave, but the cumulated acceleration will remain the same.

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ACCELERATION SPECTRUM FOR IMPERFECT SYNCHRONISM

Till now we have assumed that the stresses in the structural materials of the system do not exceed the elastic limit. If this limit is exceeded the self vibration period of the system lengthens. In Fig 4, the curve "A" indicates the self vibration period, and the curve "B", the maximum stresses in the structural steel vibrating system of Fig 3-b, subjected to growing synchronic impulses (Appendix B). The self vibration period grows up from 0.6 seconds, within the elastic limit, to 0.657 seconds, for the maximum stress, that is to say, 9.5 per cent of period increase.

Assuming the same behaviour of the structural steel, in a simple vibrating system as the rigid frame of Fig 3-a, whose self vibration period is 0.18 seconds, when within the elastic limit (Perfect synchronism with the persistent ground vibration of 0.18 seconds period), things would happen as follows:

1o.) During the initial impulses, the perfect synchronism between the ground vibration and the free vibration of the system, causes a growing acceleration in the system mass, till the stresses in its resistant members reach, and exceed the elastic limit (Appendix A).

2o.) Once the said limit has been exceeded, the self vibration period T_p of the system becomes longer than the persistent vibration period of the ground movement, creating a difference between the ground vibration frequency f_Ω and the system vibration frequency f_p. The synchronism is now imperfect, and a "beating vibration" of the system develops (Appendix A).

3o.) During the diminishing phase of the beating vibration, the stresses diminish too, and the self vibration period of the system becomes shorter. The frequencies f_Ω and f_p approaches one another, till the perfect synchronism is attained.

4o.) Once more the perfect synchronism causes an increase, a high, and a decreasement in the acceleration of the mass of the system, and so on.

The limit conditions occur when the difference of period $T_p - T_\Omega$ is maximum. Let us accept 9.5 per cent for this difference, as measured.

Since $T_\Omega = 0.18$ seconds, $f_\Omega = 5.57$ cycles/sec, and $f_p = 5.075$ cycles per second, from where: $f_\Omega - f_p = 0.495$ cycles/sec.

According to Appendix A, the following quantities, correspondant to the beating vibration, may be applied:

Persistent ground vibration period	$T_\Omega = 0.18$ seconds
" " " frequency	$f_\Omega = \frac{1}{T_\Omega} = 5.57$ cycles/sec
Factor of discrepancy	$\Delta = \pi\ (f_\Omega - f_p) = 1.555$
Angular rate	$\Omega = 2\ \pi\ f_\Omega = 35$ radians/sec

Persistent ground vibration mean wave acceleration $q = 30.4 \text{ cm/sec}^2$

Maximum amplitud $(X_{max})_o = \frac{q}{2 \Omega \Delta} = 0.28$ cm

Since the self vibration period of the system, in elasto-plastic conditions, is $T_p = 0.197$ seconds, the maximum acceleration of its mass, for the referred amplitud, is:

$$q_t' = \frac{4 \pi^2}{T_p^2} (X_{max})_o = 290 \text{ cm/sec}^2 = 0.296 \text{ g}$$

The foregoing result prove that the cumulated acceleration in the 0.18 seconds period peak, that reaches 5.19 g when perfect synchronism conditions are mantained along the full 30 seconds duration of the seism, is drastically damped if the frequency coincidence fails to be perfect. This damping can reduce the acceleration up to 0.296 g, which means 5.7 per cent of its undamped value.

Proceeding in the same way for the remainder points of the spectrum, curve "B" of Fig 1 has been drawn. It is to be noted that this reduced "B" spectrum, in addition to the very large damping exhibited in comparison with spectrum "A", reveals a trend to emphazise the importance of the peaks in the zone from 0.45 to 1.0 seconds period, while the spectrum "A" emphasises the importance of peaks in the zone from 0.1 to 0.6 seconds period.

