A STUDY ON THE OPTIMUM VALUE OF A SEISMIC COEFFICIENT

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INTRODUCTION

It is known that in the non-linear vibration phenomena of the structure caused by earthguake moticn, the fraction of critical damping, for the hysterestical restoring force due to characteristics of structures has a upper bound. That is, in the case that the yield shearing force of structure is small, the displacement and the ductility factor are large, and as the yield shearing force increases, the displacement and the ductility factor decrease. On the other hand, in the case that the yield shearing force is larger than the certain value, which is different depending upon the characteristics of structure and earthquake motions, as the yield shearing force increase, the displacement also increases and ductility factor decreases.

The yield shearing force coefficient, in case that the displacement is minimum is called the optimum yield seismic coeficicnt. This property means that in the case that the yield shearing force coeficient is larger than the optimum yield seismic coefficient, the elastic displacement is dominant, so the damping effect in the plastic range is small. While in the case that the yield shearing force is smaller than the optimum yield seismic coefficient, the damping effect in the plastic range is large and then the displacement is small.

This optimum yield seismic coefficient was discovered experiment-ally in a study on the non-linear transient response of structures caused by earthquake motions (1), (2).

H. Tajimi confirmed analytically this character in the non-linear steady state phenomena (3). H. Tajimi evaluated the response of the one and two mass systems, using "the method of slowly varing parameters" by T.K.Caughey (4). In the two mass system, however, he evaluate the response only in such a case as only either the 1st story or the 2nd story vibrates non-linearly.

On calculating the yield seismic coefficient or yield shearing force coefficient based on the non-linear response of structures, we have to notice the following points.

1) How many ratio of the plastic displacement to the yield displacement of structures is allowed? That is, in order to evaluate the above quantities, we have to know the ductility factor and the degree of failure.
2) There is a lower limit of the total displacement of structures, we have to know the optimum yield seismic coefficient.

In the experiment of repeating leading on the reinforced concreate frame of one span and one story, the following tendency, we can see (5)

1) As the compression stress in column under virtical loading increases, the ductility factor decrease (Fig.1.1).
2) The ductility factor can not take a large value.

Tho restoring force characteristics of structure is not such a idealized type as perfect plastic or bi-linear type and they are rather a spindly type. This is concerned in the effect that the resudal displacement decreases, as the compression stress in column under virtical loading increases.

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In a study on the response of one mass system, in which we assume restoring force characteristics of structure to be perfectly plastic or a bi-linear type, under rectangular shock wave as proposed by T. Kobori, the ductility factor corresponding to the optimum yield seismic coefficient is small.

From this point of view, it is desirable that the yield seismic coefficient or yield shearing force coefficient in seismic design are determined in relation with the ductility factor so as not to be below the optimum yield seismic coefficient. The optimum yield seismic coefficient found from the structural response due to the actual earthquake motions depends on a particularity of those.

In this paper, considering this property of the optimum yield seismic coefficient, we compute this value under a clear condition. We approximate the earthquake motions as a sinusoidal wave. We treate a problem of resonance in non-linear steady state vibrations. Then, if we consider only the 1st order resonance, non-linear systems with bi-linear hysteresis may be replaced approximately by linear systems with a equivalent viscous damping effect. The eigen value in such a non-linear vibration can be calculated by solving the boundary value problems in the Galerkin's method, and their displacement responses is given by applying their eigen value to conservative law of dynamical energy.

We prove there is few difference between the optimum yield seismic coefficients gained in steady state and transient vibrations state. This means that the transient vibration state can be replaced by steady state. Using this results, we shall be able to discover the optimum value from the response for a different ratio of mass, elastic rigidity and yield displacement in the case of many mass system.

METHOD OF ANALYSIS

In the case that the structures have the restoring force characteristics as indicated in Fig.2.1, we assume that the vibration with be-linear hysteresis can be approximated by the linear vibration with a equivalent viscous damping effect on diagonal line BOE which corresponds to hysteresis area ABCDEFA. Then, it is enough to consider only the first order resonance, and the effect of higher order can be neglected and the displacement of S th-story is evaluated approximately by the following equation;

$$x_S \simeq X_S \sin(\tau+\phi), \quad X_S \cos(\tau+\phi) \qquad (2.1)$$

Where x_S, X_S, ω, t and ϕ are the relative displacement of Sth-story, the maximum amplitude in steady state vibration, the circular frequency of steady state earthquake motion, the time variable and the phase angle, respectively. And τ is defined by $\tau = \omega t$.

From eq.(2.1), we know that there is only few difference between each phase angles of the relative displacements of each story. Further, we classify the non-linear vibrations into the following three case;

$$|X_s| \leqq x_{sy} \quad \text{for} \quad 1 \rightleftarrows 0 \rightleftarrows 2 \quad \text{in Fig. 2.1}$$

$$x_{sy} \leqq |X_{s\alpha}| \leqq 2x_{sy} \quad \text{for} \quad a \to b \to c \to d \to e \to f \quad \text{in Fig. 2.1}$$

$$|X_{s\beta}| \geqq 2x_{sy} \quad \text{for} \quad A \to B \to C \to D \to E \to F \quad \text{in Fig. 2.1}$$

And we assume the relation between the displacement x_s and the time τ as in Fig.2.2. Under the initial condition $x_s = 0$ for $\tau = 0$, the condition of half period, and $x_s = 0$ and $\tau = \pi$, and the condition after one period and $x_s = 0$ and $\tau = 2\pi$, the progress of a vibration on hysteresis curve is as follows;

a $\xrightarrow{\text{elastic}}$ b $\xrightarrow{\text{plastic}}$ c $\xrightarrow{\text{elastic}}$ d, and repeat cyclically every half period, for $x_{sy} \leqq |X_{s\alpha}| \leqq 2x_{sy}$

A $\xrightarrow{\text{plastic}}$ B $\xrightarrow{\text{elastic}}$ C $\xrightarrow{\text{plastic}}$ D, and repeat cyclically every half period for $|X_{s\beta}| \geqq 2x_{sy}$

Where $X_{s\alpha}$ and $X_{s\beta}$ is the relative maximum displacements of Sth-story and x_{sy} is the yield displacement of Sth-story. The equation of motion of r mass system is given as follows;

$$L_s = m_s(\ddot{x}_1 + \cdots + \ddot{x}_s) - C_{s+1}\dot{x}_{s+1} + C_s\dot{x}_s - f_{s+1}(x_{s+1}) + f_s(x_s) + m_s\ddot{Z} = 0 \quad (S = 1, 2, \cdots, r) \qquad (2.2)$$

Where m_s is the mass of the Sth-story, Cs is the viscous damping coefficient, $f_s(x_s)$ is the hysterestical restoring force of the Sth-story and $\ddot{Z}$ is the earthquake acceleration. Using $\tau = \omega t$, eq.(2.2) is rewritten as follows;

$$L_s = m_s\omega^2(\mathring{\mathring{x}}_1 + \cdots + \mathring{\mathring{x}}_s) - C_{s+1}\omega\mathring{x}_{s+1} + C_s\omega\mathring{x}_s - f_{s+1}(x_{s+1}) + f_s(x_s) + m_s\omega^2\mathring{\mathring{Z}} = 0 \quad (s = 1, 2, \cdots, r) \qquad (2.3)$$

$$\text{where} \quad \mathring{x}_s = \frac{dx_s}{d\tau}, \quad \mathring{\mathring{x}}_s = \frac{d^2x_s}{d\tau^2}$$

The solution for the 1st order resonance of eq.(2.2) is appromated by eq.(2.1). Determining the arbitrary constants by $x_s = 0$ for $\tau = 0$ and $x_s = 0$ for $\tau = \pi$, we can solve the steady state vibration as the problems of boundary values. That is, by taking $\phi = 0$ in eq.(2.1), we get a solution of eq.(2.2) as follows;

$$x_s \simeq X_{s\alpha}\sin\tau, \; X_{s\beta}\sin\tau \quad (s = 1, 2, \cdots, r) \qquad (2.4)$$

Then earthquake motion displacement Z is given by

$$Z = a\cos\tau \qquad (2.5)$$

That is because, in the resonance of linear steady state vibration with a viscous damping, phase angle difference between the earthquake displacement and the displacement of structure is about $\frac{\pi}{2}$, then we can replace the eq.(2.3), according to the Galerkin's method, by the following integral equation.

$$\int_0^{\pi} L_s \sin\tau \, d\tau = 0 \quad (s = 1, 2, \cdots, r) \qquad (2.6)$$

As the term including $\mathring{\mathring{Z}}$ vanishes all times, we are able to compute theeigen values ω^2 of $X_{s\alpha}$, $X_{s\beta}$ from eq.(2.6). Of course, ω^2 is the square of the circular frequency in the equivalent linear system.

In order to determine the steady state displacement, we utilize the conservative law of dynamical energy. That is, the equation of motion eq.(2,3) is the equation of equilibrium of dynamical force in every story of the structure. Therefore, according to a principle of virtual work, when a minute displacement is given by:

$$dx_s = X_{s\alpha}\cos\tau\, d\tau\ ,\quad X_{s\beta}\cos\tau\, d\tau \tag{2.7}$$

The work by vibrational system is nearly zero.

