DEFORMATION SPECTRA FOR ELASTIC AND ELASTOPLASTIC SYSTEMS SUBJECTED TO GROUND SHOCK AND EARTHQUAKE MOTIONS

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SYNOPSIS

A brief discussion is given of the maximum deformation experienced by single-degree-of-freedom systems subjected to ground motions of various forms. The motions considered include several simple, pulse-like excitations and a complex input corresponding to a strong-motion earthquake record. For each class of input, simple approximate rules are presented for the construction of deformation spectra for undamped and lightly damped systems from consideration of the gross characteristics of the displacement, velocity and acceleration diagrams of the ground motion. Both elastic and elastoplastic systems are investigated, and the spectra for the inelastic systems are related to those applicable to elastic systems. There is shown to exist a simple and direct relationship between the response of systems to earthquakes and to the simpler inputs.

INTRODUCTION

If the forcing function for a dynamical system is prescribed as a function of time, it is generally recognized that the response of the system can be computed in a straightforward manner by integration of the governing equations of motion, no matter how complicated the motion or the system may be. However, such computations are usually time consuming and are not appropriate for purposes of preliminary design. It is desirable, therefore, to have a body of basic information and simple approximate rules which will assist the designer to assess the effect and significance of the various factors and uncertainties involved in a given problem without the need for elaborate computations. Since the input motion in most practical problems is subject to considerable uncertainty, it is particularly important to be able to determine the sensitivity of the response to variations in the characteristics of the input motion.

The purpose of this paper is to present simple, approximate rules for estimating the maximum deformation of single-degree-of-freedom systems

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subjected to various patterns of ground motion. Primary emphasis is given to identifying the input characteristics that have a controlling influence on the response of a given system. Both elastic and elastoplastic systems, with and without damping, are investigated.

The approach used consists in investigating first the effect of pulse-like excitations of progressively increasing complexity, and, in the light of the results obtained for these inputs, examining then the response of systems to typical earthquake motions. The pulse-like excitations considered include velocity functions having from one to three half-cycles of oscillation with corresponding displacement functions having from one quarter of a cycle to one complete cycle of oscillation. These inputs approximate with reasonable accuracy the primary or dominant component of a large class of ground motions associated with nuclear explosions and some strong motion earthquakes. Consideration is first given to linearly elastic, undamped and damped systems, and then to inelastic systems of the elastoplastic type. The paper concludes with some brief comments on the relative effects of viscous damping and inelastic action.

This paper is offered as a synopsis or digest of an extensive investigation on this subject described in a report of limited distribution (1).

Throughout this discussion, the systems will be assumed to be initially at rest, and the initial values of the velocity and displacement of the ground will be taken as zero. Furthermore, unless otherwise indicated, the acceleration diagram, and hence the velocity and displacement diagrams, of the input motion will be assumed to have no discontinuities. The maximum deformation of high-frequency systems subjected to discontinuous acceleration inputs may be significantly greater than that indicated in this paper, and consequently, the information presented here should not be applied directly to the latter cases. A discussion of the effect of discontinuous inputs is available in Ref. (1).

SYSTEM CONSIDERED AND REVIEW OF FUNDAMENTAL TERMS

The single-degree-of-freedom system considered is shown in Fig. 1. It consists of a rigid mass, m, connected to the ground by a weightless spring and a dashpot exerting a restraining force that is proportional to the relative velocity between the mass and the ground. The displacement of the ground is denoted by y, and the displacement of the mass relative to the ground, the spring deformation, is denoted by u. Our interest in the following discussion will center on the maximum possible value of u.

In addition to the shape of the input motion, the maximum response of a system depends on the characteristics of the system itself. For an elastic system, these characteristics are usually specified in terms of the undamped natural frequency of the system, f, and the fraction of critical damping, β. The frequency f is expressed in cycles per second. For a system with a fixed amount of damping subjected to a given input, if the maximum deformation is plotted as a function of the natural frequency, the resulting curve constitutes what will be referred to as the <u>deformation</u>

spectrum of the system for the conditions considered.

The absolute maximum deformation of an elastic system, without regards to sign, will be denoted by the symbol U. The product of U and the undamped circular natural frequency of the system, p, the so-called pseudo-velocity of the system, will be denoted by V. In an analogous manner, the product pV will be referred to as the pseudo-acceleration of the system and will be designated by A, that is

$$V = pU \qquad \text{and} \qquad A = pV = p^2U \tag{1}$$

It is emphasized that the quantities V and A are alternative measures of the maximum spring deformation U, and that, if one of these three quantities is known, the remaining two can be computed by simple algebraic operations. It turns out that, in some cases, the maximum deformation can be expressed more conveniently indirectly in terms of V or A, rather than directly in terms of U. Furthermore, the characteristics of the deformation spectrum can more readily be approximated with the aid of all three quantities rather than in terms of any one of them. In the following discussion all three quantities will be used, and the deformation spectrum will be displayed on a logarithmic scale as shown in Fig. 2, with the ordinate representing the pseudo-velocity, V, and the abscissa the natural frequency of the system, f. On such a plot, diagonal lines extending upward to the right represent constant values of U, and diagonal lines extending downward to the right represent constant values of A. Therefore, once the diagonal scales have been established, from a plot of this type the values of U, V and A for a given frequency can be determined directly.

SPECTRA FOR ELASTIC SYSTEMS

Unless otherwise indicated, the systems considered in this section will be assumed to have no damping.

Half-Cycle Velocity Pulses. Figure 3a shows the displacement, velocity and acceleration diagrams for the first class of input motion to be investigated. This motion will be identified as a half-cycle velocity input. The acceleration trace of this input consists of a sequence of two half-cycle pulses of the same area but opposite signs, and the displacement trace increases from zero to a maximum value that is maintained indefinitely. A dot superscript denotes differentiation with respect to time, and the subscript 'o' denotes the maximum value of the function to which it is attached.

For design purposes, the deformation spectrum for undamped systems subjected to this class of input may be approximated by the diagram shown in Fig. 4. This spectrum, and the design spectra to be presented later for other classes of ground motion, are based on study of existing information (2-5), the numerical data presented in Ref. (1), and various theoretical considerations. It should be recalled that the spectrum is displayed on a logarithmic plot, on which diagonal lines extending to the right and to the left represent constant values of U and A, respectively. To a first approximation, this spectrum may be represented by the solid lines as follows:

1. Along the diagonal line ab, the deformation U is equal to the maximum ground displacement y_o.

2. Along the horizontal line bc, the pseudo-velocity V is approximately equal to 1.5 times the maximum ground velocity $\dot{y}_o$.