An objection to the precedent analysis may arise from the fact that each point of the spectrum has been treated as an isolated wave case, neglecting the effect of the neighbour waves, whose exciting forces generate acceleration. However, one must keep in mind that the studied damping effect in the peak is not caused by a mere one time change of the proper period, into another steady period, but by a transient condition of the self vibration period of the system, inherent to the elasto-plastic state of materials. This means that, when the stresses reach the plastic zone, the proper period of the system becomes elusive to the cumulating possibilities of the exciting forces. From here it results that the same damping effect of the plasticity that is present in the isolated, persistent, peak wave, is present in any other similar neighbour wave, and in every point of the spectrum. So, the consequency of applying the beating vibration criterion is to lower the entire acceleration spectrum.

DISPLACEMENT SPECTRUM

It has been shown that the mass acceleration cannot surpass a certain limit fixed by the beating vibration conditions, though severe may be the disturbing force. The plastic zone is a ceiling like limit for cumulated acceleration. However, the displacements and strains may go on cumulating, and cause the collapse of the structure, even if the acceleration has not yet reached the ultimate value. Therefore, the displacement spectrum, for elasto-plastic conditions must be known, in order to predict the displacements and strains that are likely to develop, and compare them with the displacements and strains capacity of the structure.

In Fig 5, the curve "A" is the displacement spectrum corresponding to the acceleration spectrum "A" of Fig 1, that is to say: the ordinates of the spectrum "A" in Fig 5 are equal to those of the spectrum "A" in Fig 1, multiplied by

$$\frac{T^2}{4\pi^2}$$

The curve "B" in Fig 5 is the same displacement spectrum "A" of this Fig, reduced according to the beating vibration conditions. So, the ordinates are given by the term:

$$(X_{max})_o = \frac{a}{2\,\Omega\,\Delta}$$

As observed, this curve may be attained in a very simple way if the mechanical characteristics (Curve "A" in Fig 4) of the structural materials, are known.

It is worth to point out that in the spectrum "B" of the Fig 5, the peaks of the short periods zone, as 0.18 and 0.253 seconds, for instance, are many times less than the corresponding peaks of the spectrum "A" of the same Fig, (15 and 11 times less, respectively). On the other hand, the peaks in the long periods zone, as 1.189 and 1.395 seconds, for instance, are few times less than the corresponding peaks of the curve "A" of that Fig, (2.36 and 2.0 times less, respectively). So, from the displacements and strains standpoint, the long period waves appear to be more harmful for the structures than those of short period. In fact, according to the spectrum "B" of Fig 5, for periods of 0.8 seconds and up, the displacements may be 10 or more centimeters, which exceeds the resisting capacity of most structures, while, for periods of 0.3 seconds or less, the displacements do not exceed a few millimeters, which is within the resistant capacity of nearly every structure. Provisions concerning the seismic coefficients, in the Japan building Code, as well as those in the Chilean Code, agree with this idea.

Referring to the displacement capacity of the structures, we find that the technic of limit design is advisable because of its simplicity and its plastic base.

CONCLUSIONS

1o.) The acceleration spectrum, based on accelerograms, is not a complete answer to the problem of the actual response of a structure to the seismic action. Plastic damping, which is always present in common structural materials, must be considered, due to its large damping influence.

2o.) The plastic damping may be considered by treating the structure movement as a beating vibration, provided the relation stress-period in the materials (Curve "A" in Fig 4) be known.

3o.) The collapse in structures is mostly caused by displacements cumulation, and not by stresses cumulation. Therefore, the displacements spectrum, referred to the stress-period characteristics of the materials must be known.

4o.) The earthquake-proof structures design must be carried out, in this case, by comparing the probable displacement provoked by the seism, and the structure capacity to resist it.

REFERENCES

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2. Kanai, K., "On the Spectrum of Strong Earthquake Motions", Proceedings of the First Argentine Symposium on Earthquake Engineering, April 1962 (In press).

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APPENDIX A

FORCED VIBRATION OF THE LINTEL OF A SIMPLE FRAME, SUBJECTED TO AN HORIZONTAL DISTURBING FORCE, WHOSE PERIOD IS VERY CLOSE TO THE SELF VIBRATION PERIOD OF THE FRAME (3).