However, as the eq.(2,4) is an appromimate equation, the work is not strictly zero, and we must make the average of work in one period to be zero. That is, when $X_{s\alpha},\ X_{s\beta} \neq 0$,multiplying eq.(2,3) by eq.(2,7) and integrate by τ from 0 to 2π , we get

$$\int_0^{2\pi} L_s(\ddot{Z}, x_s, \dot{x}_s, \ddot{x}_s, \tau)\cos\tau\, d\tau = 0 \qquad (s = 1, 2, \cdots, r) \tag{2.8}$$

In conclusion, we can compute the eigen value and the steady state displacement from eq.(2.6), (2.8).

ANALYSIS FOR ONE MASS SYSTEM

<u>Method of Analysis.</u> - The equation of motion for the system is given by

$$L = m\ddot{x} + c\dot{x} + f(x) + m\ddot{Z} = 0 \tag{3.1}$$

Replacing time t by $\tau = \omega t$, and applying eq.(2.5) as the earthquake motion, we get

$$L = m\omega^2\ddot{x} + c\omega\dot{x} + f(x) - ma\omega^2\cos\tau = 0 \tag{3.2}$$

Therefore, assuming that the solution of eq.(3.3) is given by eq.(2.4), we get

$$x = X_\alpha \sin\tau\ ,\quad X_\beta \sin\tau \tag{3.3}$$

From eq.(2.6) and (2.8), the equation of eigen value and the steady state displacement are introduced as follows;

$$\int_0^{\pi} L\sin\tau\, d\tau = \int_0^{\pi}\{m\omega^2\ddot{x} + c\omega\dot{x} + f(x) + m\omega^2\ddot{Z}\}\sin\tau\, d\tau = 0 \tag{3.4}$$

$$\int_0^{2\pi} L\cos\tau\, d\tau = \int_0^{2\pi}\{m\omega^2\ddot{x} + c\omega\dot{x} + f(x) + m\omega^2\ddot{Z}\}\cos\tau\, d\tau = 0 \tag{3.5}$$

Eq.(3.4) and (3.5) are calculated by using eq.(3.3). The integration of $f(x)$ is as follows.

The $f(x)$ can be given in terms of X_α and X_β in the following form.

In the interval $\quad x_y \leqq X_\alpha \leqq 2x_y$

$$\left.\begin{aligned} f(x) &= nk(X_\alpha - x_y) + kx && \text{for} \quad 0 \leqq \tau \leqq \alpha \\ f(x) &= nkx_y + k(1-n)x && \text{for} \quad \alpha \leqq \tau \leqq \tfrac{\pi}{2} \\ f(x) &= -nk(X_\alpha - x_y) + kx && \text{for} \quad \tfrac{\pi}{2} \leqq \tau \leqq \pi \end{aligned}\right\} \tag{3.6}$$

In the interval $|X_\beta| \geqq 2\chi_Y$

$$\left.\begin{aligned} f(x) &= nk\chi_Y + k(1-n)\chi && \text{for} \quad 0 \leqq \tau \leqq \tfrac{\pi}{2} \\ f(x) &= -nk(X_\beta - \chi_Y) + k\chi && \text{for} \quad \tfrac{\pi}{2} \leqq \tau \leqq \beta \\ f(x) &= -nk\chi_Y + k(1-n)\chi && \text{for} \quad \beta \leqq \tau \leqq \pi \end{aligned}\right\} \qquad (3.7)$$

where k and n are the spring constant in the elastic range and in the elastic and plastic range, respectively. Then we get

$$\int_0^{\pi} f(x)\sin\tau\, d\tau = \frac{kX_\alpha}{4}\left(2\pi - n\pi + 2n\alpha - n\sin 2\alpha + 8n\cos\alpha\frac{\chi_Y}{X_\alpha} - 4n\cos\alpha\right) \qquad (3.8)$$

$$\int_0^{2\pi} f(x)\cos\tau\, d\tau = 2knX_\alpha\left(\sin^2\alpha + 2\sin\alpha - 4\frac{\chi_Y}{X_\alpha}\sin\alpha + 1\right) \qquad (3.9)$$

$$\text{for} \quad \chi_Y \leqq |X_\alpha| \leqq 2\chi_Y$$

and

$$\int_0^{\pi} f(x)\sin\tau\, d\tau = \frac{kX_\beta}{4}\left(2\pi - 3n\pi + 2n\beta - n\sin 2\beta - 8n\cos\beta\frac{\chi_Y}{X_\beta} + 4n\cos\beta\right) \qquad (3.10)$$

$$\int_0^{2\pi} f(x)\cos\tau\, d\tau = 2knX_\beta\left(\sin^2\beta - 2\sin\beta + 4\frac{\chi_Y}{X_\beta}\sin\beta + 1\right) \qquad (3.11)$$

$$\text{for} \quad |X_\beta| \geqq 2\chi_Y$$

<u>Calculation of the eigen value.</u> - Substituting eq.(3.8), (3.10) into eq.(3.4), we get

$$X_\alpha\left\{\frac{k}{4m\omega^2}\left(2\pi - n\pi + 2n\alpha - n\sin 2\alpha + 8n\cos\alpha\frac{\chi_Y}{X_\alpha} - 4n\cos\alpha\right) - \frac{\pi}{2}\right\} = 0 \qquad (3.12)$$

$$X_\beta\left\{\frac{k}{4m\omega^2}\left(2\pi - 3n\pi + 2n\beta - n\sin 2\beta - 8n\cos\beta\frac{\chi_Y}{X_\beta} + 4n\cos\beta\right) - \frac{\pi}{2}\right\} = 0 \qquad (3.13)$$

From eq. (3.3),

$$\left.\begin{aligned} \chi &= X_\alpha\sin\alpha = 2\chi_Y - X_\alpha, && \text{for} \quad \tau = \alpha \\ \chi &= X_\beta\sin\beta = X_\beta - 2\chi_Y, && \text{for} \quad \tau = \beta \end{aligned}\right. \qquad (3.14)$$

Then, we can deduce the ductility factors μ_α, μ_β as follows,

$$\mu_\alpha = \frac{X_\alpha}{\chi_Y} = \frac{2}{1+\sin\alpha}, \quad \text{for} \quad 1 \leqq \mu_\alpha \leqq 2, \quad 0 \leqq \alpha \leqq \frac{\pi}{2} \qquad (3.15)$$

$$\mu_\beta = \frac{X_\beta}{\chi_Y} = \frac{2}{1-\sin\beta}, \quad \text{for} \quad 2 \leqq \mu_\beta \leqq \infty, \quad \frac{\pi}{2} \leqq \beta \leqq \pi \qquad (3.16)$$

Substituting eq.(3.15) and (3.16) into eq.(3.12) and (3.13) considering $X_\alpha \neq 0$, $X_\beta \neq 0$, we get

$$\frac{k}{4m\omega^2}\left(2\pi - n\pi + 2n\alpha + n\sin 2\alpha\right) - \frac{\pi}{2} = 0 \qquad (3.17)$$

$$\frac{k}{4m\omega^2}\left(2\pi - 3n\pi + 2n\beta + n\sin 2\beta\right) - \frac{\pi}{2} = 0 \qquad (3.18)$$

The square of circular frequency ω^2 in steady state resonance is computed by determining the arbitrary constant α, β from eq.(3.17) and (3.18), where ω^2 is a kind of eigen value. In order to non-dimentionalize the eq.(3.17) and (3.18), we define λ^2 by

$$\lambda^2 = \frac{\omega^2}{p^2} \qquad p^2 = \frac{k}{m} \tag{3.19}$$

where p^2 and λ^2 are called a square of undamped natural circular frequency and a frequency ratio respectively. The frequency ratio λ^2 becomes

$$\lambda^2 = \frac{1}{2\pi}(2\pi - n\pi + 2n\alpha + n\sin 2\alpha), \quad \text{for} \quad x_Y \leqq |X_\alpha| \leqq 2x_Y \tag{3.20}$$

$$\lambda^2 = \frac{1}{2\pi}(2\pi - 3n\pi + 2n\beta + n\sin 2\beta), \quad \text{for} \quad |X_\beta| \geqq 2x_Y \tag{3.21}$$

Calculation of structural response. - Substituting eq.(3.9) and (3.11) into eq.(3.5), we get

$$\frac{\pi c X_\alpha}{m\omega} + \frac{knX_\alpha}{m\omega^2}\left(\sin^2\alpha + 2\sin\alpha - 4\frac{x_Y}{X_\alpha}\sin\alpha + 1\right) - a\pi = 0 \tag{3.22}$$

$$\frac{\pi c X_\beta}{m\omega} + \frac{knX_\beta}{m\omega^2}\left(\sin^2\beta - 2\sin\beta + 4\frac{x_Y}{X_\beta}\sin\beta + 1\right) - a\pi = 0 \tag{3.23}$$

The maximum displacement X_α and X_β are computed by substituting eq.(3.15) and (3.16) into eq.(3.22) and (3.23). Defining $h = c/c_{cr}$ $(c_{cr} = 2\sqrt{km})$, the maximum displacement X_α and X_β are given as follows;

$$X_\alpha = \frac{a\pi}{\frac{2\pi h}{\lambda} + \frac{n}{\lambda^2}(1 - \sin^2\alpha)} \tag{3.24}$$

$$X_\beta = \frac{a\pi}{\frac{2\pi h}{\lambda} + \frac{n}{\lambda^2}(1 - \sin^2\beta)} \tag{3.25}$$

In order to non-dimensionalize the eq.(3.24) and (3.25), we use the relation

$$X_S = \frac{ma\omega^2}{k} \tag{3.26}$$

where X_S is the statical maximum displacement due to the statical earthquake motion. Then, the equation of $\frac{X_\alpha}{X_S}$, $\frac{X_\beta}{X_S}$ is given as follows;

$$U_\alpha = \frac{X_\alpha}{X_S} = \frac{\pi}{2\pi\lambda h + n(1 - \sin^2\alpha)} \tag{3.27}$$

$$U_\beta = \frac{X_\beta}{X_S} = \frac{\pi}{2\pi\lambda h + n(1 - \sin^2\beta)} \tag{3.28}$$

where U_α and U_β are the ratios of the dynamical maximum displacement to the statical maximum displacement.