3. Along the diagonal line cd, the pseudo-acceleration A is 1.5 times the numerical sum of the maximum and minimum ordinates of the acceleration diagram. The maximum possible value of A is therefore $3\ddot{y}_o$, and it is obtained for an acceleration function for which the positive and negative peaks are equal. Strictly speaking, the durations of the two acceleration pulses must also be of the same order of magnitude, otherwise the result will in general be conservative.

4. Along the diagonal line ef, the pseudo-acceleration A is equal to the maximum ground acceleration $\ddot{y}_o$. This means that the maximum spring force is equal to the product $m\ddot{y}_o$.

5. The frequencies corresponding to the boundaries of the transition curve de are determined approximately from the expressions given in the figure. The location of point d is related to the duration of the dominant acceleration pulse, $t_{1,a}$, (see Fig. 3a), whereas the location of point e is related to the shortest "effective" rise time of the peak acceleration, $\overline{t}_{r,a}$. As shown in Fig. 3a, the latter quantity is generally smaller than the actual rise time, $t_{r,a}$.

A somewhat more accurate representation of the central portion of the spectrum is provided by the dotted curve ghd. Point g corresponds to the frequency for which $V = \dot{y}_o$, whereas the frequency corresponding to point h may be determined from the duration of the velocity pulse, t_d, and from the peak and average ordinates of the velocity diagram, $\dot{y}_o$ and $\dot{y}_{av}$, as shown in the figure.

In Fig. 5 are shown deformation spectra determined by actual computations for systems with damping factors β in the range between zero and 100 percent critical subjected to a versed-sine velocity pulse. The input acceleration is a full-cycle sinusoidal wave. In this plot, the scales for U, V and A are normalized with respect to y_o, $\dot{y}_o$ and $\ddot{y}_o$, respectively, and the frequency axis is expressed in the dimensionless form $t_d f$, where t_d denotes the duration of the velocity half-cycle. It can be seen that the spectra for undamped systems and systems with less than 2 percent damping are in excellent agreement with the approximate spectrum presented in Fig. 4. Specifically, these spectra are bounded on the left by the line $U = y_o$, the peak value of V is of the order of $1.5\dot{y}_o$, the maximum value of A is of the order of $3\ddot{y}_o$, and, at the extreme right, the curves approach the value $A = \ddot{y}_o$. Furthermore, the locations of the maximum values of V and A, and of the point beyond which A can be taken equal to $\ddot{y}_o$ are in very good agreement with the results obtained from the expressions given in Fig. 4. The effect of damping will be considered in a later section.

Half-Cycle Displacement Pulses. This class of input is as shown in Fig. 3b. The displacement and velocity functions in this case are similar to the velocity and acceleration functions of the motion considered in Fig. 3a. The deformation spectrum for this input may be approximated by the diagram shown in Fig. 6, the characteristics of which are as follows:

1. Along the limiting lines ab and gh, the relationship between the

input and response quantities are the same as for the corresponding lines ab and ef in Fig. 4. Furthermore, the frequency corresponding to point g is the same as that for point e in Fig. 4.

2. Unlike the spectrum for the half-cycle velocity input presented in Fig. 4 which is bounded on the left by the line $U = y_o$, the present spectrum exhibits the hump bcd along which the value of U exceeds the maximum ground displacement. Line bc is defined by the expression

$$U = 2\pi(ft_d)y_{av} \tag{2}$$

in which y_{av} denotes the average ordinate of the displacement diagram. Along the diagonal line cd, $U = 1.5y_o$. It should be noted that the latter relation is the same as that between V and $\dot{y}_o$ for line bc in Fig. 4, the analogy being a consequence of the similarity of the displacement and velocity functions for the inputs considered in the two cases. It might be of interest to note that along the lines bcd, the maximum deformation occurs after termination of the pulse, whereas along the line ab, the maximum deformation and the maximum ground displacement occur at approximately the same instant.

3. Along the horizontal line de, V is equal to 1.5 times the numerical sum of the maximum and minimum ordinates of the input velocity. As a consequence, the maximum value of V is no longer limited to a value of about $1.5\dot{y}_o$, as was the case for the half-cycle velocity input considered earlier, but may be as high as $3\dot{y}_o$. This expression for line de is analogous to that for line cd in Fig. 4, the analogy being a consequence of the similarity of the velocity and acceleration functions for the inputs considered in the two cases.

4. Along the diagonal line ef, the pseudo-acceleration A is proportional to $\ddot{y}_o$, the ratio of proportionality depending on the degree of periodicity of the input acceleration function. If the durations of the individual acceleration pulses are approximately the same, then the value of A along this line may be determined from the expression

$$A = 1.5 \sum_{j=1}^{n} |(\ddot{y}_o)_j| \tag{3}$$

in which $(\ddot{y}_o)_j$ denotes the peak value of the jth half-cycle, and n denotes the number of half-cycles present. If the amplitudes of the individual waves are of the same order of magnitude but their durations differ appreciably from one another, then the result obtained from this equation will in general be conservative.

5. The frequency corresponding to point f of the transition curve fg is determined from the expression shown in the figure, where $t_{1,a}$ should be interpreted as the average duration of the two most intense acceleration half-cycles.

A somewhat better representation of the central portion of the diagram may be obtained by replacing the solid curve by the dotted curve. The expressions for the locations of points i and j in Fig. 6 are analogous to those given in Fig. 4 for points h and d, respectively. These points should not be taken outside the limits of the lines on which they are indicated in the figures.

In Fig. 7 are presented deformation spectra determined by actual computations for undamped and damped systems subjected to a half-cycle displacement pulse. The acceleration diagram of the input motion consists of a sequence of three half-sine pulses of the same amplitude and durations t_1, $2t_1$, and t_1, as shown in the inset diagram. The scales of all axes in this figure are normalized in a form analogous to that used in Fig. 5. It can be verified that the curves for lightly damped systems (of the order of 2 percent critical or less) are in good agreement with the design spectrum proposed in Fig. 6. Specifically, at the extreme left, the curves approach the value of $U = y_o$; the peak value of U is of the order of $1.5y_o$; the peak value of V is about $3\dot{y}_o$; the peak value of A is about $4\ddot{y}_o$; and, at the extreme right, the curves approach a value of $A = \ddot{y}_o$. In addition, the locations of the peak values of U, V and A, of the break in the curves at the left, and of the point at the right beyond which A can be considered to be equal to $\ddot{y}_o$ conform very closely to the approximate rules that have been presented.

Full-Cycle Displacement Pulses. An approximate design spectrum has also been developed for full-cycle displacement pulses. The general shape of these pulses is similar to that of the velocity function considered in Fig. 3b. Although the detailed features of this spectrum cannot be described because of space limitations, the following general comments can be made about the relationship of this spectrum to the one presented in Fig. 6 for half-cycle displacement pulses.