Let K_1 and K_2 be the stiffness of the columns A B and C D (Fig 3-a), respectively, against horizontal displacements of the lintel A B.

In the free vibration of the system, according to D'Alambert's principle, one may write:

1) $$m \ddot{x} + K x = 0 \text{ , being } K = K_1 + K_2$$

1') $$\ddot{x} + p^2 x = 0 \text{ , in which } p^2 = \frac{K}{m}$$

The general solution to this equation is:

2) $$x = C_1 \cos p t + C_2 \text{ sen } p t$$

a vibrational movement of period $T = \frac{2\pi}{p}$

If a disturbing force $H \text{ sen } \Omega t$ is added to the acting horizontal forces of the free vibration, equation 1) becomes:

3) $$\ddot{x} + p^2 x = q \text{ sen } \Omega t$$, in which, $q = \frac{H}{m}$ is the maximum acceleration of m, due to H.

A particular solution to equation 3) is obtained by assuming that x is proportional to sen Ω t, that is to say:

4) $$x = A \text{ sen } \Omega t$$, provided A be a constant value.

By differentiating the equation 4) twice, and combining the result with the equation 3) :

5) $$x = \frac{q \text{ sen } \Omega t}{p^2 - \Omega^2} = x_o \text{ sen } \Omega t \text{ , where } x_o = \frac{q}{p^2 - \Omega^2}$$

Adding to this solution the expresion 2) :

6) $$x = C_1 \cos p t + C_2 \text{ sen } p t + \frac{q \text{ sen } \Omega t}{p^2 - \Omega^2}$$

Assuming settling conditions for $t = 0$, it results $x = 0$, and $\dot{x} = 0$, from where

$$C_1 = 0, \text{ and } C_2 = - \frac{q \Omega}{p (p^2 - \Omega^2)} \text{ , and}$$

6′) $$x = \frac{q}{p^2 - \Omega^2} \left(\text{sen } \Omega t - \frac{\Omega}{p} \text{ sen } p t \right)$$

which means that there are: a free vibration movement of frequency $f_p = \frac{p}{2 \pi}$ and a forced vibration of frequency $f_\Omega = \frac{\Omega}{2 \pi}$.

If the frequency of the free vibration approaches very closely to that of the disturbing force (Imperfect synchronism), the factor of discrepancy, defined as $2\Delta = p - \Omega$, may be neglected in comparison with the value of p or Ω. So, from 6′) we have:

6″) $$x = \frac{q}{p^2 - \Omega^2} (\text{sen } \Omega t - \text{sen } p t) =$$
$$= \frac{2 q}{p^2 - \Omega^2} \cos \frac{(\Omega + p) t}{2} \text{ sen } \frac{(\Omega - p) t}{2} =$$
$$= - \frac{2 q \text{ sen } \Delta t}{p^2 - \Omega^2} \cos \frac{(\Omega + p) t}{2} = - \frac{q \text{ sen } \Delta t}{2 \Omega \Delta} \cos \Omega t$$

Equation 6″) expresses "Beating vibration", whose period is $T_b = \frac{2 \pi}{\Delta}$, and the variable maximum value of its Amplitude:

$$x_{max} = - \frac{q \text{ sen } \Delta t}{2 \Omega \Delta} \quad \text{(Fig 6)}$$

x_{max} reaches its maximum value when sen $\Delta t = 1$, in which case

6‴) $$(x_{max})_o = - \frac{q}{2 \Omega \Delta} \text{ , being}$$

$$q = \frac{H}{m} \text{ ; } \Delta = - (f_\Omega - f_p)\pi \text{ ; } \Omega = 2 \pi f_\Omega \text{ ; from where}$$

$$(x_{max})_o = \frac{q}{4 \pi^2 f_\Omega (f_\Omega - f_p)}$$

When the limit $\Omega = p$ is reached (Perfect synchronism), sen $\Delta t = \Delta t$ and, from 6″) :

7) $$x = - \frac{q t}{2 \Omega} \cos \Omega t$$

Table I shows the values of x versus t. As seen, displacements of the lintel increase $\pi \cdot x$ per cycle. So, in n cycles the amplitude reaches n times the amplitude reached in the first cycle. Obviously, the same is true as regards the acceleration (Fig 7).

