THE DEFLECTION OF TALL BUILDING DUE TO EARTHQUAKE

by
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SYNOPSIS

When we analyse the response of tall buildings due to earthquake, we use an analogue computer or a digital computer. Author studied the foundamental problems about the numerical analysis, the accuracy of the calculation, the effect assumption of multi-mass system, the treatment of accelerograms, and the effect of the nature of structures, by the calculation of examples. Then pointed out that the deflection of tall buildings due to earthquake motion is influenced by the damping and elasto-plastic behaviour of the structure, especially after yielding, effectively.

§ INTRODUCTION

We have carried out many studies on the dynamic analysis on the non-linear problems of tall buildings due to strong earthquake motion by using electric analogue computer. But these analogue computers have small capacity, so we have assumed the tall building to multi-mass system, four or five mass system, and tryed to calculation. Then author has analysed same problems by using electric digital computer. But on the result of calculation about the 33 storied building, he has found the great difference between 33 mass system and 5 mass system, on deflection, displacement, acceleration and shearing force. And so author has studied the foundamental problems of the linear and non-linear response analysis of multi-storied buildings due to strong earthquake motion, considering the necessity of discussion about the accuracy of calculation.

The author used following computers in this study.

1. FACOM-222 FUJI TSUSHINKI JAPAN
   Main core memory: 4000 wards
   Magnetic tape: 2 channels
2. HIPAC-103 HITACHI JAPAN
   Main core memory: 4096 wards
   Inner magnetic drum: 4096 wards
3. I.B.M.-7090 I.B.M. U.S.A.

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§ I RESPONSE ANALYSIS ON VIBRATION OF BUILDING

I.1 EQUATION OF VIBRATION OF MULTI-STORIED BUILDING

In this paper, object of the analysis limited to space frame construction, so assumed the building to multi-mass system. Assumptions are as follows;

Assumption 1 Building is fixed at the foundation to the rigid layer.

Assumption 2 All mass of one floor of building are concentrated to one mass system. See Fig.1

Assumption 3 Resisting against the lateral force of structure is shearing type, and histerisis of the frame is shown in Fig.2, that have yielding point and bi-linear type.

Assumption 4 Damping of structure is viscous damping type, and value of the damping force shown as next expression;

$$\gamma \cdot k \cdot \frac{d\delta}{dt}$$

δ: deflection of story
k: stiffness of story
γ: coefficient of damping

Assumption 5 To consider only one component, which is parallel to the frame of structure.

Then the equilibrium equation of multi-storied building is as follows;

$$\begin{cases} m_n\ddot{Y}_n+C_n\dot{\delta}_n+F(\delta_n)=0 \\ \cdots\cdots\cdots\cdots \\ m_i\ddot{Y}_i+C_i\dot{\delta}_i-C_{i+1}\dot{\delta}_{i+1}+F(\delta_i)-F(\delta_{i+1})=0 \\ \cdots\cdots\cdots\cdots \\ m_1\ddot{Y}_1+C_1\dot{\delta}_1-C_2\dot{\delta}_2+F(\delta_1)-F(\delta_2)=0 \end{cases} \quad \cdots\cdots(1)$$

m_i: mass of i-th story
Ci: damping force of i-th story depended by deflection
δ_i: deflection of i-th story (relative displacement)
suffix i: story number

Shearing force $F(\delta i)$ is shown in Fig.2 and expression(2) illustrates this value.

$$F(\delta i)=k_{is}\delta_i+f_{is} \quad \cdots\cdots(2)$$

Stiffness k_{is} is given by inclination of line of Fig.2, and same time solid friction term f_{is} is given as the intercept to F-axis of this line.

Then put $\ddot{Y}_o+\ddot{y}i=\ddot{Y}_i$, expression(1) is changed form to expression (3).

$$\begin{cases} m_n\ddot{y}_n+C_n\dot{\delta}_n+k_{ns}\delta_n+f_{ns}=-m_n\ddot{Y}_o \\ \cdots\cdots\cdots\cdots \\ m_i\ddot{y}_i+C_i\dot{\delta}_i-C_{i+1}\dot{\delta}_{i+1}+k_{is}\delta_i-k_{i+1\cdot s}\delta_{i+1}+f_{is}-f_{i+1\cdot s}=-m_i\ddot{Y}_o \\ \cdots\cdots\cdots\cdots \\ m_1\ddot{y}_1+C_1\dot{\delta}_1-C_2\dot{\delta}_2+k_{1s}\delta_1-k_{2s}\delta_2+f_{1s}-f_{2s}=-m_1\ddot{Y}_o \end{cases} \quad \cdots\cdots(3)$$

$\ddot{Y}o$: acceleration of earthquake

I.2 NUMERICAL ANALYSIS OF EQUATION

Then explains numerical analysis method of expression(3), for electric digital computer. The first, divid time axis to equal small interval ΔT sec., and derivative of the acceleration is assumed to take constant value during the small interval ΔT.

$$(A_{ij}-A_{i.j-1})/\Delta T=K \quad \text{.........(4)}$$

A_{ij}: acceleration of i-th story at j
suffix j: j-th interval (time)
suffix i: i-th story (position)

$$\ddot{y}_i=K\Delta T+A_{i.j-1}$$
$$\dot{y}_i=K\Delta T^2/2+A_{i.j-1}\Delta T+V_{i.j-1}$$
$$y_i=K\Delta T^3/6+A_{i.j-1}\Delta T^2/2+V_{i.j-1}\Delta T+D_{i.j-1} \quad \text{..........(5)}$$

Then by applying the expression(4)

$$\ddot{y}_i=A_{ij}$$
$$\dot{y}_i=(A_{ij}-A_{i.j-1})\Delta T/2+A_{i.j-1}\Delta T+V_{ij-1}$$
$$=(A_{ij}+A_{i.j-1})\Delta T/2+V_{i.j-1}$$
$$y_i=(A_{ij}-A_{i.j-1})\Delta T^2/6+A_{i.j-1}\Delta T^2/2+V_{i.j-1}\Delta T+D_{i.j-1}$$
$$=(A_{ij}/6+A_{i.j-1}/3)\Delta T^2+V_{i.j-1}\Delta T+D_{i.j-1} \quad \text{.........(6)}$$

V_{ij}: velocity of i-th story at j
D_{ij}: displacement of i-th story at j

$$\dot{\delta}_i=\dot{y}_i-\dot{y}_{i-1}$$
$$\delta_i=y_i-y_{i-1} \quad \text{............(7)}$$

Then by applying the expression(6) and (7) , general expression of the equation(3) is written as expression(8) about i-th story at j.

$$m_iA_{ij}+C_i(((A_{ij}+A_{i.j-1})\Delta T/2+V_{i.j-1})-((A_{i-1.j}+A_{i-1.j-1})\Delta T/2$$
$$+V_{i-1.j-1}))+k_{is}(((A_{ij}/6+A_{i.j-1}/3)\Delta T^2+V_{i.j-1}\Delta T+D_{i.j-1})$$
$$-((A_{i-1.j}/6+A_{i-1.j-1}/3)\Delta T^2+V_{i-1.j-1}\Delta T+D_{i-1.j-1}))-C_{i+1}((A_{i+1.j}$$
$$+A_{i+1.j-1})\Delta T^2/2+V_{i+1.j-1})-((A_{ij}+A_{i.j-1})\Delta T/2+V_{i.j-1}))$$
$$-k_{i+1.s}(((A_{i+1.j}/6+A_{i+1.j-1}/3)\Delta T^2+V_{i+1.j-1}\Delta T+D_{i+1.j-1})$$
$$-((A_{ij}/6+A_{i.j-1}/3)\Delta T^2+V_{i.j-1}\Delta T+D_{i.j-1}))+f_{is}-f_{i+1.s}$$
$$=-m_iA_{gj} \quad \text{...............(8)}$$

A_{gj}: ground acceleration at j

In these expression $A_{i.j-1}$, $V_{i.j-1}$ and $D_{i.j-1}$ are given at previous step, m_i, C_i and ΔT are constants, and A_{gj} is given by input datum at j. Then stiffness k_{is} and friction f_{is} are assumed that they take constant value during small interval ΔT. And so unknown value is Aij only. Number of unknown value is N and there are same number of equation, so Aij are given as the solution of this algbraic equation at each step. Also velocity Vij and displacement Dij are given by the expression(6). In next step these value at j are stored in memory of computer as the value of acceleration, velocity and displacement at j-1, so these solution is obtained step by step.