1. In the right-hand part of the spectrum, the relationship between the pseudo-acceleration and the input acceleration is the same as for the spectrum presented in Fig. 6.
2. In the central part, the expression for the maximum pseudo-velocity should be generalized to include the effect of all half-cycles in the velocity diagram of the input function. The resulting expression will therefore be analogous to that given in Fig. 6 for the maximum value of A.
3. The maximum value of U is no longer limited to $1.5y_o$, but is approximately equal to 1.5 times the numerical sum of the maximum and minimum ordinates of the displacement diagram. This relation is analogous to that given in Fig. 6 for the maximum value of V.
4. In the left-hand part of the diagram, the transition curve between the limiting value of $U = y_o$ and the maximum value of U can be defined accurately, but it is necessary to consider the detailed shape of the input displacement function.

Complex Inputs. From the information that has been presented it is clear that the left-hand portion of the deformation spectrum can most conveniently be related to the characteristics of the displacement trace of the input motion, whereas the central and the right-hand portions can most conveniently be related to the characteristics of the velocity and acceleration traces, respectively. The significant features of each function are the number, amplitude and duration of half-cycles present.

In order to be able to apply the approximate rules that have been developed to more complex inputs, in addition to the acceleration diagram, it is necessary to have the velocity and displacement histories of the ground motion. One must then identify on these functions the pulse or pulses that can be expected to have a controlling influence on the maximum deformation of systems in the various regions of the response spectrum. These pulses can usually be determined from consideration of their amplitudes and durations. In this connection, it should be recalled that the peak value of U is related to the duration of the dominant displacement pulse of the input motion, the peak value of V is related to the duration of the dominant velocity pulse, and the peak value of A is related to the average duration of the dominant acceleration pulses. Finally, the frequency beyond which A may be taken equal to $\ddot{y}_o$ is related to the effective rise time of the acceleration function, $\bar{t}_{r,a}$.

For the ground motions associated with the effects of blast and earthquakes, superimposed on the primary or dominant components of the input function there are secondary oscillations of small amplitudes and usually high frequencies. These oscillations are most pronounced in the acceleration and velocity curves, and their overall effect is to broaden the regions of the spectrum where V and A attain their maximum values.

As an illustration, the significant features of the deformation spectrum for an earthquake record will be estimated and compared with the results obtained by actual computations. The motion considered is the S11°E component of the Eureka, California earthquake of 21 December 1954, the acceleration, velocity and displacement diagrams of which are shown in Fig. 8. This record has been adjusted by Wiggins in the course of another study (6). While it would be obviously impossible to estimate the deformation spectrum for this input from the characteristics of the acceleration diagram alone, a reasonable estimate of the spectrum can be made with the aid of the two additional diagrams that have been presented.

The fact that the high intensity waves for all three diagrams in Fig. 8 are concentrated in the interval between 2.5 and 7 seconds suggests that this portion of the record would control the response of systems having any natural frequency. That this is indeed the case has been verified by the results of actual computations.

In the displacement trace, the dominant wave is essentially a skewed half-sine pulse with an amplitude about equal to the maximum ground displacement and a duration of $t_d \simeq 3.3$ secs., as shown by the dotted curve. In the velocity trace, the dominant wave may be expressed as the sum of two components: 1) a primary, full-cycle pulse having an amplitude of about $0.7\dot{y}_o$ and an average duration per half-cycle of $t_{1,v} \simeq 1.8$ secs., as indicated in the figure, and 2) a secondary component dominated by a sequence of three half-cycle pulses of smaller amplitudes and average duration of about 0.6 secs. Finally, detailed examination of the original acceleration trace reveals that there are from three to five closely spaced half-cycle pulses of nearly equal duration and amplitudes of the order of the maximum input acceleration. The average duration of these pulses is $t_{1,a} \simeq 0.25$ secs., and the effective rise time for the dominant acceleration pulse is about one-half as great; i.e. $\bar{t}_{r,a} = 0.125$ secs.

On the basis of this information, the deformation spectrum for systems with $\beta \leq 0.02$ would be expected to have the general shape of the diagram presented in Fig. 6, except that the middle region of the spectrum would exhibit at least two major peaks, corresponding to the maximum effects of the primary and secondary components of the dominant velocity pulse. The latter peaks would be expected at frequencies of 0.6/1.8 = 0.33 cps and 0.6/0.6 = 1 cps. In the left-hand part of the spectrum, the value of U would be expected to be of the order of y_o for frequencies for which the right-hand member of Eq. 2 is less than y_o, that is for

$$f < \frac{1}{2\pi} \frac{\pi}{2} \frac{1}{3.3} = 0.076 \text{ cps},$$

and the maximum value of $U \simeq 1.5y_o$ would be expected to occur at a frequency

$$f \simeq 0.4 \frac{\pi}{2} \frac{1}{3.3} = 0.19 \text{ cps}.$$

The maximum value of V would be estimated to be somewhat in excess of 3 times the amplitude of the primary component of the dominant velocity pulse, or a little greater than $3(0.7\dot{y}_o) = 2.1\dot{y}_o$. Similarly, the maximum value of A would be expected to be roughly of the order of $4.5\ddot{y}_o$ to $7.5\ddot{y}_o$, and to occur at a frequency of about 0.6/0.25 = 2.5 cps. For completely undamped systems, the actual maximum values of V and A may be even greater because of the influence of the low-amplitude, high-frequency oscillations which tend to produce a nearly resonant condition. Finally, the frequency beyond which A may be considered to be equal to $\ddot{y}_o$ would be expected to be about 1.25/0.125 = 10 cps.

Figure 9 presents the results of actual computations for systems with up to 40 percent critical damping. It can be verified that the spectra for systems with $\beta \leq 0.02$ are in good agreement with the estimates referred to above.

Effect of Damping. It is well known that the effect of damping is to reduce the magnitude of the maximum deformation and to smooth out the humps and undulations of the response spectra. The data presented in Figs. 5, 7 and 9 show that the reduction achieved with a given amount of damping is different in the various frequency regions and, within a given frequency region, it is different for the various inputs.

In the low-frequency region of the spectrum, the reduction is generally small because the maximum deformation of the system is reached at an early stage of the motion--usually at a fraction of its natural frequency--and the effect of damping, which requires time to exhibit itself, cannot be felt. In the high-frequency region, the effectiveness of damping depends mainly on whether the ground acceleration is a continuous or a discontinuous function. For the continuous inputs considered here, the effect is also small in this region. The greatest effect is obtained in the medium-frequency region of the spectrum where, other things being equal, the greater the periodicity of the velocity trace of the input motion, the greater is the resulting reduction in the value of the maximum deformation.