TABLE I

t	T/4	T/2	3T/4	T	5T/4	3T/2	7T/4	2T	9T/4
x	0	$-\frac{\pi}{2} x_o$	0	$+\frac{\pi}{2} 2x_o$	0	$-\frac{\pi}{2} 3x_o$	0	$+\frac{\pi}{2} 4x_o$	0

APPENDIX B

VIBRATION PERIOD OF A SIMPLE FRAME VIBRATING SYSTEM, IN THE ELASTO-PLASTIC ZONE

The self vibration period of a simple vibrating system like that of the Fig 3-b, may be expressed by:

$$T = 2\pi \sqrt{\frac{m\ h^3}{\alpha\ E\ I}}$$

where m is the mass, assumed concentrated in the lintel, E is the coefficient of elasticity of the materials, I is the moment of inertia of the columns and α is a coefficient depending on the radio between the stiffness of the columns and that of the lintel.

If the amplitude of the swings is large enough to cause stresses greater than the elastic limit stress, the coefficient of elasticity E becomes smaller and, consequently, the period T lengthens.

In Fig 4, the curve "A" gives the value of the vibration period of the system, along a process of 75 seconds of duration, in which the lintel received axial, steadily increasing, horizontal impulses, synchronized with the instantaneous proper period of the system. During the next 20 seconds of the process, the intensity of the impulses decreased uniformly till cero.

The experiment was made in a structural steel simple frame (Fig 3-c), similar to that of Fig 3-b, in which the column C D was inverted in order to avoid the effect of the eccentricity of the weight, concentrated in B and C. The length and stiffness of the lintel was equal to the length and stiffness of the columns. The elastic limit of the steel was 2 800 kg/cm^2 and the ultimate stress, 4 200 kg/cm^2.

Within the elastic limit, the self vibration period of the system was 0.6 seconds. This value, maintained till the stress reached 2 750 kg/cm^2, at 42 seconds time, began to increase. For 4 200 kg/cm^2, at 75 seconds time, the period was 0.657 seconds. Beyond that point the synchronic impulses diminished uniformly to cero.

The curve "B" in Fig 4 gives the stresses corresponding to curve "A". The curve Cu is similar to curve "A", for a copper frame.