I.3 ACCURACY OF THE NUMERICAL SOLUTION 1

As one of the problems on the accuracy of the numerical solution author picked up the time interval ΔT. In determination of the time interval ΔT, the smaller value is the better. But number of calculation steps is inversly proportionate to the time interval ΔT, and also occupied time of computer must be required long time. So it is important to find the optimum value of ΔT. In this problem, there are two points to be considered, one is a period of the building and the other is a period of the acceleration of earthquake motion. But the later is more influenced on this problem, because we are considering the tall buildings and they have a long period of the natural vibration.

Then author picked up a 10 story building, for example, and tried to calculation the response of structure due to an actual strong earthquake motion, changing the time interval ΔT. He chose 4 cases, ΔT=0.005 sec., 0.01 sec., 0.02 sec. and 0.03 sec. as the time interval. Fig.3 shows these results of calculation, the case of ΔT=0.03 sec. is omitted, by reason of its strange result. In this figure the case of ΔT= 0.005 sec. is considered most probable and the case of ΔT=0.01 sec. is very resemble to the case of ΔT=0.005 sec. but the other one is not so likely. According these facts, auther determind to use the value of ΔT= 0.01 sec., as optimum value for this earthquake motion.

I.4 ACCURACY OF THE NUMERICAL SOLUTION 2

Second problem is a comparison to result of calculation by using analogue computer. In Fig.4 shows the comparison of the result of calculation by author's method of digital computation and by the analogue computer, that is named SERAC and is set in Tokyo Univercity. An example is a response analysis of 33 story building, that is now planing in Tokyo Japan, during the earthquake EL CENTRO 1940NS. This figure shows us, the result of this method is similar to the result of computation by using analogue computer SERAC.

I.5 ACCURACY OF THE NUMERICAL SOLUTION 3

Next author comparate with a modal analysis method, the problem is limited elastic one. In the modal analysis

$$y_i=\sum_{s=1}^{n} u_{is} q_s \qquad \text{................(9)}$$

Equation(3) is transformed as follows;

$$\ddot{q}_s+2h_s w_s \dot{q}_s+w_s^2 q_s=-\beta_s \ddot{Y}_o \qquad \text{............(10)}$$

h_s: damping ratio of s-th mode
w_s: natural frequency of s-th mode
u_{is}: normal function
q_s: normal coordinate
β_s: amplification of s-th mode
suffix i: story number
suffix s: mode

$$2h_s w_s = \frac{\Sigma Ci(u_{is}-u_{i-1,s})^2}{\Sigma m_i u_{is}^2}$$

$$w_s^2 = \frac{\Sigma ki(u_{is}-u_{i-1\cdot s})^2}{\Sigma m_i u_{is}^2}$$

$$\beta_s = \frac{\Sigma m_i u_{is}}{\Sigma m_i u_{is}^2} \qquad \text{..............(11)}$$

Equation(10) is same form to the equation of one mass system, so that q_s is obtain by same method as expression(6). Then y_i are got by superposition of each mode. See Fig.5 In this case, input data have 6 significant figures and the accuracy of conversion of algebraic equation is also have 6 significant figuers, as far as these accuracy author has not found the error.

§ II TREATMENT OF STRONG EARTHQUAKE ACCELERATION RECORD

Author has got the digital data of strong earthquake *3, which were sent to Tokyo Univercity from Prof. G.V.Berg, and used them in this study, but these data were recorded only maximum and minimum points or singular points of accelerogram. Then he assumed two methods of interpolation, one is linear interpolation method and the other is sine-curve interpolation method, and studied the difference of these two methods, as the result of the response analysis.

In Fig.6 illustrated these two methods, curved line shows an interpolation method by sine-curve and broken line shows a linear one, and points a and b show the recorded points. In Fig.7 shows the actual example, black points are recorded points. Fig.8 shows the difference of these two methods by response spectrum, in this case damping ratio are given as h=0 and h=0.05, and curve A is sine-curve interplation and curve B is linear one. As the result the difference are about 10% between these methods.

The author shows other example in Figures 9,10, and 11, that are the response analysis of 33 storied building *1. In Fig.9 shows the result of 5-mass system and in Fig.10 shows the result of 33-mass system, these are same building, damping coefficient γ is 0.06 and its histerisis is elasto-plastic type. Fig.11 shows same building, but damping coefficient γ is 0.02 and bi-linear type structure which have inclination of F-δ curve after yielded, 10% of elastic inclination. In these cases more large difference appears, in the lower part and upper part of the building, between two methods of interpolation and the difference is 50% in maximum. And then required the accuacy of the strong earthquake motion records.

§ III THE EFFECT OF THE ASSUMPTION OF MULTI-MASS SYSTEM

For the reason of the capacity of the computer or the occupied time of the computer for the calculation, we had assumed the multi-mass system, which have few number of mass, very often, for example the 30 storied building is assumed to the 5 mass system. In these cases there are many difficult problems on the method of one mass corresponds to several stories, especially to calculate the non-linear response of the building due to strong earthquake motion. Then author picked up following 4 methods

I One mass corresponding to one story.

II One mass corresponding to several stories, in this case the mass of assumed system is given as the total mass of the several stories.

III One mass corresponding to several stories, as the same method of SERAC *2.

IV To use the method II in the part of the building, but other part of the building are applied the method I.

Then author shows the example, 33 storied building *1 that is shown the vibrational data of each story in Table 1 type A and the mode and the natural periods in Fig. 12, on the difference of the reduction about the vibration mode and the results of response analysis of storng earthquake motion.

Table 2: The reduction system of multimass system.

Table 3: Period, damping ratio and amplification of each mode.

Figures 14, 17, 18, and 19: The comparison of the method I, II and III.

Figures 15 and 20: The comparison of the method I, IVa and IVb.

Also in Fig.16 shows the response of thedeflection and comparison ofthe method I and II about the other 20 storied building.

As the result it is not adequate to use these reduction, so the multi-mass system must be maked as one mass corresponds to the one story.

§ IV THE EFFECT OF DAMPING AND PLASTIC BEHAVIOUR

We have few data about the damping coefficient and plastic behaviour of building, including frame, slab, wall, and other secondary elements, therefor author carried out trial calculations to finds the effects of these unknown factors.

About the plastic behaviour author changed the value of KB/KA, KA is the elastic stiffness of the structure and KB is the inclination of F-δ curve after yielding, at first he chose 4 cases, KB/KA=0, 0.1, 0.5 and 1 then applied to the model building that is 33storied building *1(Table 1 typeA) and damping coefficient C/k=0.06. In Fig.21 the results of the response analysis are shown.

Damping coefficient C/k is influenced by the secondary elements of the building, especially in tall buildings with steel frame. The value of damping coefficient C/k may be taken from 0.005 to 0.06. Then author chose 5 cases, C/k=0.005, 0.01, 0.02, 0.04 and 0.06, and used the same model building (Table 1 type A, 33 storied building KB/KA=0.1) tried the analysis. The results of the analysis are shown in Fig.22.

About the other building, it is also 33 storied building (Table 1 type B , and in Fig.13 shows the modes and periods of this building), author picked up several cases, and he chang the earthquake motion *3, tried to the analysis and shown the results of deflection in Figures 23, 24, 25, 26 and 27.