RESPONSE OF ELASTOPLASTIC SYSTEMS

The study reported in this section is an extension of an earlier investigation (7). The resistance-deformation relationship of the systems considered is shown in Fig. 10a. The yield levels in the two directions of deformation are considered to be the same, and unloading from a point of maximum deformation is assumed to take place along a line parallel to the initial elastic portion of the diagram. The yield point deformation is denoted by u_y, and the yield force by Q_y.

The problem at hand is to determine the relationship between the yield level of the system and the absolute maximum deformation that the system will undergo when subjected to a given ground motion. The latter quantity is denoted by the symbol u_m.

Maximum Deformations of Inelastic and Elastic Systems. It is instructive to relate the maximum deformation of the elastoplastic system to that of an elastic system having the same stiffness as the initial stiffness of the inelastic system. The resistance diagrams of the two systems are shown in Fig. 10b, where u_o denotes the numerical value of absolute maximum deformation of the elastic system, and Q_o denotes the corresponding spring force.

One might be inclined to expect that the maximum deformation of the inelastic system, u_m, will always be greater than of the associated elastic system, u_o, and that these two quantities will be related in such a way that the areas under the load-deformation diagrams for the two systems up to their respective point of maximum deformation are equal. However, it turns out that this result, which may be interpreted to be a consequence of the principle of conservation of energies, is valid only under very restrictive conditions.

In Fig. 11 the maximum deformation of an elastoplastic system without damping is compared with the maximum deformation of the associated elastic system. The input motion is a parabolic velocity half-cycle, as shown in the inset diagram, and the yield level of the inelastic system is considered to be one-fourth of that required for elastic behavior. The results are presented in a dimensionless form as a function of the frequency parameter $t_d f$, in which the natural frequency for the inelastic system is determined from the slope of the initial, elastic portion of its resistance diagram.

It can be seen from this figure that the relationship between u_m and u_o is not constant, but differs significantly in the different ranges of the frequency parameter. In the right-hand portion of the diagram, corresponding to systems with high natural frequencies, the values of u_m are indeed greater than u_o, but even, within this region, the relationship between u_m and u_o does not conform to that obtained on the basis of the concept of preservation of energies. In the left-hand part of the diagram, corresponding to low-frequency systems, the maximum deformations of both the elastic and the inelastic systems are for all practical purposes equal to the maximum ground displacement. And more important, in the central part of the diagram, the maximum deformation of the elastoplastic systems is smaller than of the associated elastic systems.

In Fig. 12 are given similar plots for systems with 2 percent critical damping subjected to the Eureka earthquake referred to earlier. The damping factor for the inelastic system is based on the stiffness of the elastic range of its resistance diagram. The essential trends of these curves can be seen to be similar to those of the curves presented in the preceding figure for the simple input.

Spectra for Yield Deformation. In Fig. 13 are presented response spectra for elastoplastic systems without damping subjected to the half-cycle velocity pulse shown in the inset diagram. The results presented are for ductility factors, μ, in the range between $\mu = 1$ and $\mu = 10$, where μ is defined as

$$\mu = u_m/u_y \tag{4}$$

As before, a four-way logarithmic plot is used, except that on the diagonal scale on the left is plotted the yield deformation of the system, u_y, instead of the maximum deformation u_m. Furthermore, the pseudo-velocity, V, and pseudo-acceleration, A, which are plotted on the vertical and the right-hand diagonal scales are redefined as follows:

$$V = pu_y \quad \text{and} \quad A = pV = p^2 u_y \tag{5}$$

Accordingly, the three response quantities used in this plot are alternative measures of the yield deformation, and the resulting curves give the value of the yield deformation or yield resistance which, for the given input, is necessary to limit the maximum deformation of the system to a specified multiple of its yield deformation. This form of presentation is particularly convenient for purposes of design. Having evaluated u_y, the value of u_m may be determined by taking the product μu_y. It should be noted that the above definitions of V and A for the elastoplastic system are consistent with those used for elastic systems, since the yield deformation of an elastic system may be considered to be equal to the maximum deformation u_o.

Similar data have been evaluated for numerous other inputs, including both pulse-like excitations and several earthquake motions. In Fig. 14 are given the results obtained for systems with 2 percent critical damping subjected to the Eureka earthquake record referred to previously. It can be seen that the average relationships between the spectra for the inelastic and the elastic systems ($\mu = 1$) for the various ranges are quite similar to those applicable to the simple input considered in Fig. 13.

Approximate Design Rules. On the basis of the information that has been presented in this paper and the additional data given in (1) and (8), the following approximate rules have been developed for relating the spectra for the yield deformation of elastoplastic systems to the deformation spectra of elastic systems.

1. In the left-hand, low-frequency region of the spectrum for which the maximum deformation of the elastic system may be taken equal to the maximum ground displacement, the maximum deformation of the inelastic and elastic systems may be considered to be the same, i.e. $u_m = u_o = y_o$. Therefore, the

ordinates of the spectra for the yield deformation of inelastic systems are $1/\mu$ times those of the associated elastic systems.

2. In the extreme right-hand part of the spectrum where the pseudo-acceleration A for the elastic systems is of the order of the maximum ground acceleration, the value of A for both the elastic and the elastoplastic systems may be considered to be the same. In other words, if the yield deformation of the elastoplastic system is less than the maximum deformation of the associated elastic system, the inelastic component of the resulting deformation may be excessive. Hence, both the elastic and the inelastic systems must be designed for a static force equal to the product of the mass of the system and the maximum ground acceleration.

3. In the intermediate frequency range, the relationship between the two systems is generally more complex, but it may be stated approximately in terms of the characteristics of the elastic spectrum as follows:

(a) For natural frequencies extending from the low-frequency region considered under paragraph 1 to a value located at approximately 1/3 the distance between the frequency for which the elastic spectrum dips sharply to the right and the frequency beyond that point for which $V = \dot{y}_o$, the maximum deformation of the inelastic and the associated elastic systems may be considered to be the same, and the expression $1/\mu$ may be used as indicated previously under paragraph 1. This relation is fairly accurate for half-cycle velocity inputs for which the maximum values of U and V are of the order of y_o and $1.5\dot{y}_o$ respectively. However, when the maximum value of U is of the order of $1.5y_o$ and the maximum value of V is of the order of $3\dot{y}_o$, as is the case with half-cycle displacement pulses and many earthquake motions, the maximum deformation of the inelastic systems will in general be less than that of the corresponding elastic systems. The difference may be appreciable for ductility ratios in the range between $5 \leq \mu \leq 10$ and natural frequencies close to those associated with the maximum values of U and V. In general, the greater the maximum values of U/y_o and $V/\dot{y}_o$, the more conservative are the values of u_y determined from the expression u_o/μ. In the latter cases, a somewhat better estimate of the yield deformation may be obtained from the expression $u_y = y_o/\mu$, that is by taking the maximum deformation of the elastoplastic system equal to the maximum ground displacement. This estimate may not be conservative, however.