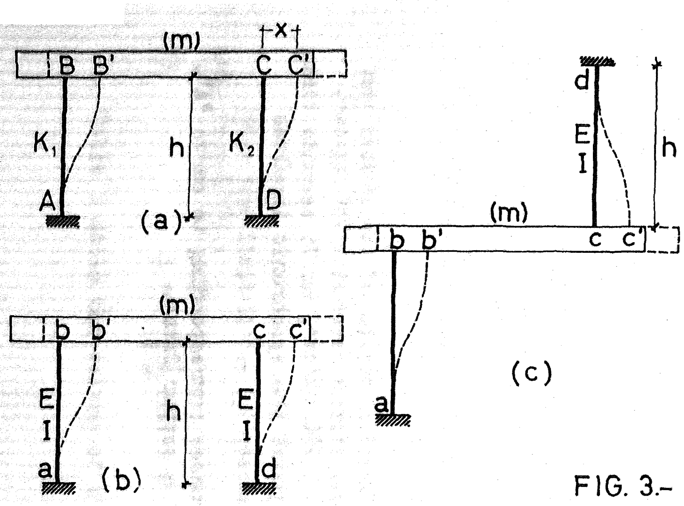


FIG. 3.-

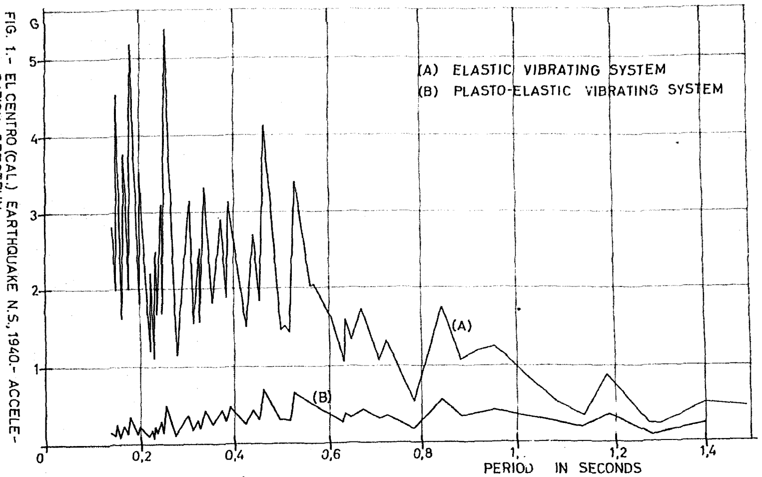


FIG. 1.- EL CENTRO (CAL.) EARTHQUAKE N.S., 1940.- ACCELERATION SPECTRUM.

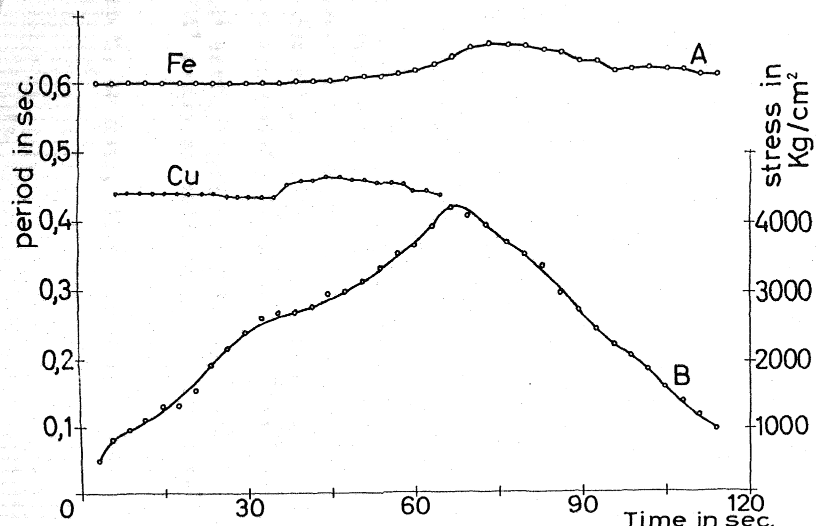


FIG. 4.-

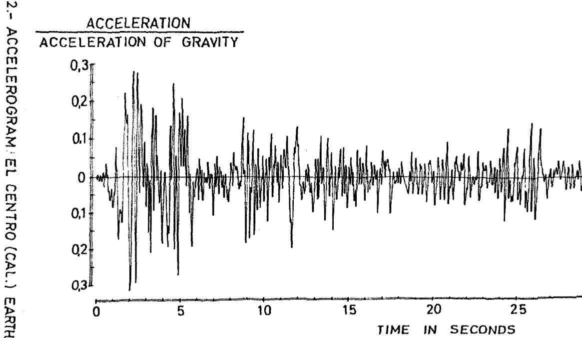


FIG. 2.- ACCELEROGRAM: EL CENTRO (CAL.) EARTHQUAKE, N.S.