These problems give us any conclusion not yet, but these figures give us some suggestion, that we must research these facts for calculate the response of tall building in future, and we must take care of to determind the data of buildings to the response analysis.

§ CONCLUSION

By this study author gets some conclusion as follows;

1. Numerical analysis method for the response analysis of space frame structure due to strong earthquake motion is completed.
2. In this calculation time interval ΔT is very impotant and as this value $\Delta T=0.01$ sec. is adequate.
3. Interpolation and read method of accelerogram is influence to the result of calculation and so acceleration records of earthquake must be more accurate in time.
4. Multi-storied buildings can be analysed multi-mass system but the number of the masses must be taken the same number of the story of building.
5. The damping coefficient and plastic behaviour after yielding of structure influence to the result of analysis, and so take care of to determind these data for the response analysis.

ACKNOWLEGMENTS

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REFERENCES

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2. Ditto , SERAC Report No.2, p.24, June 1962.

3. G.V.Berg; The acceleration records of strong earthquake of U.S.A. (I.B.M. cards)

TABLE 1

story	TYPE A mass ton	TYPE A height cm	TYPE A stiffness ton/cm	TYPE A yielding strength ton	TYPE B mass ton	TYPE B height cm	TYPE B stiffness ton/cm	TYPE B yielding strength ton
33	4639	360	1300	1058	3926	384	1316	895
32	1979	360	1300	1459	2402	384	1316	1399
31	1979	360	1380	1833	2435	384	1316	1867
30	1979	360	1520	2178	2410	384	1328	2302
29	1979	360	1520	2489	2410	384	1328	2690
28	1979	360	1610	2772	2465	384	1482	3049
27	1979	360	1770	3023	2447	384	1656	3385
26	1979	360	1770	3245	2447	384	1656	3665
25	1979	360	1890	3436	2486	384	1759	3936
24	1979	360	1930	3599	2471	384	2035	4144
23	1979	360	1930	3795	2487	384	2035	4400
22	1979	360	2200	3971	2512	384	2106	4635
21	1979	360	2270	4127	2499	384	2244	4843
20	1979	360	2270	4264	2499	384	2244	5026
19	1979	360	2390	4380	2537	384	2287	5189
18	1979	360	2460	4477	2519	384	2384	5324
17	1979	360	2460	4555	2519	384	2384	5434
16	1979	360	2550	4628	2562	384	2429	5524
15	1979	360	2680	4683	2546	384	2538	5636
14	1979	360	2710	4719	2546	384	2538	5675
13	1979	360	2780	4741	2579	384	2538	5747
12	1979	360	2860	4762	2563	384	2723	5740
11	1979	360	3050	4784	2563	384	2723	5766
10	1979	360	3050	4806	2598	384	2746	5953
9	1979	360	3050	4828	2575	384	2790	5761
8	1979	360	3070	4849	2575	384	2790	5859
7	1979	360	3070	4871	2612	384	2822	5882
6	1979	360	3070	4893	2602	384	2881	5959
5	1979	360	3140	4915	2602	384	2881	5952
4	1979	360	3140	4937	2622	384	2937	6008
3	2223	360	3140	4961	2582	384	3039	5970
2	2873	360	4720	5176	2567	384	3956	6163
1	3321	360	4720	5426	2497	384	3303	6350

TABLE 2

method	I	II			III	IVa	IVb
number of mass	33	17	11	8	5	30	29
story							
33	33	17	11	8	5	30	29
32	32	16				29	28
31	31						27
30	30	15	10			28	26
29	29						25
28	28	14		7		27	24
27	27		9		4		23
26	26	13				26	22
25	25					25	21
24	24	12	8	6		24	20
23	23					23	19
22	22	11				22	18
21	21		7		3	21	17
20	20	10		5		20	16
19	19					19	15
18	18	9	6			18	14
17	17					17	
16	16	8		4		16	13
15	15		5			15	
14	14	7			2	14	12
13	13					13	
12	12	6	4	3		12	11
11	11					11	
10	10	5				10	10
9	9		3			9	9
8	8	4		2		8	8
7	7				1	7	7
6	6	3	2			6	6
5	5					5	5
4	4	2		1		4	4
3	3		1			3	3
2	2	1				2	2
1	1					1	1

TABLE 3

number of mass	mode	33	17	11	8	5	30	29
natural period	1	3.73	3.79	3.86	3.94	3.76	3.75	3.75
sec.	2	1.38	1.40	1.45	1.50	1.43	1.40	1.38
	3	0.83	0.85	0.88	0.83	0.94	0.84	0.83
damping ratio	1	0.050	0.050	0.048	0.048	0.050	0.050	0.050
h	2	0.137	0.134	0.130	0.125	0.131	0.134	0.137
	3	0.227	0.223	0.214	0.203	0.200	0.226	0.229
amplification	1	1.35	1.34	1.34	1.34	1.34	1.34	1.34
	2	-0.53	-0.52	-0.51	-0.49	-0.50	-0.52	-0.52
	3	0.30	0.29	0.27	0.21	0.23	0.28	0.31

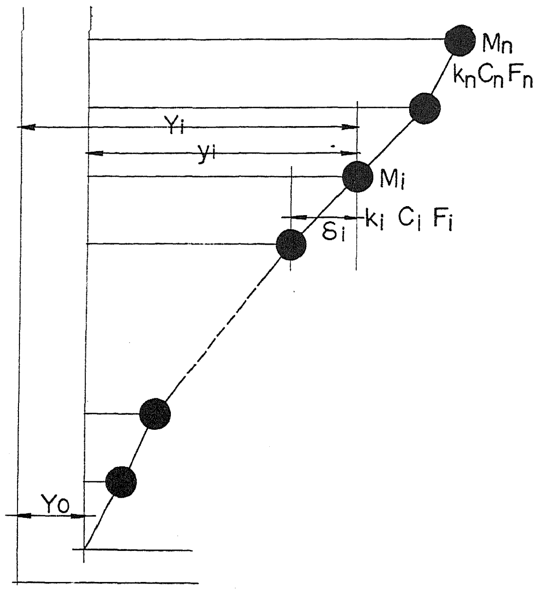


FIG. 1

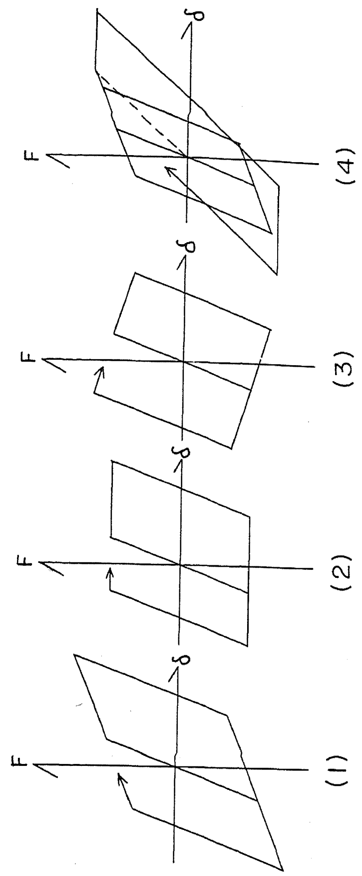
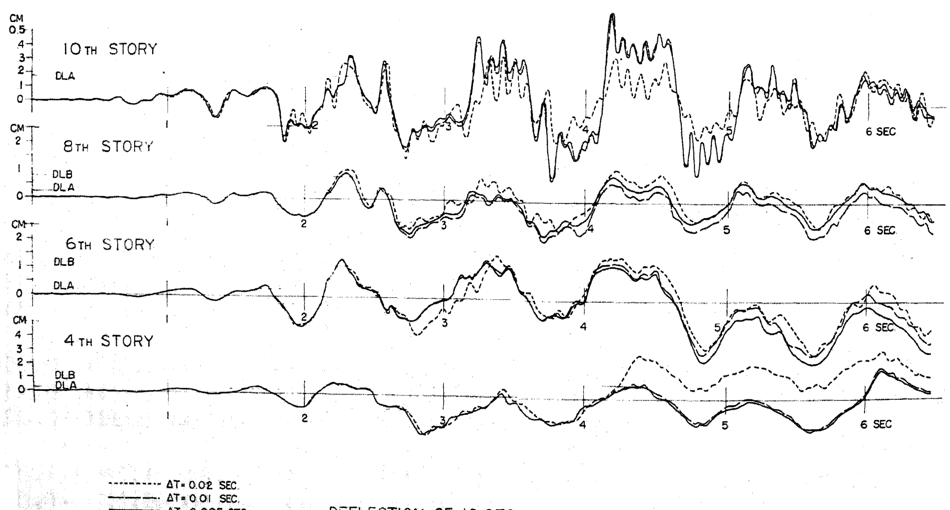


