

EARTHQUAKE RESISTANT DESIGN - THE PULSE METHOD

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Synopsis:

This paper attempts to introduce an approach to earthquake resistant design which, as far as the author is aware, has not been expounded before. In this approach, instead of considering the response of a structure to a whole earthquake, as in the spectrum approach and energy methods, only the worst pulses of an earthquake are considered, and the maximum transient response determined for the possible initial conditions.

In this paper results are presented of an analytical investigation into the transient response of a single mass system with an idealised elasto-plastic force deflection characteristic.

It is concluded that the approach may give a valuable insight into the behaviour of structures in future earthquakes.

Introduction:

The approach to earthquake engineering has changed over the years.

The earliest approach was to assume that buildings were rigid structures and design accordingly, the maximum ground motion being the main criterion required for the design.

As the engineer's understanding grew, it became apparent that building structures in general were not rigid but had a certain amount of flexibility. It thus became obvious that, if the problem of earthquake design was to be treated rigorously, it would have to be treated as a problem of dynamics. The problem was further complicated by the fact that the ground motion, which excited the building, was very irregular.

A break-through was made by Biot (1) with his introduction of the concept of an earthquake spectrum, which defined the maximum response of an elastic oscillator to a particular earthquake. This concept was explored thoroughly and applied by numerous investigators, and in particular by Housner, Hudson and their colleagues at Caltech (2,3,4). As a result of these studies it became apparent that the natural period of the building is an important factor in the design of buildings.

Also it became apparent that buildings did not necessarily remain elastic during earthquake vibrations. This further complication was treated in several ways. Housner (5) and Blume (6) attempted to relate the response to the elastic spectrum analyses by invoking energy considerations. Penzien (7), on the other hand, determined new spectra

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taking into account the fact that the elastic limit of a structure had been exceeded. This introduced new criteria again to be considered in design. This is that the post elastic behaviour of a structure is important in earthquake resistant design. This property is often loosely described as ductility.

As the post elastic behaviour of structures varies tremendously, different mathematical models appeared. Penzien (7) treated the ideal elasto-plastic case; Iwan (8) treated the bilinear system; and Jennings treated a generalised curved resistance-deformation relationship. Blume (6) postulated a method to take into account the actual resistance-deformation relationship of structures.

All this work was still centred around the spectrum concept although Caughey (10), Iwan (8) and Jennings (9) had also done work on the periodic response of different hysteretic non-linear systems to a sinusoidal exciting force without attempting to extend these results to the earthquake response problem.

The main objection to the response spectrum technique is that it is dependent entirely on the records of past earthquakes that man has been fortunate enough to record on strong motion instruments. As the number caught is limited, and most of these are of a similar nature, it is not possible, using the spectra at present available, to postulate what would happen to the same structures in earthquakes of differing characteristics.

This paper represents an attempt to overcome this objection.

Approach:

In this paper the problem is tackled by considering the transient response of an idealised vibrating system to a sinusoidal exciting force. The object was to start from a position of rest, apply the sinusoidal force and calculate the amplitude when the system again came to rest. By varying the phase of the exciting force it was possible to obtain the maximum response of the system from the particular starting point for the particular sinusoidal force.

The idealised system considered was that of a concentrated mass suspended by a weightless spring with an elasto-plastic characteristic - see figure 1. The main interest centred around the maximum possible response, and intuitively it was felt that this would be obtained with the initial starting point being at the elastic limit - i.e. when the energy stored in the system is a maximum.

Thus the initial conditions are as shown in figure 2 with the system at rest. The results in this paper are for the maximum amplitude of the next position of rest as the initial phase of the exciting force is varied.

The reasoning behind this approach is that an earthquake record can be considered as a succession of pulses of various magnitudes and frequencies. If one selects the worst of these pulses, then the analysis presented in this paper should yield an approximate upper limit to the response they are likely to incur. The author is thus postulating a method of analysis based on the magnitude and period of the worst pulses in an earthquake and saying that this will give an upper limit to the expected response.