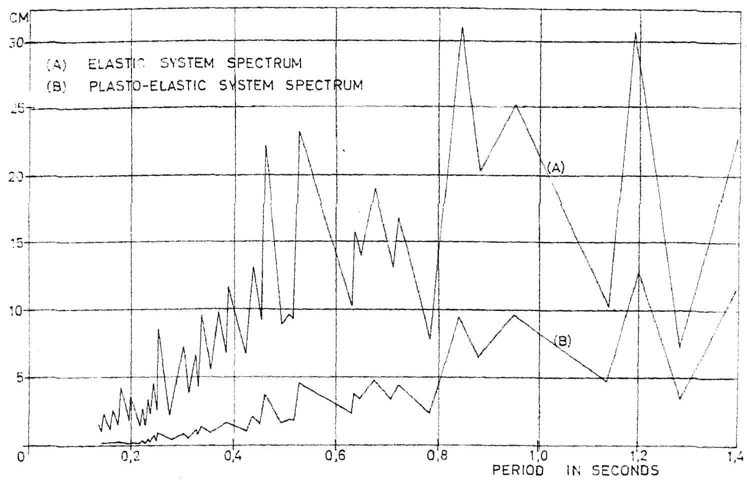


FIG. 5 - EL CENTRO (CAL.) EARTHQUAKE 1940 - DISPLACEMENT SPECTRUM.

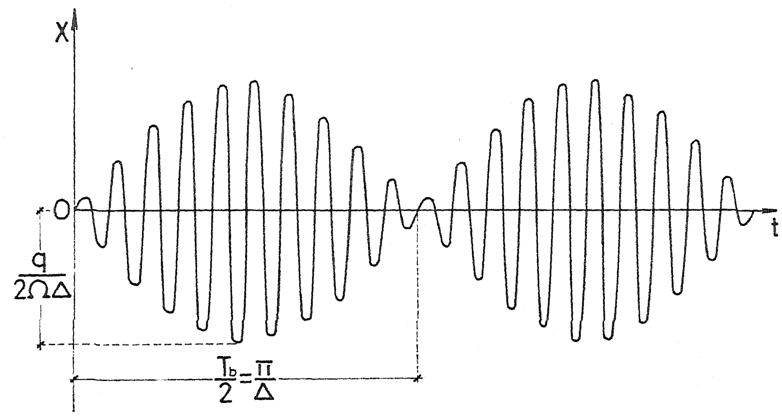


FIG. 6.- Imperfect synchronism $p-\Omega=2\Delta$ (beating vibration)