FIG. 2



DEFLECTION OF 10 STORY BUILDING
EL CENTRO 1940 NS

FIG. 3

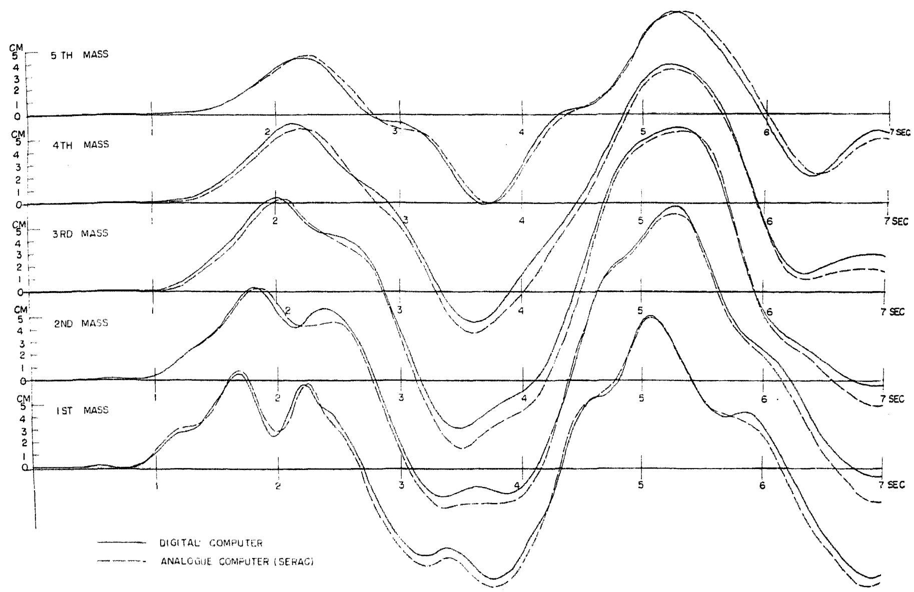


FIG. 4 DEFLECTION OF 33 STORY BUILDING
5 MASS EL CENTRO 1940 NS

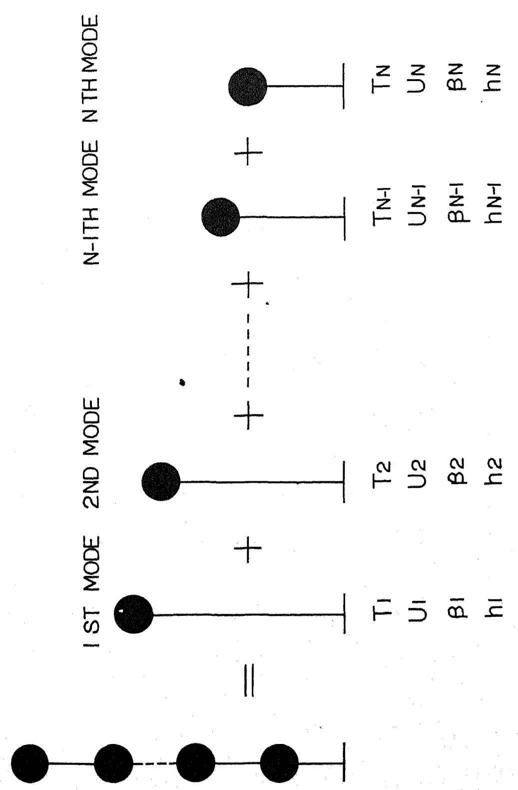


FIG. 5

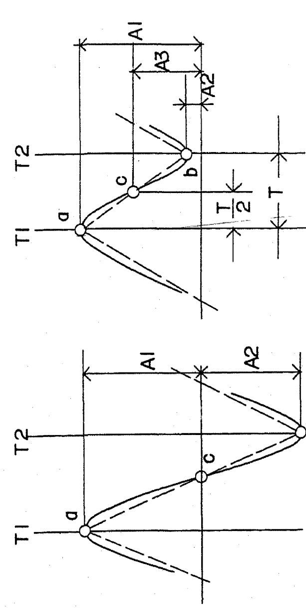
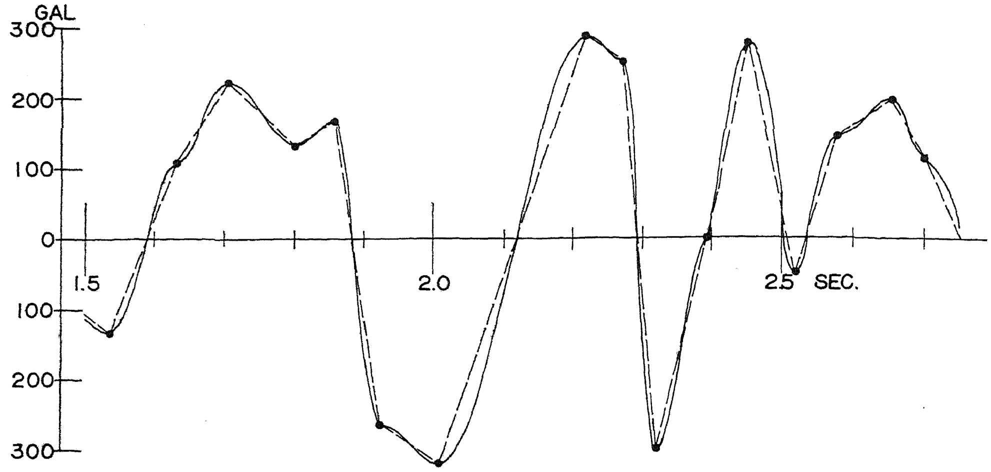
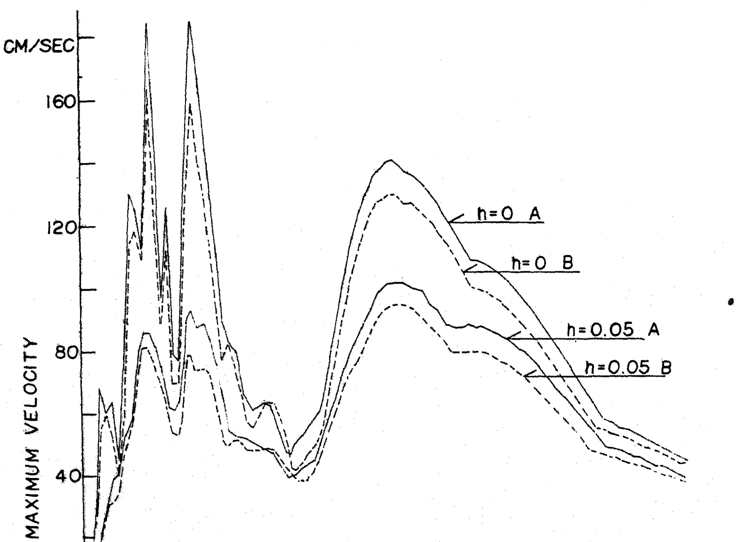