(b) For the range of the moderately high natural frequencies included between the ranges considered in paragraphs 3a and 2, the relations between the spectra for the inelastic and elastic systems is sensitive to changes in the value of the natural frequency. However, the spectrum curves within this region can usually be determined by drawing smooth transition curves between the curves established in paragraphs 2 and 3a. When this cannot be done readily, the following equation, expressing the concept of preservation of energies, may be used as a guide.

$$\frac{u_y}{u_o} = \frac{1}{\sqrt{2\mu - 1}}$$

This relation may be considered to be valid within the relatively narrow frequency range for which the pseudo-velocity of the associated elastic system is from about 1.0 to 0.8 times the maximum ground velocity.

In the application of these rules, it is recommended that the spectrum for elastic systems be represented by a smooth "mean" curve, without the undulations that are characteristic of response spectra.

Limitations. The approximate relations presented above are proposed for elastoplastic systems with damping factors of the order of $\beta \leq 0.05$, and ductility factors of the order of $\mu \leq 10$. Furthermore, the ground motions are considered to be such that the maximum possible deformation of elastic systems is no more than $2.5y_o$, the maximum pseudo-velocity is no more than $3.5\dot{y}_o$, and the maximum pseudo-acceleration is less than about $4.5\ddot{y}_o$. Included in these inputs are ground motions dominated by a half-cycle or full-cycle velocity pulse, a half-cycle or full-cycle displacement pulse, and many strong motion earthquake records. Finally, for earthquake motions, these rules should also be limited to systems having a minimum of one percent critical damping, otherwise they may prove to be conservative near the regions of the spectrum where V and A are maximum.

Relative Effects of Inelastic Action and Damping. The results presented show that the general effect of inelastic action is to reduce the load for which a structure must be designed to resist a prescribed ground motion. In this sense, this effect is similar to that of viscous damping. However, from a comparison of the data presented in Figs. 5, 7 and 9 with those given in Figs. 13 and 14, it can be seen that the relative effectiveness of these factors is quite different in the various regions of the spectrum. In particular, in the low-frequency range of the spectrum for which the effect of damping may be considered to be negligible, the effect of inelastic action is extremely important. These results show clearly that, in general, the effect of inelastic action cannot be considered in terms of a fixed amount of "equivalent damping".

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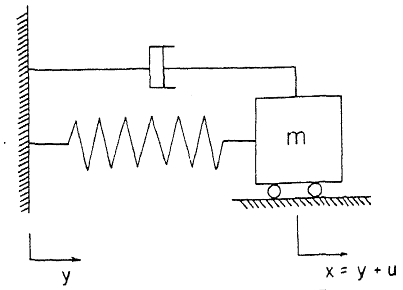