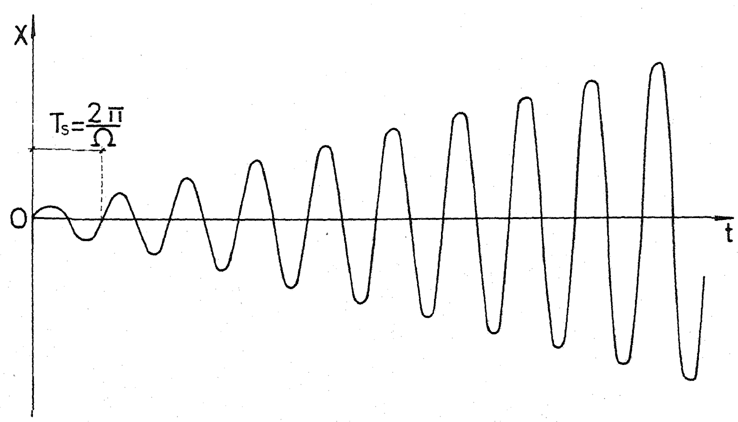
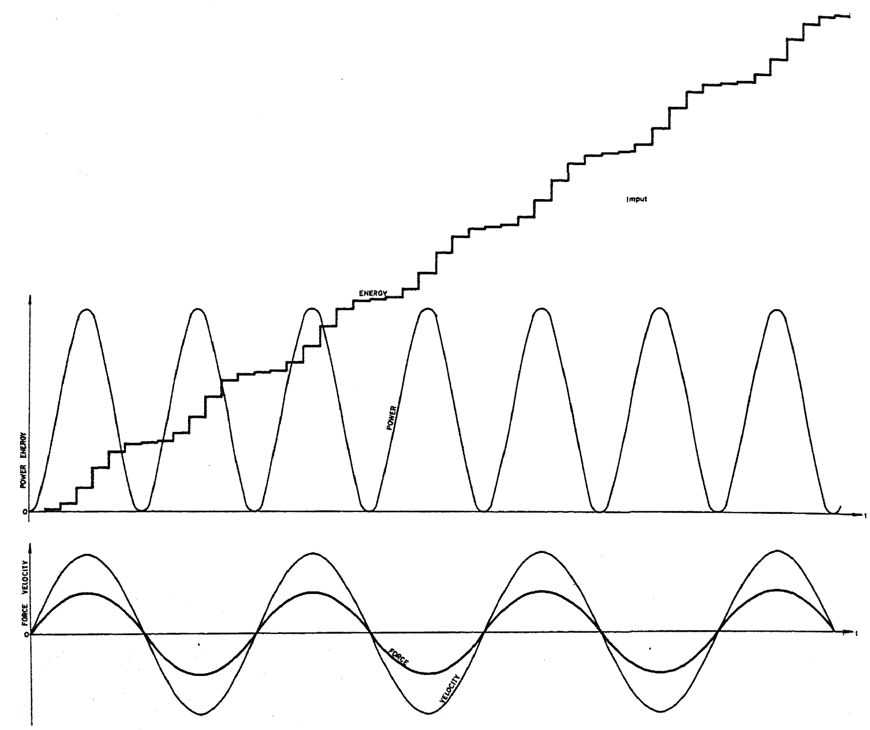
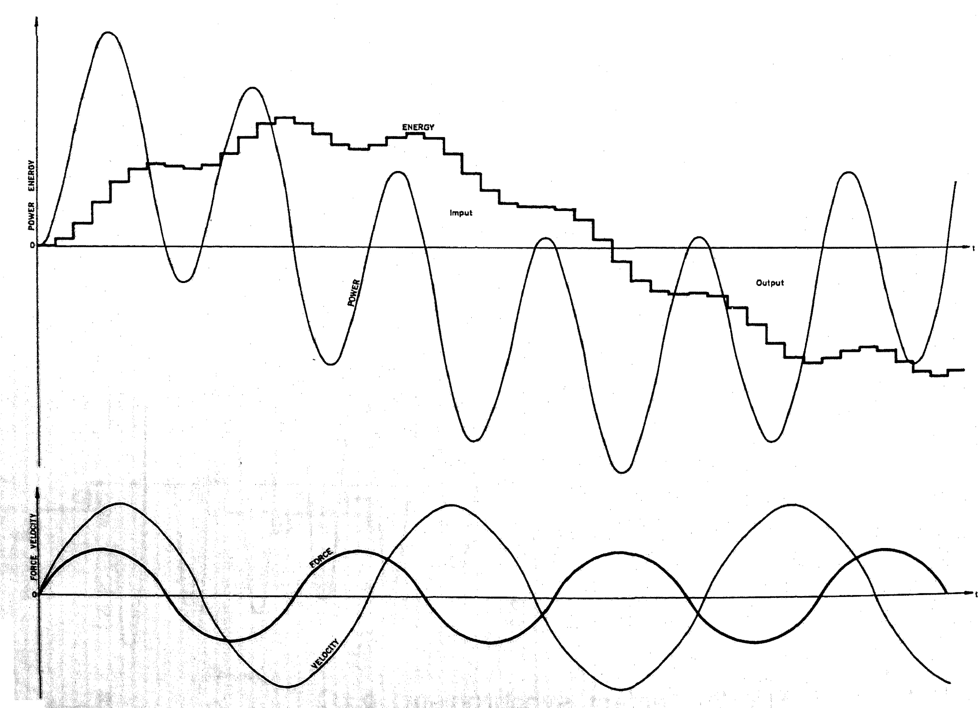


FIG. 7.- Perfect synchronism: $p=\Omega$ (synchronous vibration)


Imput
ENERGY
POWER
POWER ENERGY
0
FORCE VELOCITY
FORCE
VELOCITY
FIG. 8.-


ENERGY
Imput
Output
POWER
POWER ENERGY
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FORCE VELOCITY
FORCE
VELOCITY
FIG. 9.-

E R R A T A

DISPLACEMENT SPECTRUM IN THE DYNAMIC RESPONSE OF INELASTIC STRUCTURES, FOR DESIGN PURPOSES.