FIG. 6



EL CENTRO 1940 NS
FIG. 7



RESPONSE SPECTRUM
EL CENTRO 1940 NS
FIG. 8

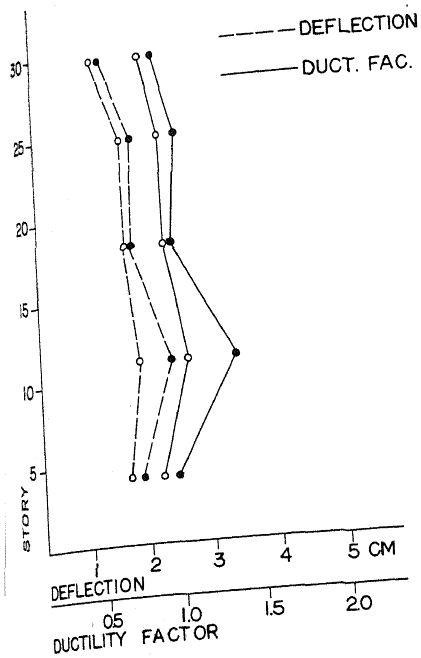


FIG. 9

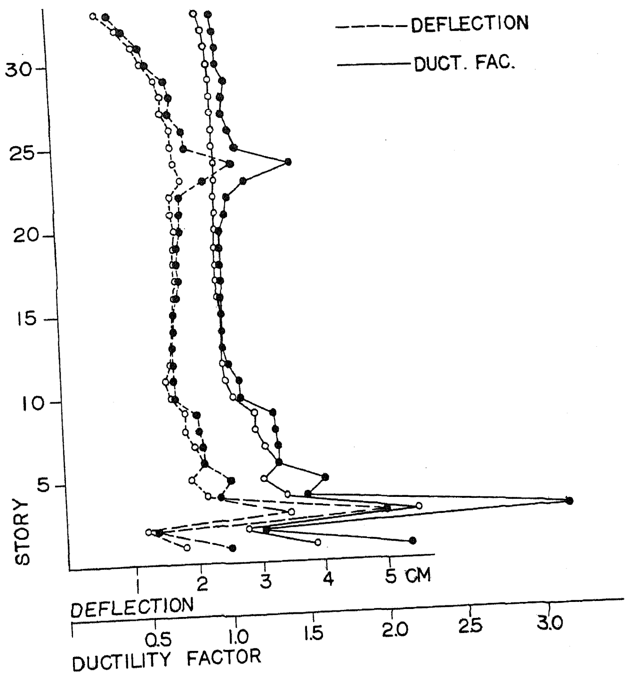


FIG. 10

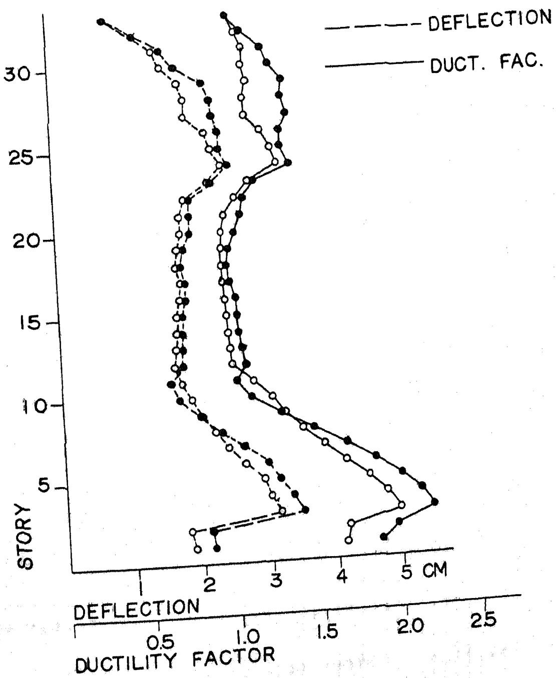
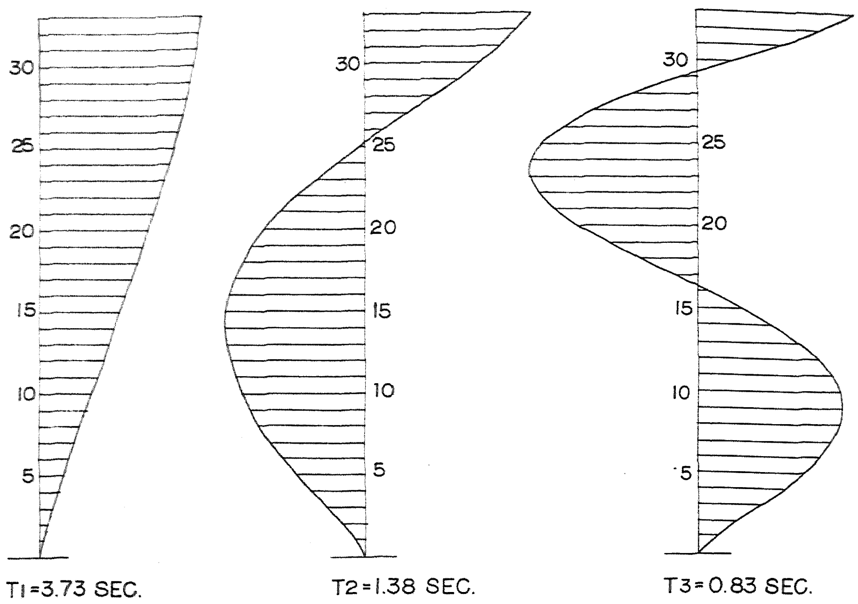
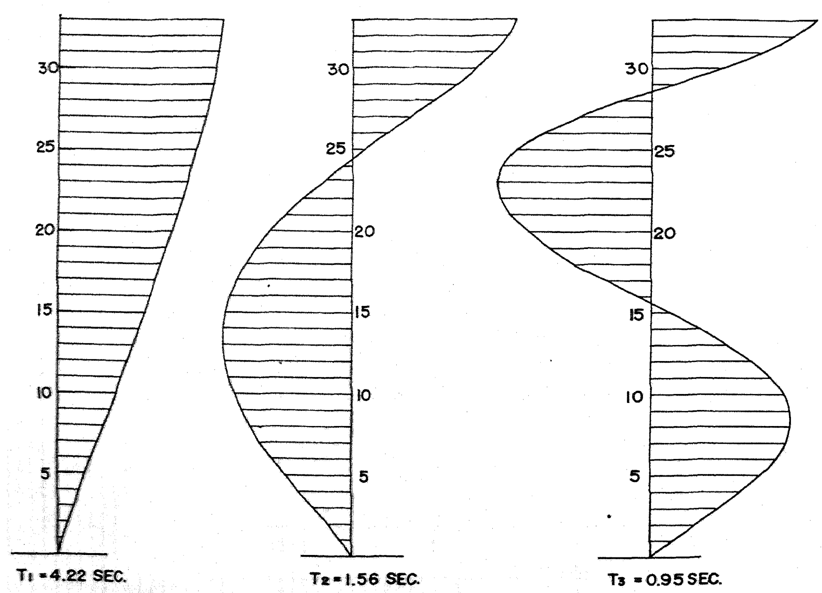