Analysis:

Consider the system shown in figure 1. The equations of motion are :

(1) in the elastic region

$$\ddot{m}\dot{x} = F \cos (\omega t + \phi) - kx$$

(2) in the plastic region

$$\ddot{m}\dot{x} = F \cos (\omega t + \phi) - kx_e \text{ moving in a positive direction}$$

$$\ddot{m}\dot{x} = F \cos (\omega t + \phi) + kx_e \text{ " " " negative direction}$$

- where
- m = mass of concentrated mass
 - x = displacement relative to unstrained position of cantilever
 - F = amplitude of forcing function
 - ω = frequency of forcing function
 - ϕ = initial phase of forcing function
 - k = elastic spring stiffness of cantilever
 - x_e = maximum elastic displacement of the mass.

The initial conditions are: $t = 0; x = x_e; \dot{x} = 0.$

The solutions of the above differential equations were obtained in analytical form and an IBM 1620 FORTRAN program written to evaluate the displacement when \dot{x} was next equal to zero.

The method used was to search for the value of t which would satisfy the required final condition of $\dot{x} = 0$. Where there was a transition from the elastic phase to the plastic phase the value of t corresponding to this transition had to be found and the velocity at this point determined, and these became the initial values for calculations in the plastic region.

For each combination of F and ω the value of ϕ was varied by 0.1π and the maximum determined by assuming a parabolic curve through the peak value and the two adjacent values.

Results:

The program was written so that the results were obtained in a dimensionless form.

The parameters in which the result is expressed are

$$\mu = \left\{ \frac{x}{x_e} \right\}_{\max} \text{ i.e. the maximum response for any particular value of } F \text{ and } \omega.$$

$$\alpha = \frac{F}{kx_e} = \text{a measure of the amplitude of the exciting force.}$$

$$\beta = \omega \sqrt{\frac{m}{k}} = \text{the ratio of the frequency of the exciting force to the natural elastic frequency of the oscillator.}$$

The results obtained from the computer analysis are plotted in figure 3 which shows μ as a function of β for different α .

Discussion of Results:

If instead of a force being applied to the mass an acceleration is applied to the base of the form $A \sin(\omega t + \phi)$ then it can be shown that the same results as above are obtained where μ and β have the same meaning and

$$\alpha = A/g \frac{W}{F_y}$$

Where A = the amplitude of ground acceleration
W = mg = weight of concentrated mass
Fy = the limiting static horizontal force that can be applied
to the mass before it yields.

If a coefficient C_u is defined such that

$$C_u W = F_y$$

then C_u can be referred to as the ultimate seismic coefficient, and will be a measure of the lateral strength of the structure.

Thus

$$\alpha = A/g / C_u$$

Interpreted in this manner figure 3 now becomes a 'damage' chart giving the maximum yield which could occur for any combination of the ratio of amplitude of ground acceleration to ultimate seismic coefficient, and the ratio of building period to period of the maximum pulse.

When looked at in this light, the results indicate that for structures of uniform lateral strength - i.e. same C_u - small earthquakes will affect structures of different periods to approximately the same degree, while large earthquakes will have a far more drastic effect on small period structures than on large period structures. As small period structures often have a higher lateral strength than larger period structures this may explain why small period structures suffer most near the centre of an earthquake while longer period structures seem to suffer worse than small period structures at some distance from the earthquake.

This may mean that the explanation that this phenomenon is caused by filtering out of the short period components during the passage of the earthquake waves, as suggested by Housner (11), may only be of secondary importance.

In figure 4, the results of figure 3 are plotted for the El Centro earthquake taking the maximum acceleration as 0.22g with a period of 0.65 secs. These values were estimated from records presented by Penzien in his paper. (7). The maximum response is plotted against building period for different values of ultimate seismic coefficient. These are compared to Penzien's results from the same paper, which give the spectrum response of an identical oscillator to the El Centro record, replotted

to the parameters used by the author. The author considers the agreement remarkable.