FIG. 1 SYSTEM CONSIDERED

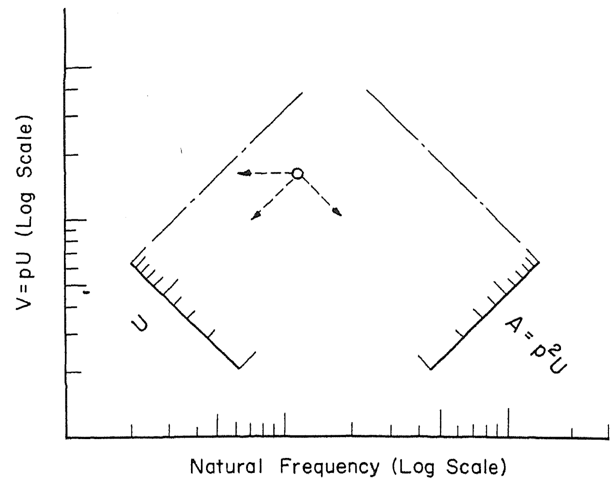

FIG. 2 FORMAT OF DEFORMATION SPECTRA

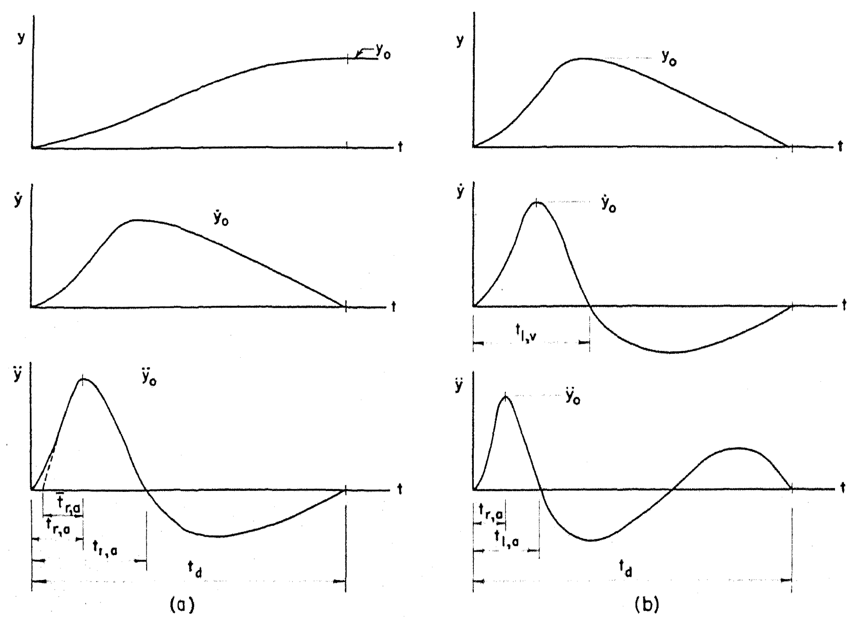

FIG. 3 HALF-CYCLE VELOCITY AND DISPLACEMENT PULSES

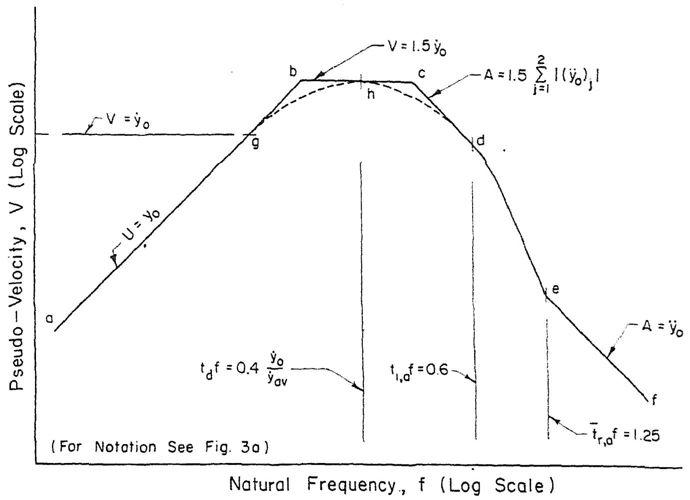

FIG. 4 TYPICAL DEFORMATION SPECTRUM FOR HALF-CYCLE VELOCITY PULSES

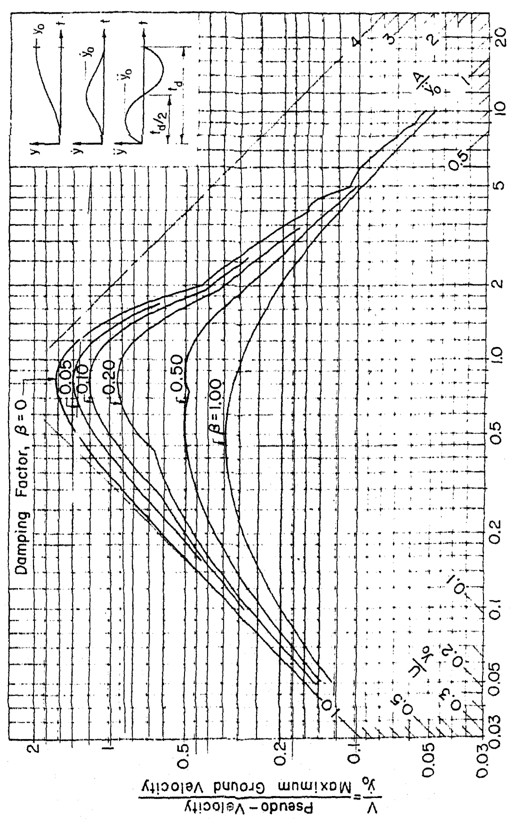

FIG. 5 DEFORMATION SPECTRA FOR VERSED-SINE VELOCITY PULSE

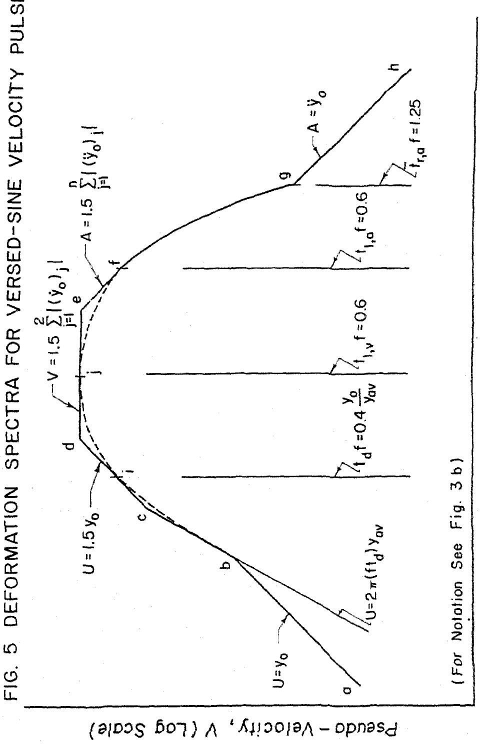

FIG. 6 TYPICAL DEFORMATION SPECTRUM FOR HALF-CYCLE DISPLACEMENT PULSES

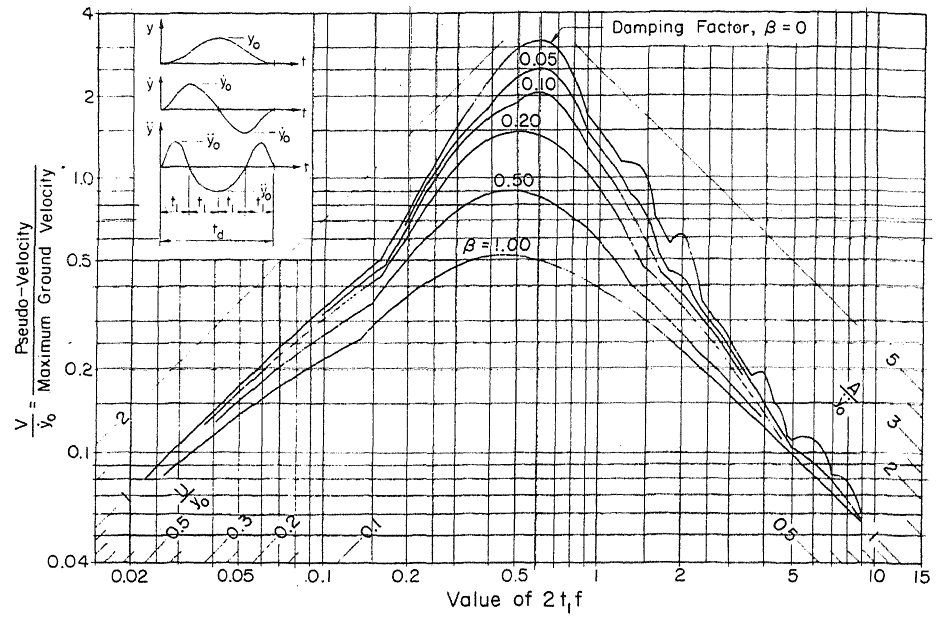