FIG. 11



MODE OF 33 STORY BUILDING

FIG. 12



MODE OF 33 STORY BUILDING

FIG. 13

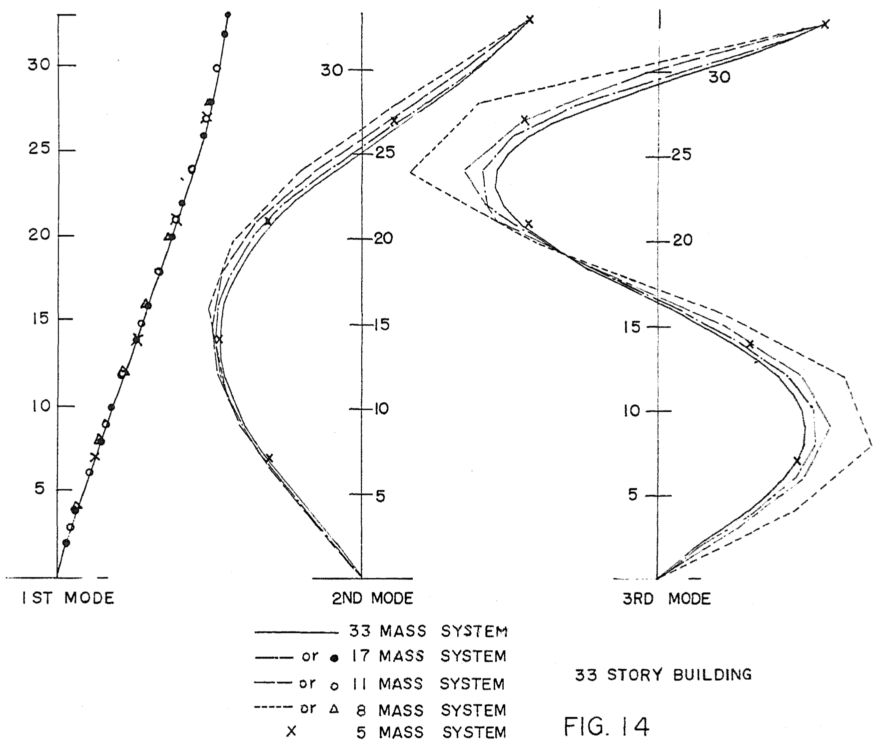

30
25
20
15
10
5
1ST MODE
30
25
20
15
10
5
2ND MODE
30
25
20
15
10
5
3RD MODE
33 MASS SYSTEM
or ● 17 MASS SYSTEM
or ○ 11 MASS SYSTEM
or △ 8 MASS SYSTEM
X 5 MASS SYSTEM
33 STORY BUILDING

FIG. 14

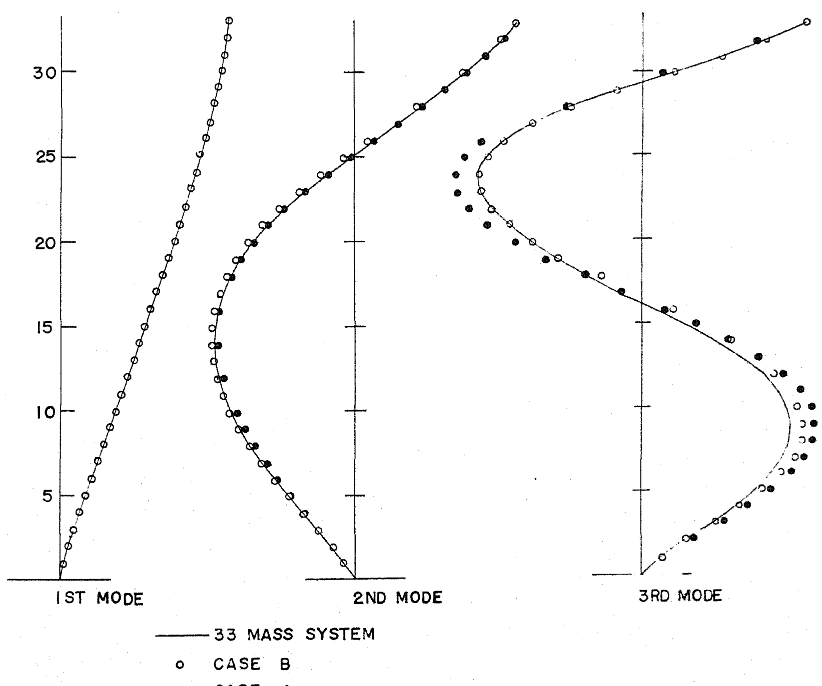
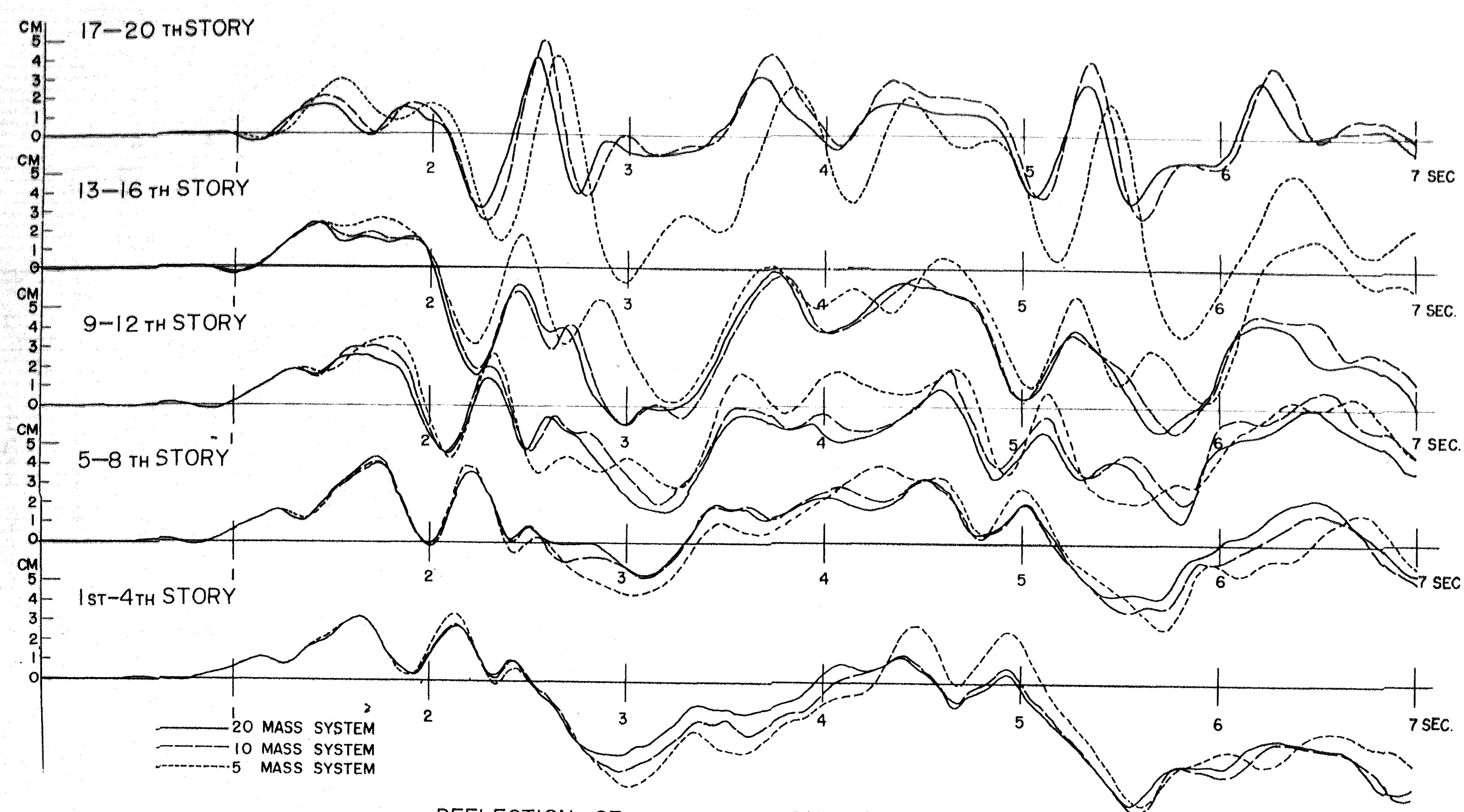
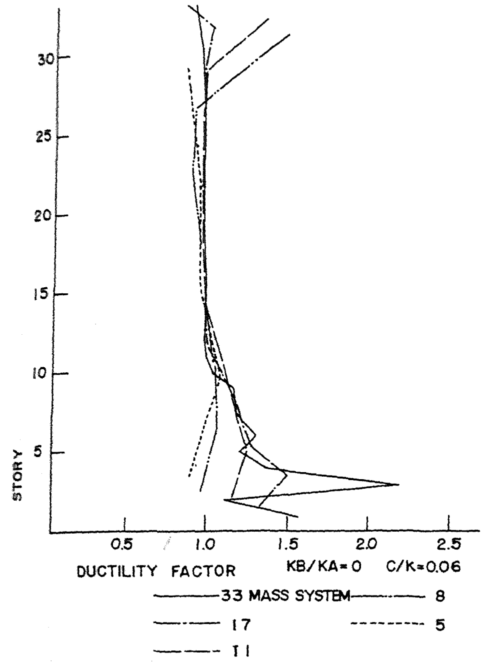