In figure 5, the results of figure 3 are replotted to give $\frac{1}{\alpha}$ as a function of β for different values of μ . This has been labelled as the Ultimate Design Coefficient Chart. For any particular earthquake pulse it shows the value of ultimate design coefficient for which the structure must be designed, given its ductility and period properties. This conforms with the commonly accepted criteria that low period buildings require higher design coefficients.

In figure 6, the results of figure 5 are plotted for the El Centro earthquake taking the maximum acceleration as 0.22g with a period of 0.65 secs. This is compared with the SEOAC design curve. As the SEOAC code is in terms of 'working' loads, the values have to be multiplied by a load factor to obtain the ultimate design coefficient curve. SEOAC curves are shown for load factors of 1.5 and 3, as most buildings probably lie within this range, with steel structures probably tending to the lower value and concrete structures to the higher value. Blume, Newmark and Corning (12) suggest that concrete buildings designed by the SEOAC code should have a ductility factor of at least 4. Curves representing the intermediate zone of the proposed code for New Zealand are also drawn for comparison.

Limitations:

In considering the results presented in this paper the following limitations of the analysis must be remembered.

- (1) An ideal elasto-plastic system has been considered. In practice buildings normally possess a curved resistance deformation curve after yield, may exhibit strain-hardening and often have no purely plastic region.
- (2) No viscous damping has been included. In practice one would expect at least a small proportion of frequency dependent damping of which viscous damping is the simplest to analyse.
- (3) Only a single story structure has been considered. In practice it is the design of multistory buildings with which the earthquake engineer is mostly concerned.
- (4) Nothing has been said about the possibility of more than one excursion into the plastic region and its effect on design.
- (5) A sinusoidal waveform has been considered. In practice the waveform may be closer to parabolic or a more complicated form.

Conclusions:

In spite of the above limitations the author believes the results have considerable significance.

The following are the most significant results: -

- (1) Although in all earthquakes small period structures are more prone to damage than larger period structures assuming both have the same lateral strength, the difference is only slight for smaller intensities but becomes much more marked as the intensity of the earthquake increases, even assuming the character of the earthquake remains the same.
- (2) It has been recognised for some time that the seismic design coefficient depends on the period of the building and its ductility. This paper indicates what may be a practical design method to take these two factors into account. Before this becomes possible, however, much more will need to be known about the post-elastic behaviour of typical earthquake resistant structures.
- (3) This analysis may lead, in particular, to significant results concerning the behaviour of inverted pendulum type of structures, as these probably come closest to the idealised system considered.

Acknowledgement:

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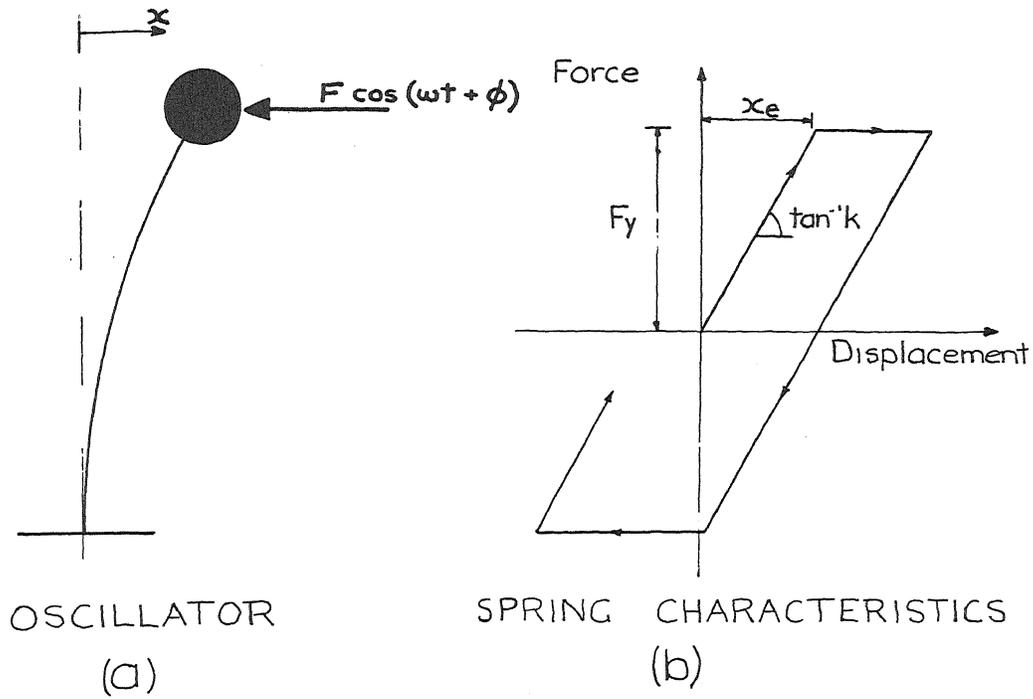