BY J. IBANEZ

PAGE 213: After Table I, add as follows:

"Fig. 8 shows the force, velocity, power and energy developed at the contact area of the ground and the foundation, for perfect synchronism conditions. Force and velocity act always in the same direction and sense. In consequence, their product, or, what is the same, the power, is always positive, and a steady flow of energy occurs from the ground to the structure.

Fig. 9 shows the force, velocity, power and energy developed at the contact area of the ground and the foundation, for imperfect synchronism conditions. Force and velocity act, alternatively, in the same and in opposite senses and, consequently, the power is alternatively positive and negative. So, the energy flows to the structure for a while (acceleration time), and is given back to the ground in the next time.

The amount of energy taken from the ground is limited."

DISPLACEMENT SPECTRUM IN THE DYNAMIC RESPONSE OF INELASTIC STRUCTURES, FOR DESIGN PURPOSES

BY J. IBANEZ

QUESTION BY: R.J.P. GARDEN - NEW ZEALAND

The de-tuning action described appears to be a powerful safeguard. The soft strata effects given in Fig. 1 would cause considerable variety in the train of ground waves arriving at the many buildings affected. The many buildings will also display great variety. The effectiveness of de-tuning will presumably vary considerably and it seems not unlikely that there would even be occasions when lengthening period of ground motions would match the lengthening period of the building. This paper may give us a new appreciation of the randomness of earthquake damage. It suggests to the questioner that analyses based on generalised earthquake motions could be of doubtful practical use in building design.

AUTHER'S REPLY: Since the various strata of soft soil, lying on the rocky bed, are supposed to behave as a multi-mode of vibration building, having as many floors as the number of strata, we think that a lengthening of the period of the ground motion may occur, if the stress-strain proportion of the soil disappears. However the foregoing fact does not imply that the said lengthening could match the lengthening of the building period, because this one is of an elusive nature.

QUESTION BY: J. KRISHNA - INDIA

For qualitative results, a generalised approach on the kind of earthquakes and buildings should be permissible, but for quantitative results for a specific case, a separate analysis will be necessary. The method of approach will, however, be indicated by the method suggested by the author.

AUTHOR'S REPLY: Since the acceleration spectrum, as well as the displacement spectrum in this paper, have been derived from a specific accelerogram (that of the "El Centro"), for a specific structure (that of Fig. 3 c), the quantitative results do not apply to other cases. The paper presents only a method of approach.

QUESTION BY: P.W. TAYLOR - NEW ZEALAND

It was suggested by a previous contributor to the discussion (Mr. Garden) that it is illogical to use the same acceleration spectrum for design of buildings on both soft deposits and hard rock. We recognise this, in a general way, when we prefer stiff structures on soft soils and flexible structures on rock. It would be better to modify the design acceleration spectrum according to foundation conditions. Not only the maximum value but also the shape of the curve should be adjusted for local foundation conditions.

AUTHOR'S REPLY: We think that the basic ideas contained in the paper are quite in agreement with what has been suggested by Mr. Taylor. In fact, the seismic phenomenon is treated in the paper as a free, damped, vibration movement of the multi-degree of freedom system formed by the local geological layers, this movement being provoked by the seismic shocks of the rocky bottom.

The self vibration movements, started by every shock, last as long as the damping effect permits, and are wholly integrated in the acceleration and displacement spectra. Peaks in the spectrum must correspond to modes in the soil. Amplification, diminution, and randomness in the accelerograms and seismograms, must be related with the interaction of modes as well as with the occurrences in the seismic focus.

In consequence, this seems consistent with Mr. Taylor's proposal to "modify the design acceleration spectrum according to foundation conditions."