30
25
20
15
10
5
1ST MODE
2ND MODE
3RD MODE
33 MASS SYSTEM
○ CASE B
● CASE A
33 STORY BUILDING

FIG. 15



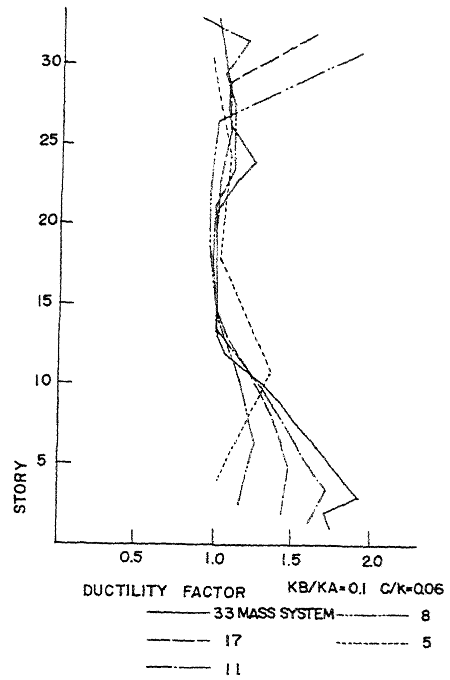
DEFLECTION OF 20 STORY BUILDING
EL CENTRO 1940 NS

FIG. 16



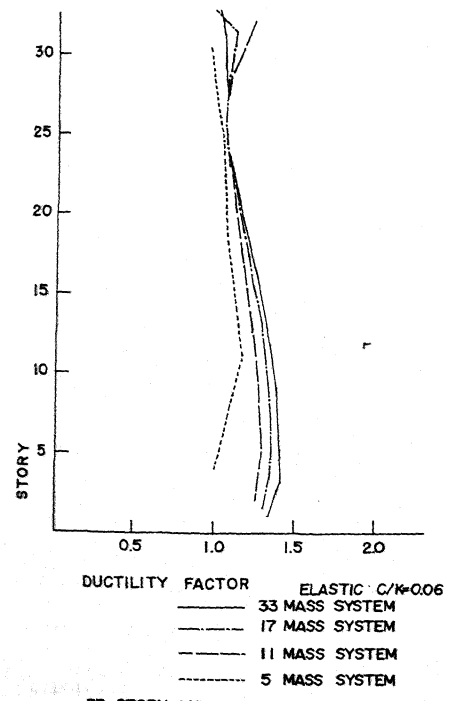
33 STORY BUILDING EL CENTRO 1940 NS (STR)