The relation between the maximum displacement U and the ductility factor μ is as follows;

$$\frac{g_Y}{K_E} = \frac{\frac{kx_Y}{mg}}{\frac{a\omega^2}{g}} = \frac{x_Y}{\frac{ma\omega^2}{k}} = \frac{x_Y}{X_S} = \frac{U_\alpha}{\mu_\alpha}, \ \frac{U_\beta}{\mu_\beta} \tag{3.29}$$

where the yield shearing force coefficient g_Y and the seismic coefficient of the earthquake motions K_E are given by

$$g_Y = \frac{k x_Y}{mg} \quad , \qquad K_E = \frac{a\omega^2}{g} \tag{3.30}$$

This relation means that the scale of structural strength for earthquake motion is equal to the ratio of U to μ .

Response curve for one mass system.- In order to compute the response curve, the numerical calculation is carried out. The parameters in numerical calculation are a fraction of critical damping, h in the elastic range, and the spring constant ratio, n in the elastic and plastic range. The process of numerical calculation is the following;

(1) We calculate the eigen value λ^2 by eq.(3.20) and (3.21) for n . Then, if we determine α , β suitably with the condition of $0 \leq \alpha \leq \frac{\pi}{2}$, $\frac{\pi}{2} \leq \beta \leq \pi$ in range of α, β , we can compute λ^2 always.

(2) The ductility factor μ_α , μ_β are computed by eq.(3.15) and (3.16) by using α and β which are determined in calculation of λ^2.

(3) The maximum displacement U_α and U_β are computed by eq.(3.27) and (3.28) using α, β and λ .

(4) The structural strength $\frac{g_Y}{K_E}$ are computed by eq.(3.30) by making use of μ_α, μ_β, U_α and U_β .

The results of response curve are shown in Figs.3.1, 3.2 and 3.3. The relation between the structural strength $\frac{g_Y}{K_E}$ and the maximum displacement U for n = 1, 0.75, 0.5 and h=0, 0.05, 0.1, 0.2 is shown in Fig.3.1.

The maximum displacement U and ductility factor μ in these curve change with parameters n, h and we notice that the displacement U has the minimum value when the $\frac{g_Y}{K_E}$ is some value. The U and $\frac{g_Y}{K_E}$ decrease with increase of parameters n, h, so we see that the profitable condition in the seismic design is to make the viscous damping effect as much as possible and the type of restoring force characteristic to be plastic perfect type.

When U and $\frac{g_Y}{K_E}$ is minimum, the ductility factor μ has almost constant value which is about two. From these results, we see that the damping effect becomes most effective and it happens in the case of $\mu \simeq 2$.

Fig.3.2 shows the relations between $\frac{g_Y}{K_E}$ and μ for n=1, 0.75, 0.5 and h = 0, 0.05, 0.1, 0.2. The Fig.3.3 shows the relation between U , $\frac{g_Y}{K_E}$, μ and the spring constant ratio n for the optimum value.

On the stabilily of the solution. Eq.(3.2) has been solved by assuming that the solution of its equation is eq.(3.3). This means that the solution of eq.(3.3) is assumed as steady state solution and stable. Therefore, we must search the condition that the solution of eq.(3.3) can be approximated by the steady solution (3.4).

After long tedious calculation, we get the stability condition of the solution as follows;

$$\rho \leqq 0, \text{ that is, } \quad n\cos^2\alpha ,\ n\cos^2\beta \leqq 2\pi\lambda h \tag{3.31}$$

where

$$\rho = \frac{1}{2\pi\lambda}\left(\frac{n}{\lambda}\cos^2\alpha,\ \beta - 2\pi h\right)$$

The stability condition eq.(3.31) is satisfied when the spring constant ratio n is small, that is, the system is almost linear, α and β is nearly equal to $\frac{\pi}{2}$, λ is nearly equal to one and h is large.

In general, the state of resonance has tendency of becoming unstable vibration in either linear or non-linear vibrations. In the resonance of non-linear steady state vibration with a hysteresis characteristic , the condition of stability is very strict as known from eq.(3.31).

Comparision between transient and steady state response.- The following figures shows comparision between responses by author's theorem and responses by the other method.

(1) Comparision between the author's analysis with the other one in steady state response.

The response calculated by author's theorem are shown in Fig.3.4, in comparision with the responses by H. Tajimi(3) and N. Ando(7). These curves are for the spring constant ratio n = 1.0(perfect plastic type) in the elastic and plastic range and the fraction of critical damping h = 0, 0.05, 0.1 in the elastic range.

From these curves, we see that the three analytical results agree perfectly with each other.

(2) Comparision between the responses calculated by author's and the steady state and transient response caused by stationary waves.

We compute the steady state response and the transient (maximum) response by the electrical analog computer NEAC T-100 under the action of four kinds of stationary waves as shown in Fig.3.5(8).

We compare these responses computed by NEAC T-100 with the analytical values by authors in Fig.3.6 (a) (the transient response) and in Fig.3.6 (b) (the steady state response). From these curves, we see that the author's theoritical responses agree fairly well even with the transient response.

(3) Comparision between the responses calculated by author's and the transient response caused by one step of rectangular shock wave.(6)

We have calculated the most disadvantageous transient response of one mass systems caused by the one step of rectangular shock wave.

The results of comparision between the most disadvantageous transient response and the author's theoritical steady response are shown in Fig.3.7. These curves agree fairly well with each other.

(4) Comparision between the transient response caused by actual earthquake motions and caused by the one step of rectangular shock waves.

We show the relation between the seismic coefficient δ_y and the maximum displacement x in Fig.3.8, based on the transient response caused by the El-Centro, California May 18,1940 which N.M.Newmark and G.V.Berg have computed(9),(10) and on the one step of rectangular wave which authors have computed(6).

These two response curves almost agree with each other at the natural period $Ts \leqq 0.4^{sec}$. At the natural period $Ts > 0.8$ only the optimum yield seismic coefficient agree fairly well with each other. In the comparision with these responses, we see that the optimum yield seismic coefficient computed in case of the non-linear steady state vibration are almost coincident with that computed in case of the non-linear transient vibration.

ANALYSIS FOR TWO MASS SYSTEM

Method of analysis.- The equations of motion for the two mass system are given by the following equation:

$$L_1 = m_1\ddot{x}_1 - c_2\dot{x}_2 + c_1\dot{x}_1 - f_2(x_2) + f_1(x_1) + m_1\ddot{z} = 0 \qquad (4.1)$$

$$L_2 = m_2\ddot{x}_1 + m_2\ddot{x}_2 + c_2\dot{x}_2 + f_2(x_2) + m_2\ddot{z} = 0 \qquad (4.2)$$

Replacing time t by $\tau=\omega t$, and using eq.(2.5) as the earthquake motions $\ddot{z}$, the above equations become as follows;

$$L_1 = m_1\omega^2\overset{\circ\circ}{x}_1 - c_2\omega\overset{\circ}{x}_2 + c_1\omega\overset{\circ}{x}_1 - f_2(x_2) + f_1(x_1) - m_1 a\omega^2\cos\tau = 0 \qquad (4.3)$$

$$L_2 = m_2\omega^2\overset{\circ\circ}{x}_2 + m_2\omega^2\overset{\circ\circ}{x}_1 + c_2\overset{\circ}{x}_2 + f_2(x_2) - m_2 a\omega^2\cos\tau = 0 \qquad (4.4)$$

Therefore, if the solutions of eq.(4.3) and eq.(4.4) are assumed to be eq.(2.4), they become:

$$x_1 = X_{1\alpha}, X_{1\beta}\sin\tau\ , \quad x_2 = X_{2\alpha}, X_{2\beta}\sin\tau \qquad (4.5)$$

From eq.(2.6) and eq.(2.8), we get the equation of the eigen value and steady-state displacement. The equation of the eigen value is

$$\int_0^{\pi} L_1\sin\tau d\tau = 0\ , \quad \int_0^{\pi} L_2\sin\tau d\tau = 0 \qquad (4.6)$$

The steady-state displacement is

$$\int_0^{2\pi} L_1\cos\tau d\tau = 0\ , \quad \int_0^{2\pi} L_2\cos\tau d\tau = 0 \qquad (4.7)$$

In order to calculate $\int_0^{\pi,2\pi} f_s(X_{s\alpha}, X_{s\beta}\sin\tau)\sin\tau d\tau$ and $\int_0^{\pi,2\pi} f_s(X_{s\alpha}, X_{s\beta}\sin\tau)\cos\tau d\tau$ in eq.(4.6) and eq.(4.7), we classify the range of $|X_s|$ as follows:

(A) $x_{sy} \leqq |X_{s\alpha}| \leqq 2x_{sy} \quad (s=1,2)$

(B) $|X_{1\beta}| \geqq 2x_{1y}\ , \quad x_{1y} \leqq |X_{2\alpha}| \leqq 2x_{2y}$