FIG. 7 DEFORMATION SPECTRA FOR HALF-CYCLE DISPLACEMENT PULSE

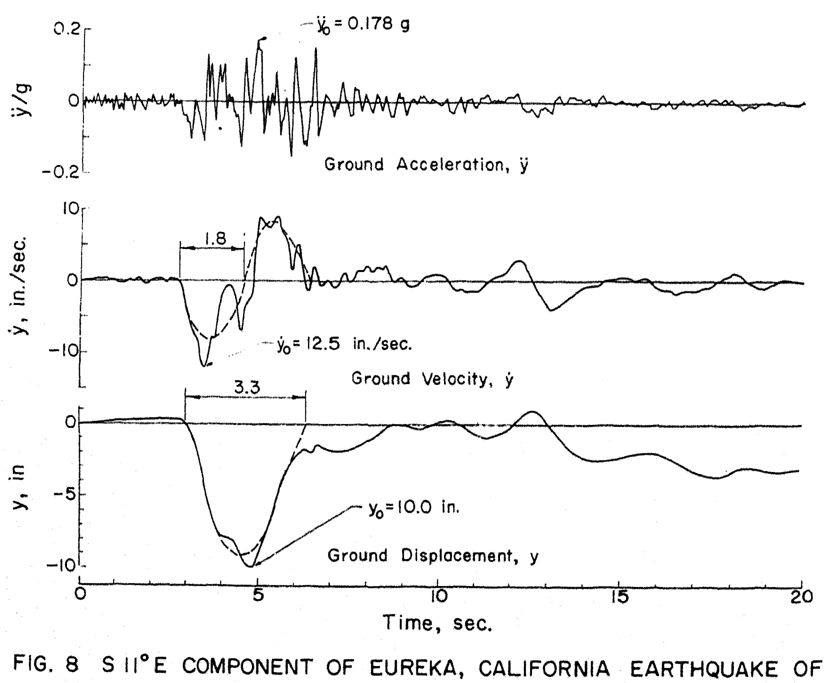

FIG. 8 S 11° E COMPONENT OF EUREKA, CALIFORNIA EARTHQUAKE OF DEC. 21, 1954

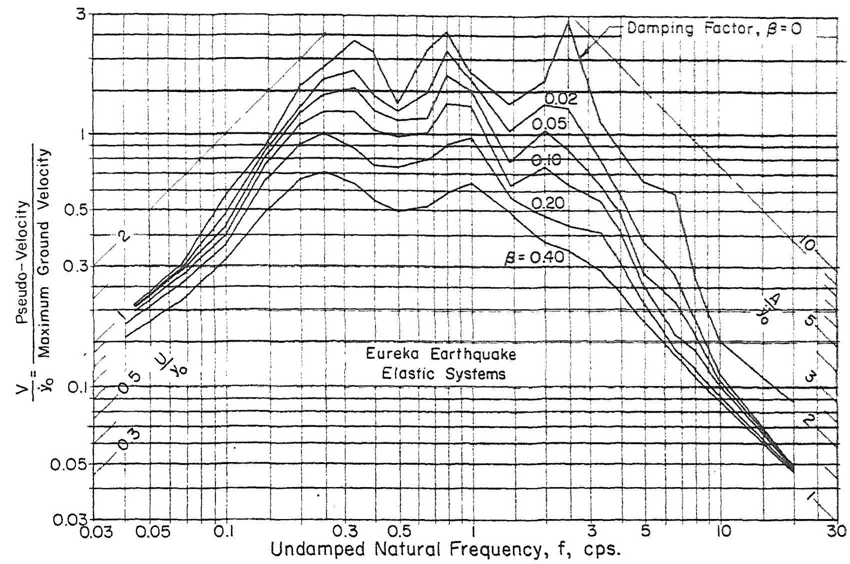

FIG. 9 DEFORMATION SPECTRA FOR EUREKA EARTHQUAKE

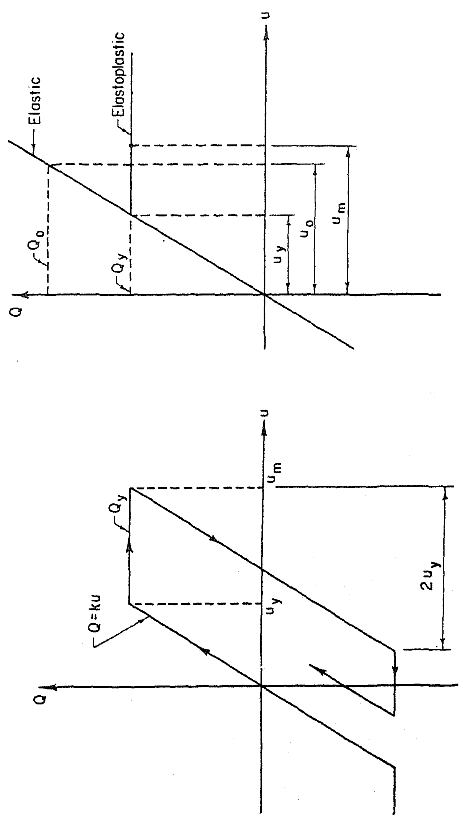

FIG. 10 RESISTANCE-DEFORMATION DIAGRAMS FOR ELASTOPLASTIC SYSTEMS

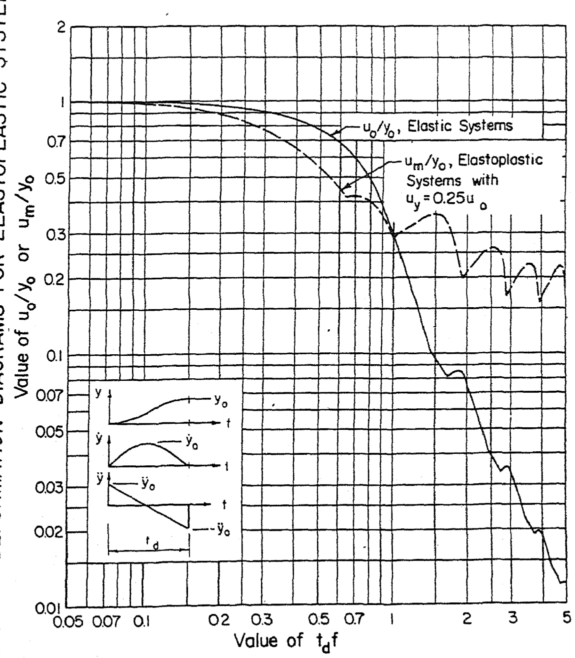

FIG. 11 COMPARISON OF MAXIMUM DEFORMATIONS FOR ELASTOPLASTIC AND ELASTIC SYSTEMS — Parabolic Velocity Half-Cycle