FIG. 17



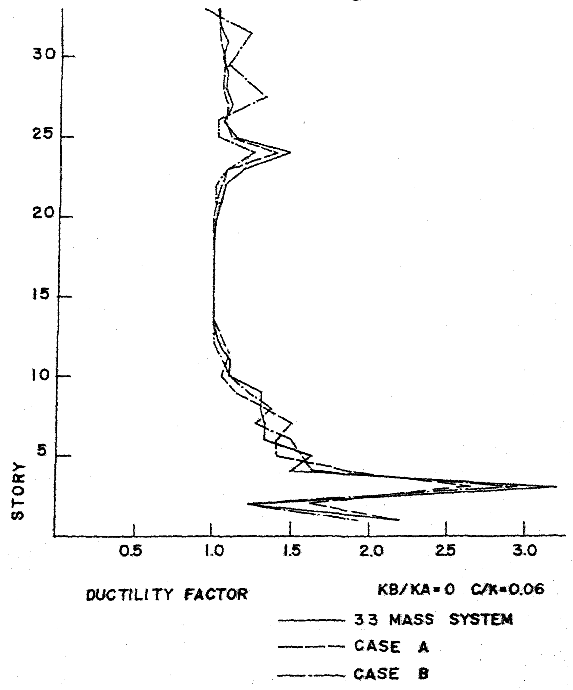
33 STORY BUILDING EL CENTRO 1940 NS

FIG. 18



33 STORY BUILDING EL CENTRO 1940 NS

FIG. 19



33 STORY BUILDING EL CENTRO 1940 NS

FIG. 20

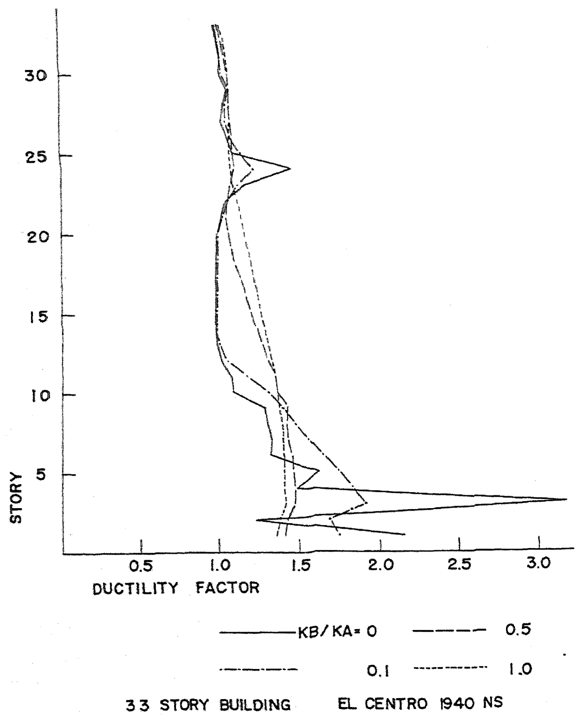


FIG. 21

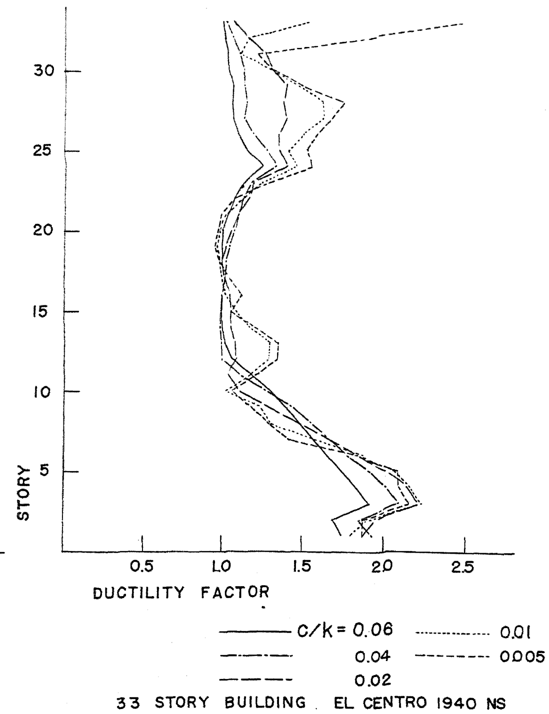


FIG. 22

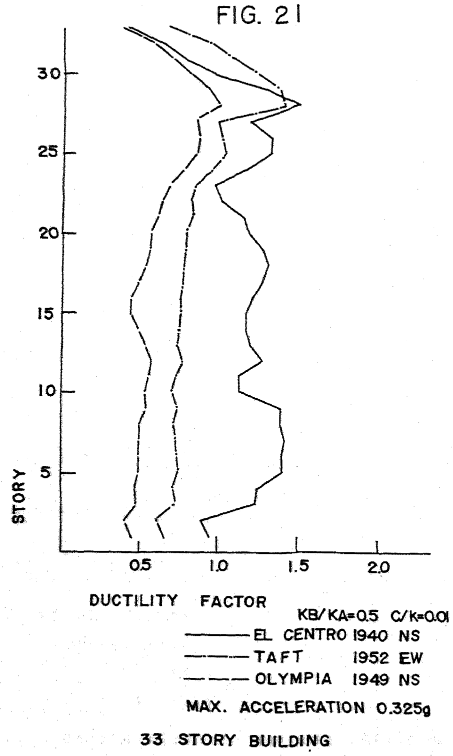


FIG. 23

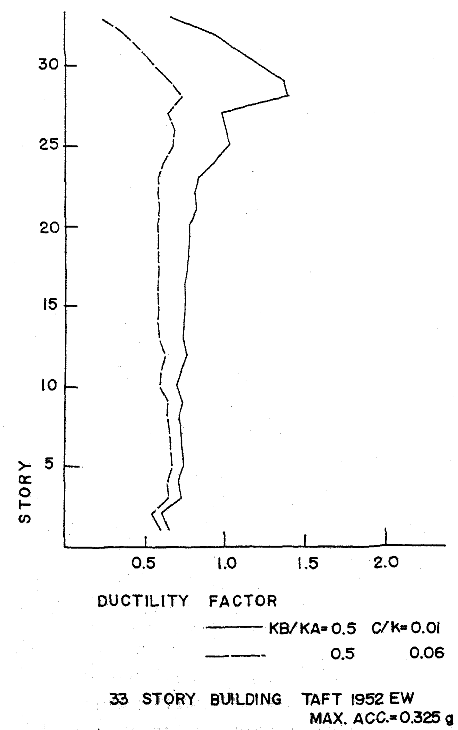


FIG. 24

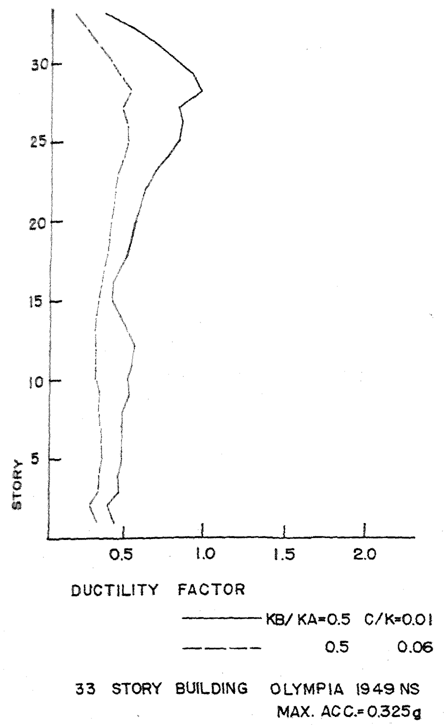


FIG. 25

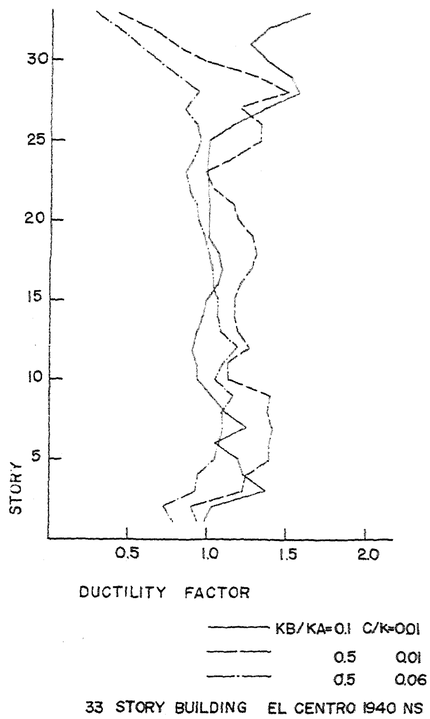


FIG. 26

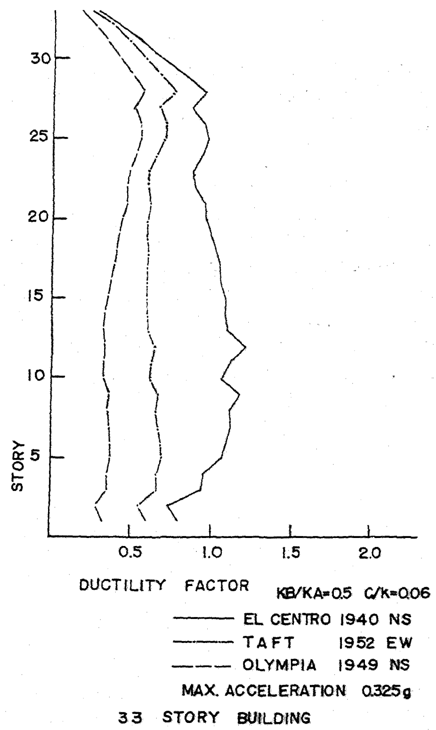


FIG. 27

THE DEFLECTION OF TALL BUILDING DUE TO EARTHQUAKE
BY H. KOBAYASHI

QUESTION BY: K.L. BENUSKA - U.S.A.

In Fig 22, you snow the resulting ductility factors with the varying damping coefficient. There is an irregularity in the sense that as you increase the damping coefficient you have crossover parts. In some stories you are increasing the ductility requirements and some decreasing. It is not a regular decrease. Could you comment on what the graph of regular story displacement would look like without making the correction to get ductility. Would it also be an irregular type of variation? It is my understanding that you definition of yield displacement varies as you go up the structure. Is that correct? i.e. If you increase damping does the sidesway response decrease in a uniform manner or does it also exhibit a rather erratic response. If you had recorded displacement rather than ductility factor would the curves have decreased with increased damping in a more regular manner than the story ductility factor did?

AUTHOR'S REPLY: You asked me to comment on the graphs of relative storey displacement, however, these graphs have similar trends to Fig. 22. I suppose you are concerned with elastic problems. In elasto-plastic problems, it is often shown that, if the lower part of the building reaches yield, seismic effects are not propagated to the upper part of the structure.

QUESTION BY: R.W. CLOUGH - U.S.A.

You made a comment that you tried two different methods of interpolation of the given accelerogram data. One of these was a sine curve interpolation. Could you explain what you did in this type of interpolation? Are you introducing zero slope corrections at the intermediate points in Fig 6? For instance, does your 2nd point from the left on the graph have zero slope at that point?

AUTHOR'S REPLY: In Fig. 6 a and b accelerogram data are given. Acceleration is given by the following expressions.

From a to c; case (1): $ACC = A1 . \cos\left(\frac{\pi}{2} . \frac{t-T1}{Tc-T1}\right)$

case (2): $ACC = A3 + (A1-A2) \quad . \cos\left(\frac{\pi}{2} . \frac{t-T1}{T/2}\right)$

From c to b; case (1): $ACC = A2 . \sin\left(\frac{\pi}{2} . \frac{t-TC}{T2-Tc}\right)$

case (2): $ACC = A3 + (A1-A2) \quad . \cos\left(\frac{\pi}{2} . \frac{t-T1}{T/2}\right)$

Tc: Time at c   t; Time