(C) $x_{1y} \leqq |X_{1\alpha}| \leqq 2x_{1y}\ , \quad |X_{2\beta}| \geqq 2x_{2y}$

(D) $|X_{s\beta}| \geqq 2x_{sy} \quad (s=1,2)$

The results of integration of eq.(4.6) and eq.(4.7) for the case of (A), (B), (C) and (D) is as follows:

$$\left.\begin{aligned} &-\frac{\pi}{2}X_{1\alpha,\beta} - \frac{k_2}{4m_1\omega^2}X_{2\alpha,\beta}F_2(\alpha_2,\beta_2) + \frac{k_1}{4m_1\omega^2}X_{1\alpha,\beta}F_1(\alpha_1,\beta_1) = 0 \\ &-\frac{1}{2}X_{1\alpha,\beta} - \frac{\pi}{2}X_{2\alpha,\beta} + \frac{k_2}{4m_2\omega^2}X_{2\alpha,\beta}F_2(\alpha_2,\beta_2) = 0 \end{aligned}\right\} \qquad (4.8)$$

$$\left.\begin{aligned} &-\frac{\pi c_2}{m_1\omega}X_{2\alpha,\beta} + \frac{\pi c_1}{m_1\omega}X_{1\alpha,\beta} - \frac{k_2 n_2}{m_1\omega^2}X_{2\alpha,\beta}G_2(\alpha_2,\beta_2) + \frac{k_1 n_1}{m_1\omega^2}X_{1\alpha,\beta}G_1(\alpha_1,\beta_1) - a\pi = 0 \\ &\frac{\pi c_2}{m_2\omega}X_{2\alpha,\beta} + \frac{k_2 n_2}{m_2\omega^2}X_{2\alpha,\beta}G_2(\alpha_2,\beta_2) - a\pi = 0 \end{aligned}\right\} \qquad (4.9)$$

where in case of $x_{sy} \leqq |X_{s\alpha}| \leqq 2x_{sy}$; $X_{1\alpha,\beta} = X_{1\alpha}$, $X_{2\alpha,\beta} = X_{2\alpha}$

$F_1(\alpha,\beta) = F_1(\alpha_1)$, $F_2(\alpha,\beta) = F_2(\alpha_2)$, $G_1(\alpha,\beta) = G_1(\alpha_1)$ and $G_2(\alpha,\beta) = G_2(\alpha_2)$

in case of $|X_{s\beta}| \geqq 2x_{sy}$; $X_{1\alpha,\beta} = X_{1\beta}$, $X_{2\alpha,\beta} = X_{2\beta}$, $F_1(\alpha,\beta) = F_1(\beta_1)$

$F_2(\alpha,\beta) = F_2(\beta_2)$, $G_1(\alpha,\beta) = G_1(\beta_1)$ and $G_2(\alpha,\beta) = G_2(\beta_2)$

$$\left.\begin{aligned} F_s(\alpha_s) &= 2\pi - n_s\pi + 2n_s\alpha_s - n_s\sin 2\alpha_s + 8n_s\cos\alpha_s \frac{x_{sy}}{X_{s\alpha}} - 4n_s\cos\alpha_s \\ F_s(\beta_s) &= 2\pi - 3n_s\pi + 2n_s\beta_s - n_s\sin 2\beta_s - 8n_s\cos\beta_s \frac{x_{sy}}{X_{s\beta}} + 4n_s\cos\beta_s \\ G_s(\alpha_s) &= \sin^2\alpha_s + 2\sin\alpha_s - 4\sin\alpha_s \frac{x_{sy}}{X_{s\alpha}} + 1 \\ G_s(\beta_s) &= \sin^2\beta_s - 2\sin\beta_s + 4\sin\beta_s \frac{x_{sy}}{X_{s\beta}} + 1 \end{aligned}\right\} \quad (4.10)$$

Calculation for eigen value.- In order to non-dimensionize the eq.(4.8) we define the following quantities;

$\zeta = \dfrac{x_{2y}}{x_{1y}}$; relative yield displacement ratio

$\nu = \dfrac{k_2}{k_1}$; spring constant ratio in the elastic range.

$\varepsilon = \dfrac{m_2}{m_1}$; mass ratio

${}_1p^2 = {}_1s^2 p_1^2$, $p_1^2 = \dfrac{k_1}{m_1}$; undamped natural frequency in the elastic range

$${}_1s^2 = \frac{1}{2\epsilon}\left[\{\nu + \epsilon(1+\nu)\} - \sqrt{\{\nu+\epsilon(1+\nu)\}^2 - 4\epsilon\nu}\right]$$

${}_1\lambda^2 = \dfrac{\omega^2}{{}_1p^2}$; frequency ratio

The ductility factors $\mu_{s\alpha}$ and $\mu_{s\beta}$ are computed by the same method as for one mass system as follows;

$$\left.\begin{aligned} \mu_{s\alpha} &= \frac{X_{s\alpha}}{x_{sy}} = \frac{2}{1+\sin\alpha_s} \\ \mu_{s\beta} &= \frac{X_{s\beta}}{x_{sy}} = \frac{2}{1-\sin\beta_s} \end{aligned} \quad (S=1,2)\right\} \qquad (4.11)$$

Substituting eq.(4.11) into eq.(4.8), the frequency ratio becomes

$$\begin{aligned} {}_1\lambda^2 &= \frac{\nu\zeta}{2\pi\epsilon_1 s^2}\,\frac{\{1 \pm \sin(\alpha_1,\beta_1)\}\,F_2(\alpha_2,\beta_2)}{(1+\zeta) + \{\pm\zeta\sin(\alpha_1,\beta_1) \pm \sin(\alpha_2,\beta_2)\}} \\ {}_1\lambda^2 &= \frac{1}{2\pi\,{}_1s^2}\left[F_1(\alpha_1,\beta_1) - \frac{\nu\zeta\{1 \pm \sin(\alpha_1,\beta_1)\}}{1+\sin(\alpha_2,\beta_2)}F_2(\alpha_2,\beta_2)\right] \end{aligned} \qquad (4.12)$$

where in case of $x_{sy} \leqq |X_{s\alpha}| \leqq 2x_{sy}$;

$\sin(\alpha_1,\beta_1) = \sin\alpha_1$, $\sin(\alpha_2,\beta_2) = \sin\alpha_2$, $F_1(\alpha_1,\beta_1) = F_1(\alpha_1)$, $F_2(\alpha_2,\beta_2) = F_2(\alpha_2)$

in case of $|X_{s\beta}| \geqq 2x_{sy}$;

$\sin(\alpha_1,\beta_1) = -\sin\beta_1$, $\sin(\alpha_2,\beta_2) = -\sin\beta_2$, $F_1(\alpha_1,\beta_1) = F_1(\beta_1)$, $F_2(\alpha_2,\beta_2) = F_2(\beta_2)$

$$\left.\begin{aligned} F_s(\alpha_s) &= 2\pi - n_s\pi + 2n_s\alpha_s + n_s\sin 2\alpha_s \\ F_s(\beta_s) &= 2\pi - 3n_s\pi + 2n_s\beta_s + n_s\sin 2\beta_s \end{aligned} \quad (S=1,2)\right\} \quad (4.13)$$

In the eq.(4.12), the two equation being equal each other, we get the transcendental equation for calculating α_s and β_s . The equation includes two unkown quantities α_s and β_s , so that, if either α_s or β_s is determined, the other variable β_s or α_s is computed. If α_s and β_s are given, eigen value ${}_1\lambda^2$ is computed easily by eq.(4.12).

Calculation of structural response.- Using eq.(4.11), $G_s(\alpha_s)$ and $G_s(\beta_s)$ in eq.(4.10) is given as

$$\left.\begin{aligned} G_s(\alpha_s) &= 1 - \sin^2\alpha_s \\ G_s(\beta_s) &= 1 - \sin^2\beta_s \end{aligned} \quad (s=1,2)\right\} \qquad (4.14)$$

The maximum displacements $X_{s\alpha}$ and $X_{s\beta}$ are computed by eq.(4.9) as follows;

$$\left.\begin{aligned} X_{2\alpha,\beta} &= \frac{\pi m_2 a \omega^2}{\pi C_2 \omega + k_2 n_2 \{1-\sin^2(\alpha_2,\beta_2)\}} \\ X_{1\alpha,\beta} &= \frac{\pi (m_1+m_2) a \omega^2}{\pi C_1 \omega + k_1 n_1 \{1-\sin^2(\alpha_1,\beta_1)\}} \end{aligned}\right\} \qquad (4.15)$$

where in case of $x_{sy} \leqq |X_{s\alpha}| \leqq 2x_{sy}$; $\sin^2(\alpha_1,\beta_1)=\sin^2\alpha_1$, $\sin^2(\alpha_2,\beta_2)=\sin^2\alpha_2$, and in case of $|X_{s\beta}| \geqq 2x_{sy}$; $\sin^2(\alpha_1,\beta_1)=\sin^2\beta_1$, $\sin^2(\alpha_2,\beta_2)=\sin^2\beta_2$

In order to non-dimensionize the eq.(4.15), we use

$$x_{2s} = \frac{m_2 a \omega^2}{k_2} \quad , \quad x_{1s} = \frac{(m_1+m_2) a \omega^2}{k_1} \qquad (4.16)$$

where x_{2s} and x_{1s} are the statical maximum displacements of the 2nd mass and the 1st mass caused by the statical earthquake motions, respectively. Then the equation of $\frac{X_2}{x_{2s}}$ and $\frac{X_1}{x_{1s}}$ become

$$U_2 = \frac{X_2}{x_{2s}} = \frac{X_{2\alpha}}{x_{2s}},\ \frac{X_{2\beta}}{x_{2s}} = \frac{\pi}{2\pi h_{2,\delta}\sqrt{\frac{\epsilon}{\nu}}\,\lambda + n_2\{1-\sin^2(\alpha_2,\beta_2)\}} \qquad (4.17)$$

$$U_1 = \frac{X_1}{x_{1s}} = \frac{X_{1\alpha}}{x_{1s}},\ \frac{X_{1\beta}}{x_{1s}} = \frac{\pi}{2\pi h_{1,\delta}\,\lambda + n_1\{1-\sin^2(\alpha_1,\beta_1)\}} \qquad (4.18)$$

where $h_1 = \frac{C_1}{2\sqrt{m_1 k_1}}$ and $h_2 = \frac{C_2}{2\sqrt{m_2 k_2}}$ are fraction of critical damping in the elastic range for the 1st mass and the 2nd mass, respectively and n_1 and n_2 are the spring constant ratio in the elastic and plastic range for the 1st mass and the 2nd mass, respectively.