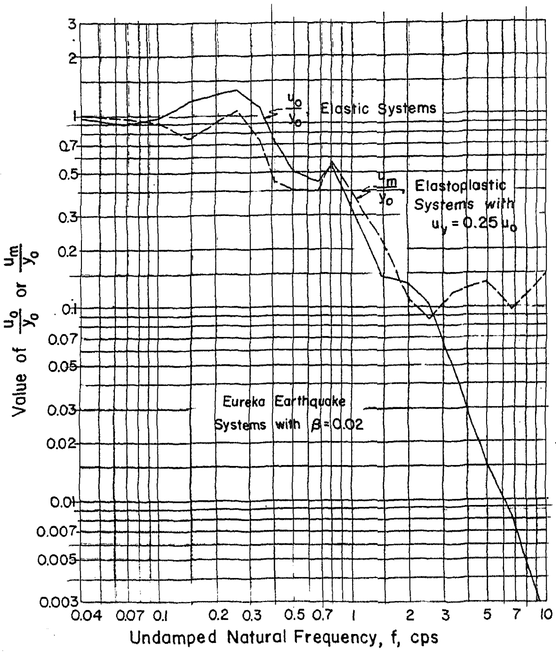

FIG. 12 COMPARISON OF MAXIMUM DEFORMATIONS FOR ELASTOPLASTIC AND ELASTIC SYSTEMS —— Eureka Earthquake

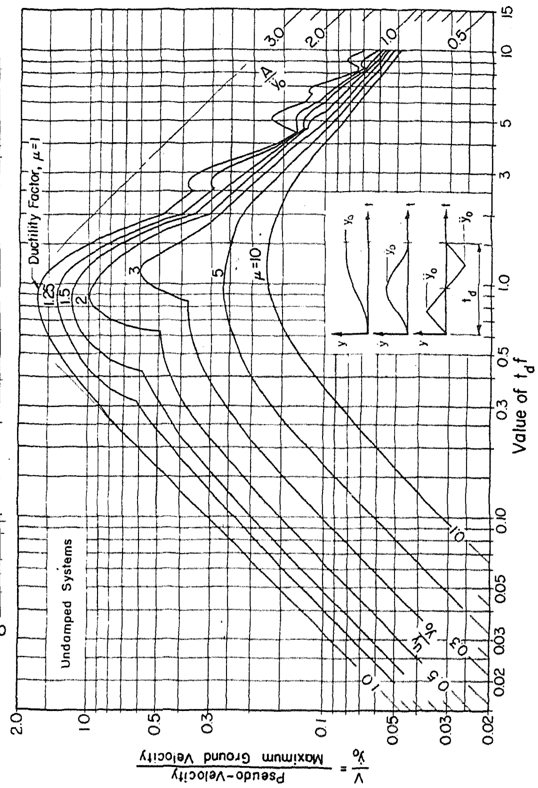

FIG. 13 DEFORMATION SPECTRA FOR ELASTOPLASTIC SYSTEMS SUBJECTED TO A HALF-CYCLE VELOCITY PULSE

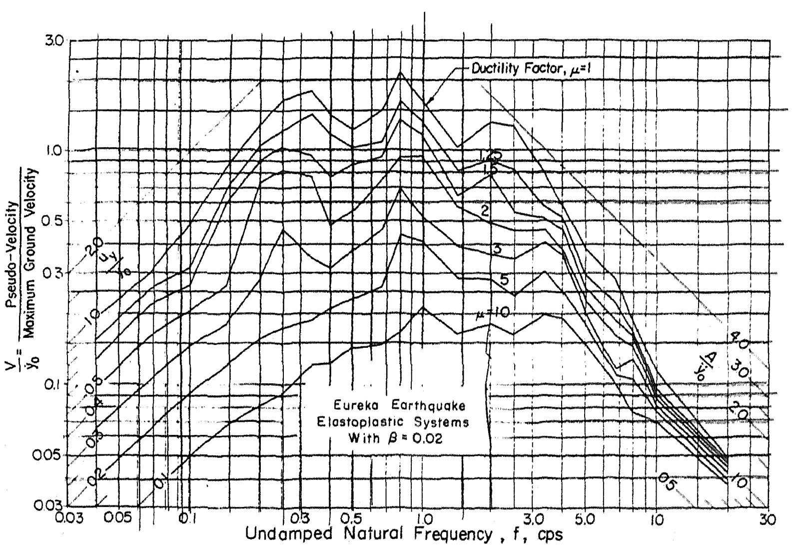

FIG. 14 DEFORMATION SPECTRA FOR ELASTOPLASTIC SYSTEMS SUBJECTED TO EUREKA EARTHQUAKE

DEFORMATION SPECTRA FOR ELASTIC AND ELASTOPLASTIC SYSTEMS

BY A.S. VELETSOS, N.M. NEWMARK AND C.V. CHELAPATI.

QUESTION BY: F.J. BORGES - PORTUGAL.

The results obtained in the present paper, although mainly concerning simple types of pulses, compares well with those obtained considering random noise vibrations with a power spectrum conveniently limited in frequency, and with those obtained considering recorded accelerograms. It seems so that we may be confident in applying these results for practical purposes.

REPLY BY: A.S. VELETSOS

I am in complete agreement with your evaluation.

It might be worth emphasizing on this occasion that the results obtained in this study are limited by the assumptions on which they are based, and that care should be exercised in applying the relationsips presented to systems of greater complexity than the single-degree-of-freedom systems considered. This is particularly true of the conclusions drawn with the regard to the response of systems in the elastic range of deformation. For elastic systems, the response of multi-degree-of-freedom systems can, of course, be evaluated by superposition of the responses of a series of single-degree-of-freedom systems. Also, whereas in the low-frequency and the medium-frequency ranges of the spectrum, the relationship between the maximum deformations of the inelastic and the associated elastic systems may, for most practical purposes, be considered to be independent of the time-history of the input motion. In the high-frequency region, this relationship depends greatly on whether the input acceleration is a continuous or a discontinuous function. This difference should be kept in mind because the acceleration diagrams used in some of the probabilistic studies that have been made were discontinuous functions. The boundaries between the medium-frequency and high-frequency ranges of the spectrum are defined in the paper.

QUESTION BY: T. HATANO - JAPAN.

Nearly all of the papers presented in this conference on the seismic analysis of buildings stand on the assumption of the shearing vibration and fluid viscosity (Fig. 1) considering miscellaneous types of inelastic resistance-deformation relations (Fig. 10). But were these assumptions made from the observed facts on the prototype buildings or not? If the equations are made from the philosophical assumption even the solutions made from computers may be useless.

I think, we must start from the observations on the real structures and make equations to be able to explain the real behaviour of the structures.

REPLY BY: A.S. VELETSOS

The characteristics of the model considered in this study represent an idealization of those for an actual structure behaving essentially as a single-degree-of-freedom system. The greatest uncertainty in this idealization is the resistance-deformation relationship used and particularly the conditions assumed under reversals of loading from regions of inelastic deformation. As far as it is known, there are no data available from tests on actual structures to evaluate the adequacy of these assumptions. However, the limited data available from static and dynamic tests on structural components and simple structural frames suggest that the assumed relationships are reasonable representations of actual conditions. Furthermore, the analytical studies reported in Ref. 1 and the results of additional studies now in progress indicate that, within a wide range of natural frequencies, the maximum deformation of inelastic systems is relatively insensitive to major variations in the characteristics of the resistance diagram. In other words, the uncertainties referred to above appear to be of small practical consequence, with the result that the information presented may be applied to a fairly wide range of practical problems.

Possibly the greatest value of fundamental studies of the type presented in this paper is the insight that they provide into the behaviour of systems of still greater complexity than those analyzed. The results of such studies enable one to identify the important parameters of a problem, and provide a convenient frame of reference for the interpretation of the results of more elaborate analyses that may have to be carried out in a given practical case.