The maximum displacement ratio U_1 and U_2 are computed from the eigen value λ^2. U_1 and U_2 have such a character that these value are minimum in $\alpha_s = 0$, $\beta_s = \pi$ as in one mass system. In general, however, the maximum displacement $X = X_1 + X_2$ does not happen always in around $\alpha_s = 0$, $\beta_s = \pi$, because α_s and β_s are determined arbitrarily for each mass. Since the relative displacement for each mass occurs at the same time as mentioned previously, the maximum displacement X is gained by adding the relative maximum displacement of each mass:

$$X = X_1 + X_2 = \frac{\pi (m_1+m_2) a \omega^2}{\pi C_1 \omega + k_1 n_1\{1-\sin^2(\alpha_1,\beta_1)\}} + \frac{\pi m_2 a \omega^2}{\pi C_2 \omega + k_2 n_2\{1-\sin^2(\alpha_2,\beta_2)\}} \qquad (4.19)$$

Non-dimensionize the eq.(4.19) by defining

$$X_s = x_{1s} + x_{2s} = \frac{(m_1+m_2) a \omega^2}{k_1} + \frac{m_2 a \omega^2}{k_2} \qquad (4.20)$$

the maximum displacement ratio $\frac{X}{x_s}$ becomes

$$U = \frac{X}{x_s} = \frac{\nu(1+\epsilon)}{\epsilon + \nu(1+\epsilon)} \left\{ U_1 + \frac{\epsilon}{\nu(1+\epsilon)} U_2 \right\} \qquad (4.21)$$

The yield shearingforce coefficient for each mass is as follows:

$$g_{1Y} = \frac{k_1 x_{1Y}}{(m_1+m_2) g} \quad , \quad g_{2Y} = \frac{k_2 x_{2Y}}{m_2 g} \qquad (4.22)$$

Using eq.(4.22) and (3.30), we get the relations between yield shearing force coefficients, g_{1Y}, g_{2Y} and earthquake seismic coefficient, K_E, namely,

$$\frac{g_{2Y}}{K_E} = \left(\frac{1+\epsilon}{\epsilon}\frac{U_1}{\mu_1} + \frac{U_2}{\mu_2}\right)\frac{\nu\zeta}{1+\nu\zeta} \qquad (4.23)$$

$$\frac{g_{1Y}}{K_E} = \left(\frac{1+\epsilon}{\epsilon}\frac{U_1}{\mu_1} + \frac{U_2}{\mu_2}\right)\frac{\epsilon}{(1+\epsilon)(1+\nu\zeta)} \qquad (4.24)$$

The following relation is also introduced from the eq.(4.23) and eq.(4.24);

$$\frac{g_{2Y}}{g_{1Y}} = \frac{\zeta\nu(1+\epsilon)}{\epsilon} \tag{4.25}$$

Calculation of response curve.- We get the different response curves depending on the structure parameters h_s, ν, ζ, ϵ and n_s. When we compute the response curve, we must consider the condition of each mass which depends on the rigions of $X_{s\alpha}$ and $X_{s\beta}$. That is, we must notice that the eigen value, ${}_{\prime}\lambda^2$ are calculated in the following regions;

(A) $\quad x_{sY} \leqq |X_{s\alpha}| \leqq 2x_{sY} \qquad (s=1,2)$

(B) $\quad |X_{1\beta}| \geqq 2x_{1Y}, \quad x_{2Y} \leqq |X_{2\alpha}| \leqq 2x_{2Y}$

(C) $\quad x_{1Y} \leqq |X_{1\alpha}| \leqq 2x_{1Y}, \quad |X_{2\beta}| \geqq 2x_{2Y}$

(D) $\quad |X_{s\beta}| \geqq 2x_{sY} \quad (s=1,2)$

When the eigen value, ${}_{\prime}\lambda^2$ is given in the case of (A),(B),(C) and (D), U_1 and U_2 are computed by eq.(4.17) and eq.(4.18), and U also is gained by eq.(4.21). The ductility factor μ_s for each mass and the seismic coefficient ratio $\frac{g_{sY}}{K_E}$ are computed by eq.(4.11), eq.(4.23) and eq.(4.24).

Example.- We approximated the system to be perfect plastic, that is, $n_1 = n_2 = 1$. We can classify the transition from the plastic to the elastic range in the following three cases.

Case I:

(1) $\beta_s = \frac{\pi}{2} \{ \mu_s = \infty, \ {}_{\prime}\lambda^2 \equiv 0 \}$

(2) $\beta_1 > \beta_2 \ \{ \pi > \beta_1 > \frac{\pi}{2} (\infty > \mu_1 > 2), \ \pi > \beta_2 > \frac{\pi}{2} (\infty > \mu_2 > 2) \}$

(3) $\beta_1 > \beta_2 \ \{ \beta_1 = \pi \ (\mu_2 = 0), \ \pi > \beta_2 > \frac{\pi}{2} (\infty > \mu_2 > 2) \}$

(4) $\alpha_1 < \beta_2 \ \{ \frac{\pi}{2} > \alpha_1 \geqq 0 (2 \geqq \mu_1 > 1), \ \pi > \beta_2 > \frac{\pi}{2} (\infty > \mu_2 > 2) \}$

(5) $\alpha_1 < \beta_2 \ \{ \frac{\pi}{2} > \alpha_1 > 0 (2 > \mu_1 > 1), \ \beta_2 = \pi, \ \alpha_2 = 0 \ (\mu_2 = 2) \}$

(6) $\alpha_1 > \alpha_2 \ \{ \frac{\pi}{2} > \alpha_1 > 0 (2 > \mu_1 > 1), \ \frac{\pi}{2} > \alpha_2 \geqq 0 \ (2 \geqq \mu_2 > 1) \}$

Case II:

(1) $\beta_s = \frac{\pi}{2} (\mu_s = \infty, \ {}_{\prime}\lambda^2 \equiv 0)$

(2) $\beta_1 = \beta_2 \ \{ \pi > \beta_1 > \frac{\pi}{2} (\infty > \mu_1 > 2), \ \pi > \beta_2 > \frac{\pi}{2} \ (\infty > \mu_2 > 2) \}$

(3) $\beta_1 = \beta_2 \ \{ \beta_1 = \pi (\mu_1 = 2), \ \beta_2 = \pi (\mu_2 = 2) \}$

(4) $\alpha_1 = \alpha_2 \ \{ \alpha_1 = 0 \ (\mu_1 = 2), \ \alpha_2 = 0 \ (\mu_2 = 2) \}$

(5) $\alpha_1 = \alpha_2 \ \{ \frac{\pi}{2} > \alpha_1 > 0 \ (2 > \mu_1 > 1), \ \frac{\pi}{2} > \alpha_2 > 1 \ (2 > \mu_2 > 1) \}$

(6) $\alpha_1 = \alpha_2 \ \{ {}_{\prime}\lambda^2 \equiv 1, \ \alpha_1 = \frac{\pi}{2} (\mu_1 = 1), \ \alpha_2 = \frac{\pi}{2} \ (\mu_2 = 1) \}$

Case III:

(1) $\beta_s = \frac{\pi}{2} \{ \mu_s = \infty, \ {}_{\prime}\lambda^2 \equiv 0 \}$

(2) $\beta_1 < \beta_2 \ \{ \pi > \beta_1 > \frac{\pi}{2} (\infty > \mu_1 > 2), \ \pi > \beta_2 > \frac{\pi}{2} \ (\infty > \mu_2 > 2) \}$

(3) $\beta_1 < \beta_2 \ \{ \pi > \beta_1 > \frac{\pi}{2} (\infty > \mu_1 > 2), \ \beta_2 = \pi \ (\mu_2 = 2) \}$

(4) $\beta_1 > \alpha_2 \ \{ \pi > \beta_1 > \frac{\pi}{2} (\infty > \mu_1 > 2), \ \frac{\pi}{2} > \alpha_2 \geqq 0 \ (2 \geqq \mu_2 > 1) \}$

(5) $\beta_1 > \alpha_2 \{ \beta_1 = \pi, \ \alpha_1 = 0 \ (\mu_2 = 1), \ \frac{\pi}{2} > \alpha_2 > 0 \ (2 > \mu_2 > 1) \}$

(6) $\alpha_1 < \alpha_2 \ \{ \frac{\pi}{2} > \alpha_1 \geqq 0 \ (2 \geqq \mu_1 > 1), \ \frac{\pi}{2} > \alpha_2 > 0 (2 > \mu_2 > 1) \}$

The condition that the non-linear vibration occurs in the Case I, II, III, will be computed by eq.(4.26),(4.27) and (4.28) for the Case I, the Case II and the Case III, reppectively.

$$\frac{\nu\zeta}{\epsilon}\frac{F_2(\beta_2)}{1+\zeta-\sin\beta_2}=\pi-\frac{\nu\zeta}{1-\sin\beta_2}F_2(\beta_2),\quad \frac{\pi\nu\zeta}{\epsilon}\frac{1+\sin\alpha_1}{1+\zeta+\zeta\sin\alpha_1}=F_1(\alpha_1)-\pi\nu\zeta(1+\sin\alpha_1) \tag{4.26}$$

$$\frac{\nu\zeta}{\epsilon(1+\zeta)}=1-\nu\zeta \tag{4.27}$$

$$\frac{\pi\nu\zeta}{\epsilon}\frac{1-\sin\beta_1}{1+\zeta-\zeta\sin\beta_1}=\pi-\frac{\nu\zeta}{1-\sin\beta_2}F_2(\beta_2),\quad \frac{\nu\zeta}{\epsilon}\frac{F_2(\alpha_2)}{1+\zeta+\sin\alpha_2}=\pi-\frac{\nu\zeta}{1+\sin\alpha_2}F_2(\alpha_2) \tag{4.28}$$

The conditions are shown in Fig.4.1 and Fig.4.2. These curves show the relations between ν, α_s and β_s in the case of $\epsilon=1$ and $\zeta=$ 0.5, 1.0, 1.5, 2.0$_s$. U, μ_s, $\frac{g_{1Y}}{K_E}$ and $\frac{g_{1Y}}{g_{2Y}}$ is also computed from the curves.

An explanation of the figures.- When $\nu=2$, $\zeta=0.5$(point(a) in the Fig.4.1), we get the value $\beta_2\cong 142°$, which is found from the $\zeta=0.5$ curve of Fig.4.1 in the case of $\beta_1=180°(\mu_1=2)$
Using this value of β_2, we get $U\cong 3.5$, $\mu_2\cong 5.2$, $\frac{g_{1Y}}{K_E}\cong 1.04$ and $\frac{g_{1Y}}{g_{2Y}}\cong 0.45$.
On the other hand, as α_1 for $\alpha_2=0$, $\beta_2=180°$ ($\mu_2=2$) can not be found, the non-linear vibration process is possible only in the range of $180°>\beta_2\geqq 142°$.
The minimum value U exists in the range $3.5>U>3.14$, which corresponds to $2>\mu_1>1$, $5.2>\mu_2>2$, $1.5>\frac{g_{1Y}}{K_E}>1.04$ and $0.83>\frac{g_{1Y}}{g_{2Y}}>0.49$.

From the preceding explanation, in $\epsilon=1.0$, $n_s=1.0$ and $h_s=0$, U and μ_s change with ν and ζ as parameters, and the relation between ν, ζ and ϵ which makes U minimum is as follows;

$$\frac{\nu\zeta}{\epsilon(1+\zeta)}=1-\nu\zeta \tag{4.27}$$

In the Case II, the non-linear vibration process is possible in whole range from . In the case I and III the non-linear vibration process is possible only in range from to some value of , .

On the stability of the solution.- The stability condition of the solution of eq.(4.3) and eq.(4.4) is introduced in the same way as in the one mass system, and is given by

$$\begin{aligned}&4\pi^2\sqrt{\frac{\epsilon}{\nu}}h_1h_{2,}s_1^2\lambda^2-2\pi h_{1,}s_1\lambda n_2\cos^2\alpha_2,\beta_2\\&\quad-2\pi\sqrt{\frac{\epsilon}{\nu}}h_{2,}s_1\lambda n_1\cos^2\alpha_1,\beta_1+n_1n_2\cos^2\alpha_1,\beta_1\cdot\cos^2\alpha_2,\beta_2\geqq 0\end{aligned} \tag{4.29}$$

CONCLUSION

The analysis of this paper examines the approximate method for determining the optimum yield seismic coefficient in dynamic steady state respo nse of non-linear systems, and we applied this method to one and two mass system. As the results, the following conclusions can be drawn:
1) The displacement of structures have the minimum value for the some yield shearing force. It means that the damping effect based on bi-linear hysteresis properties of structures has a limit and the damping effect becomes maximum when structures possess the some yield strength.
2) The ductility factor is not always minimum even if the displacement of structures is minimum, or regardless of viscous damping and hysteresis properties, it is always two.
3) As it is assumed that the non-linear vibration is always in the state of resonance, its vibration is apt to be unstable. Particuraly, in the neighbourhood where the displacement of structures is the smallest, its vibration is almost unstable, except in the case that the system has the large viscous damping effect, or is approximated by the linear one.
4) The results of the two mass system is, qualitatively, similar to these of one mass system. For the coupling effect of upper and lower story, the story yield seismic coefficient and ductility factor which make the displacement of structures minimum are different each other.

The story ductility factors of each story are equal each other and the displacement of structures becomes the smallest, provided that the relation between the mass ratio, spring constant ratio in the elastic range and the yield displacement ratio of each story is given in eq.(4.27).
5) So long as conditional formula in eq.(4.27) is satisfied, the vibrational transition from the plastic to the elastic range, in steady state resonance, is possible, and when these conditions are not satisfied, there does not happen the linear resonance (Figs. 4-1, 4-2).
6) Stability condition in the two mass system is not so strict as in the one mass system.
7) The optimum yield seismic coefficient computed in the case of the non-linear steady state vibration are almost coincident with that computed in the case of non-linear transient vibration.

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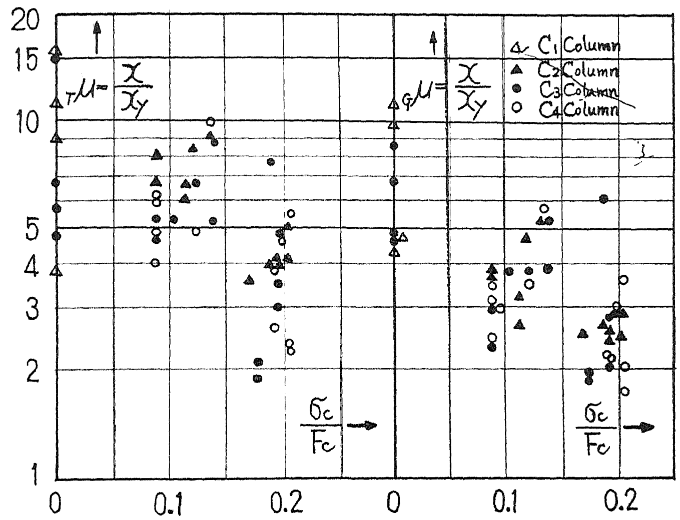

Fig.1.1 Relation between the ductility factor (μ) and the Compression Stress in Column (σ_c/F_c)

F_c ; 28-day concrete cylinder strength

σ_c ; compression stress in column

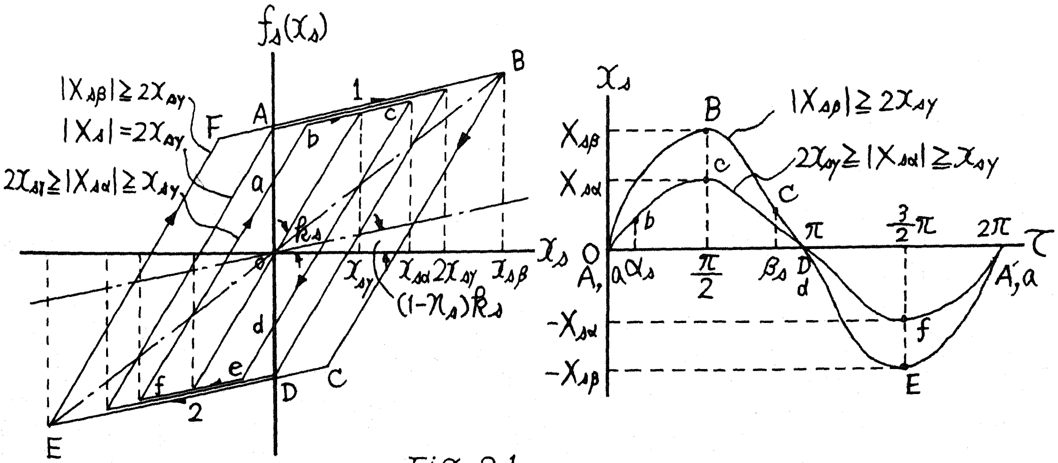

Fig. 2.1

(a) Relation between the restoring force $f(x_s)$ and displacement (x_s)

(b) Relation between displacement (x_s) and time variable (τ)

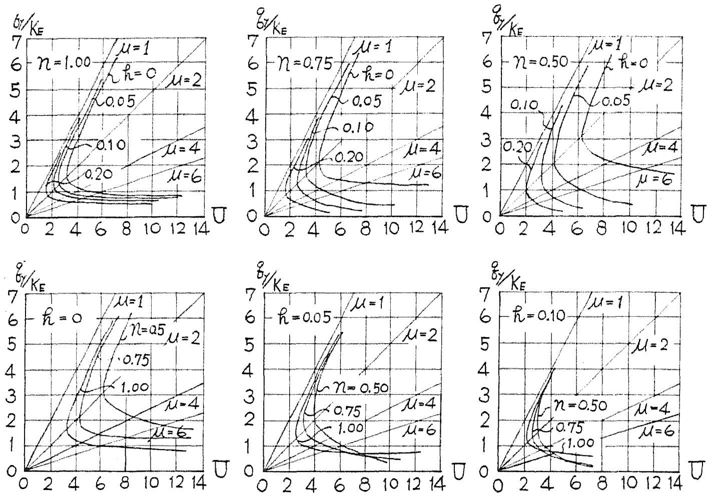

Fig. 3.1 Relation between yield strength ratio ($\frac{q_y}{K_E}$) and displacement ratio ($\bar{U}$) as parameters n, h

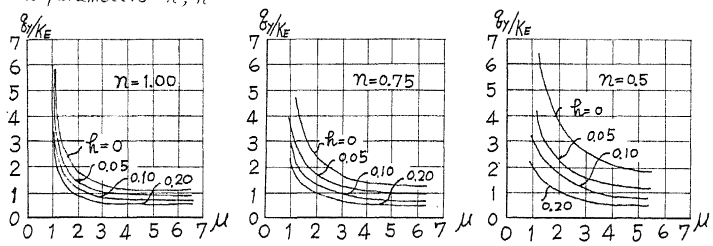

Fig. 3.2 Relation between yield·strength ratio ($\frac{q_y}{K_E}$) and ductility factor (μ) as parameters, n, h

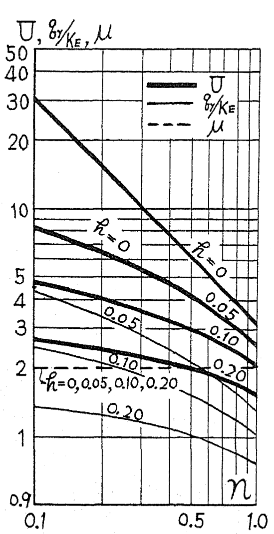

Fig.3.3 Relation between ($\bar{U}$), ($\frac{q_y}{K_E}$), (μ) and Spring constant ratio in elastic range and plastic range(n) in case of optimum.

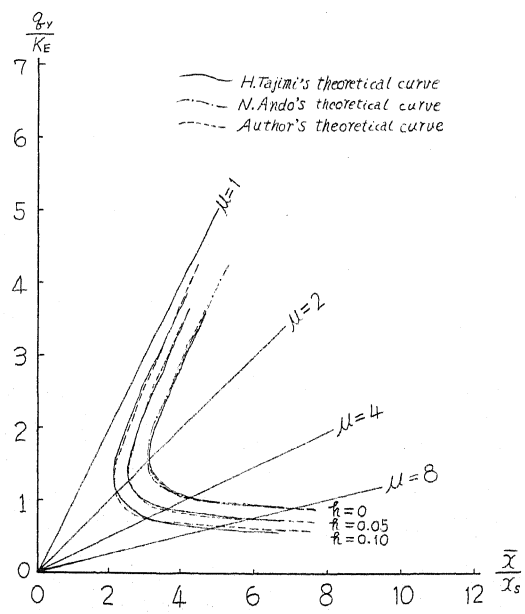

Fig.3.4 Comparison between the author's analysis with the other one in steady state response in case of $n = 1.0$

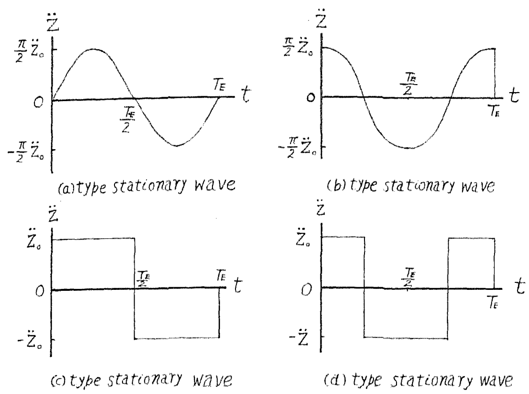

Fig.3.5 Four kinds of stationary waves

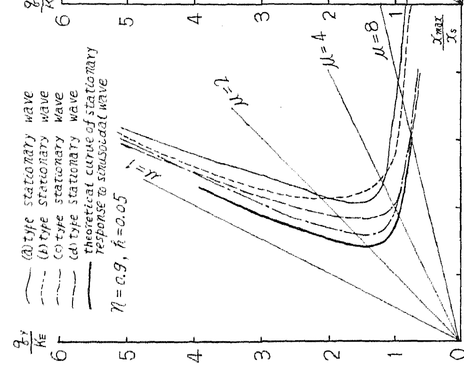

Fig.3.6(a) Comparison between the response calculated by author's and transient response caused by stationary waves

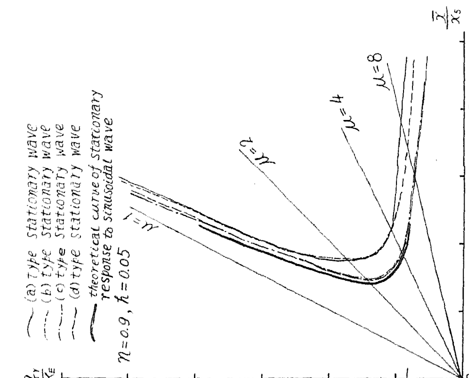

Fig.3.6(b) Comparison between the response calculated by author's and steady state response caused by stationary waves

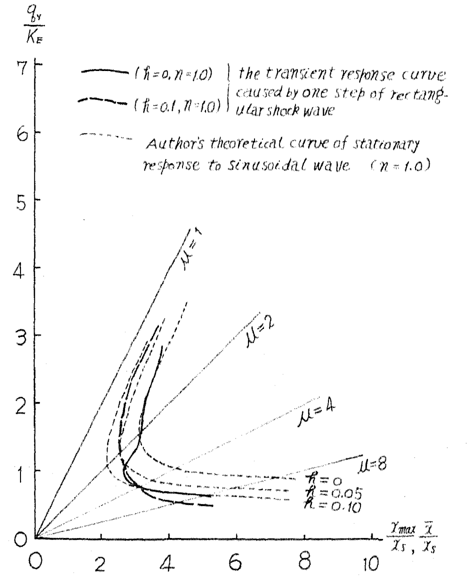

Fig.3.7 Comparison between the responses calculated by author's and the transient responses caused by one step of rectangular shock wave

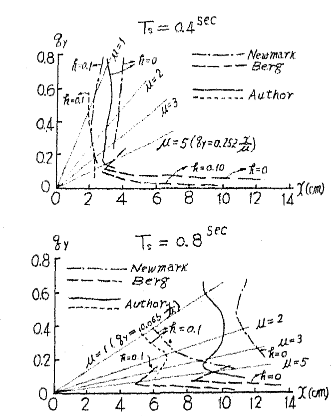

Fig.3.8 Comparison between the transient response caused by actual earthquake motions and caused by the one step of rectangular shock waves

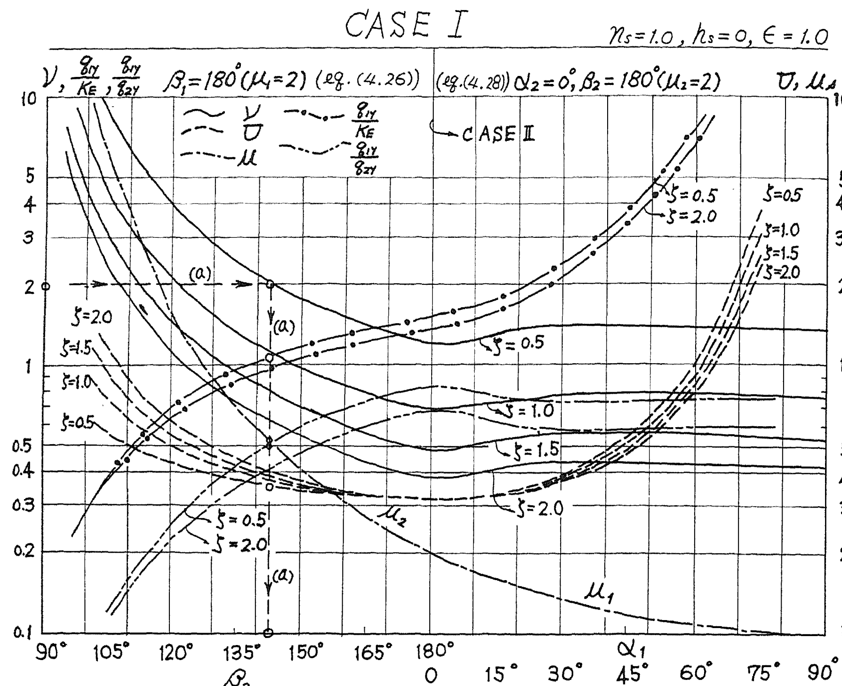

Fig. 4.1 Nomograph for determining β_s, α_s, U, μ_s, $\frac{g_{1Y}}{K_E}$ and $\frac{g_{1Y}}{g_{2Y}}$ from given parameters ν, ζ

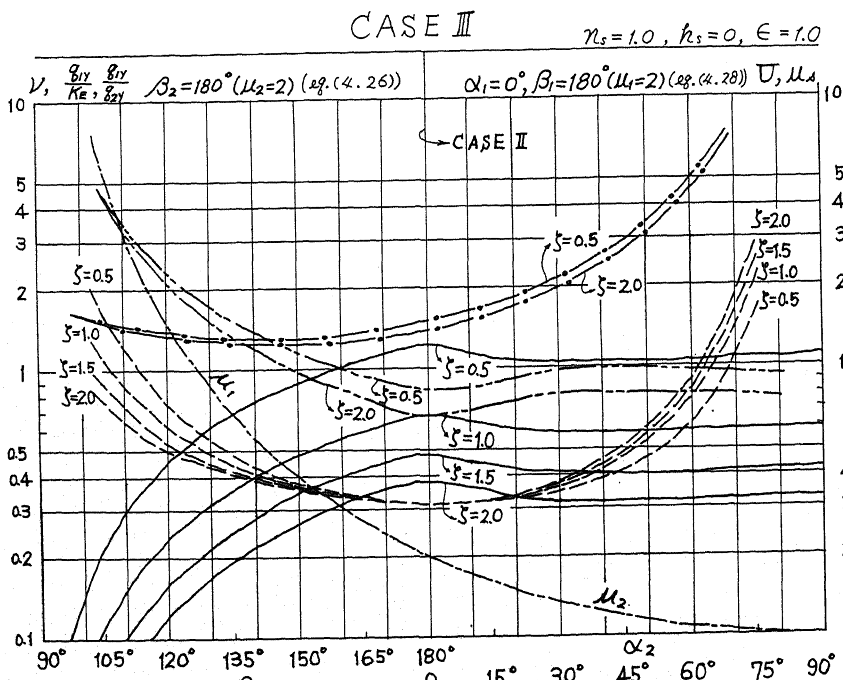

Fig. 4.2 Nomograph for determining β_s, α_s, U, μ_s, $\frac{g_{1Y}}{K_E}$ and $\frac{g_{1Y}}{g_{2Y}}$ from given parameters ν, ζ

E R R A T A

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PAGE 411: lines 19 – 22, should read:

"In the Case II, the non-linear vibration process is possible in whole range from

$$ {}_{,}\lambda^2 = 0 \ (\mu_s = \infty) \ \text{to} \ {}_{,}\lambda^2 = 1 \ (\mu_s = 1) $$

In the Case I and III, the non-linear vibration process is possible only in range from ${}_{,}\lambda^2 = 0 \ (\mu_s = \infty)$ to some value of"

$$ {}_{,}\lambda^2, \ 0 < {}_{,}\lambda^2 < 1 \quad (\mu_s > 1) $$

PAGE 414: Replace Fig. 3.3 by the following:

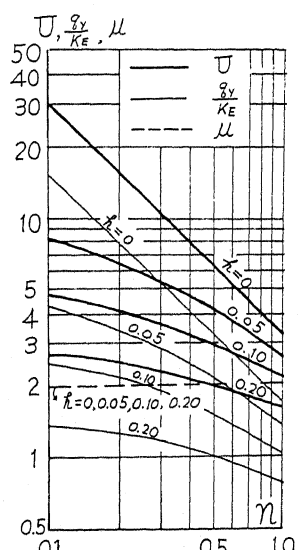

Fig. 3.3 Relationship between (U), (qY/KE), (μ) and spring constant ratio in elastic range and plastic range (n) in case of optimum

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ADDITIONAL COMMENTS *

For the two-mass system, the yield shearing force coefficient ratio (q_{2y}/q_{1y}) in each story becomes the following from equation (4.25).

$$q_{2y}/q_{1y} = \frac{1}{\varepsilon}\left\{\sqrt{(V+\varepsilon V-\varepsilon)^2+4\varepsilon^2 V}-(V+\varepsilon V-\varepsilon)\right\}$$

From this equation, we can find the optimum yield shearing force coefficient ratio, if we determine the mass ratio (ε), and the spring constant ratio (V) for each story.

The figure shows the relation between q_{2y}/q_{1y} and V as a parameter ε in case of optimum. From this figure, $q_{2y}/q_{1y} < 2$ and when $\varepsilon = 1$, $V = 1$, $q_{2y}/q_{1y} = 1.23$.

Many things remain to be solved in the future. For instance, in order to calculate the yield shearing force coefficient based on the non-linear response of structures, we have to notice the following point as we mentioned in this paper.

(1) How much ductility factor or extent of damage is allowed?

(2) As there is a lower limit of the maximum displacement of structures, we have to know the optimum yield shearing force coefficient.

For the 1st. point, we found that the shearing force characteristics for a reinforced concrete portal frame have the following tendency in our experimental study.

(i) The ductility factor does not become very large and the actual restoring force characteristics of a structure are not so simple as can be expressed by perfect plastic or bi-linear type.

(ii) As the compression stress in a column increases, the ductility factor decreases.

The restoring force characteristics are nearly of the bi-linear type when the compression stress in a column is zero, and is characterised by a double spindle type curve when the compression stress in a column is large enough.

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* These comments were made at the Conference by the authors in elaboration of certain points in their paper.

According to the relation between the ductility factor (μ) and the ratio of compression stress (σ_c) to 28-day cylinder strength (F_c) in a column (σ_c/F_c), we can see that the ductility factor decreases with increasing compression stress in a column.

For the 2nd. point, we found that when the yield shearing force coefficient of an upper story is different from the yield shearing force coefficient of a lower story, the relative discrepancy of the ductility factor increases.

From these results, in order to keep the maximum displacement as small as possible, it is preferable not to have abrupt changes of the yield shearing force coefficient distribution along the height. Indeed, the base shear coefficient should be determined in relation to the natural period of the structure.

According to the relation between the yield shearing force coefficient ratio and the ductility factor ratio in the upper story to lower story, we can see that the yield shearing force coefficient ratio is determined in accordance with the ductility factor ratio.

For example, when we wish to make the ductility factor of each story to be equal, the maximum yield shearing force coefficient should be almost twice the magnitude of the base shear coefficient and the distribution of the yield shearing force coefficient should be exponential in form.

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Transaction of A.I.J., No. 106, Dec. 1964 (Japanese)

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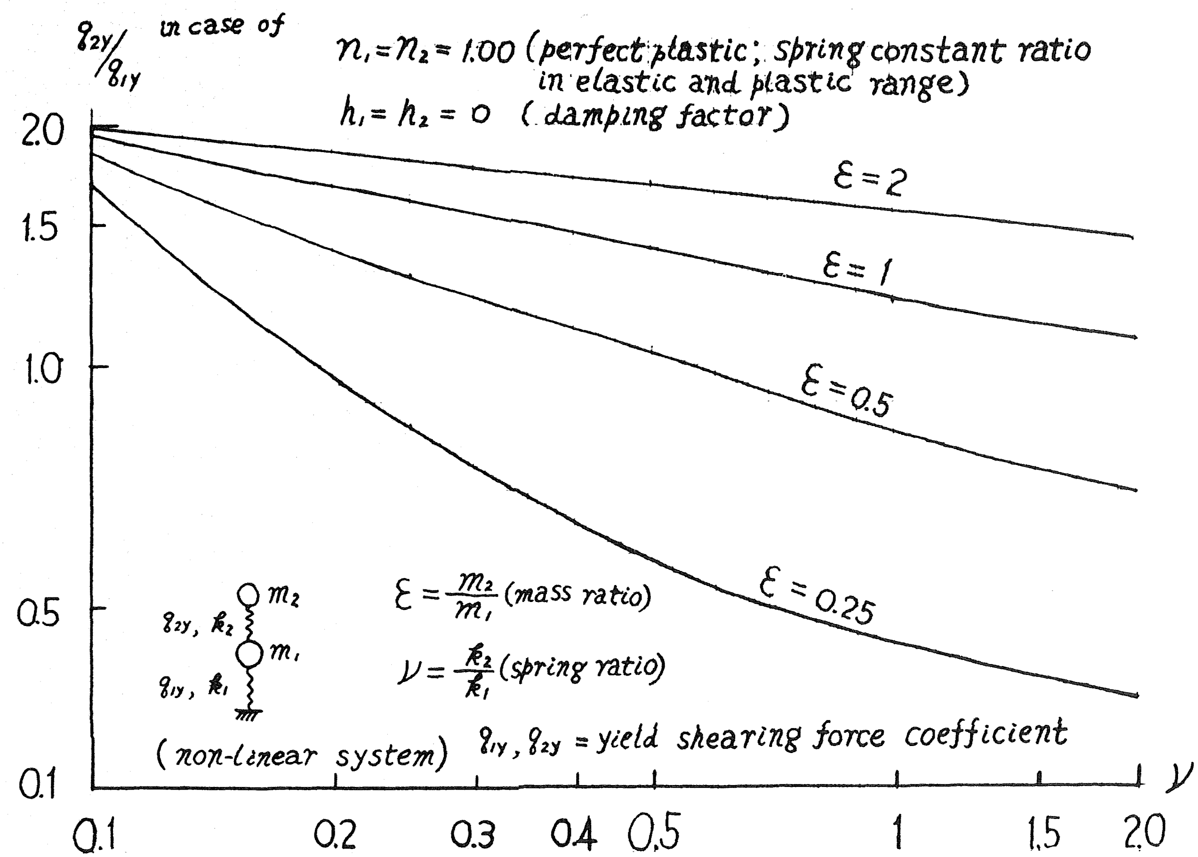

Relationship between q_{2y}/q_{1y} and ν as parameter ε in case of Optimum