FIGURE 1

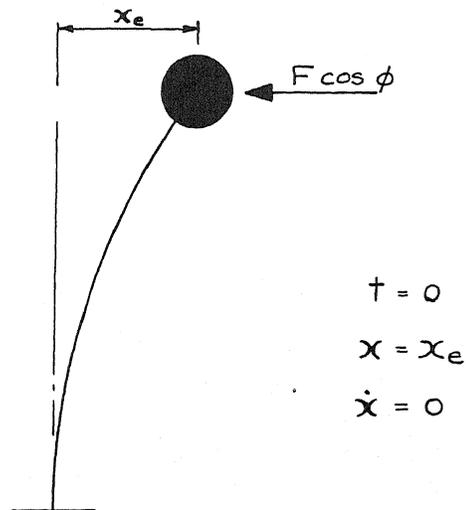


FIGURE 2 - INITIAL CONDITIONS

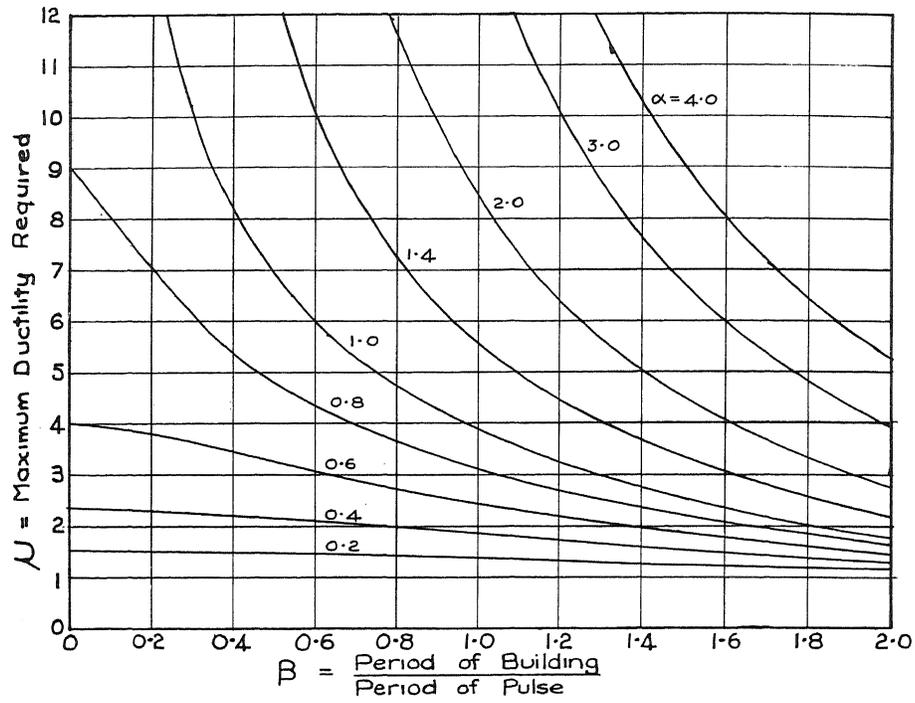


FIGURE 3 - DAMAGE CHART

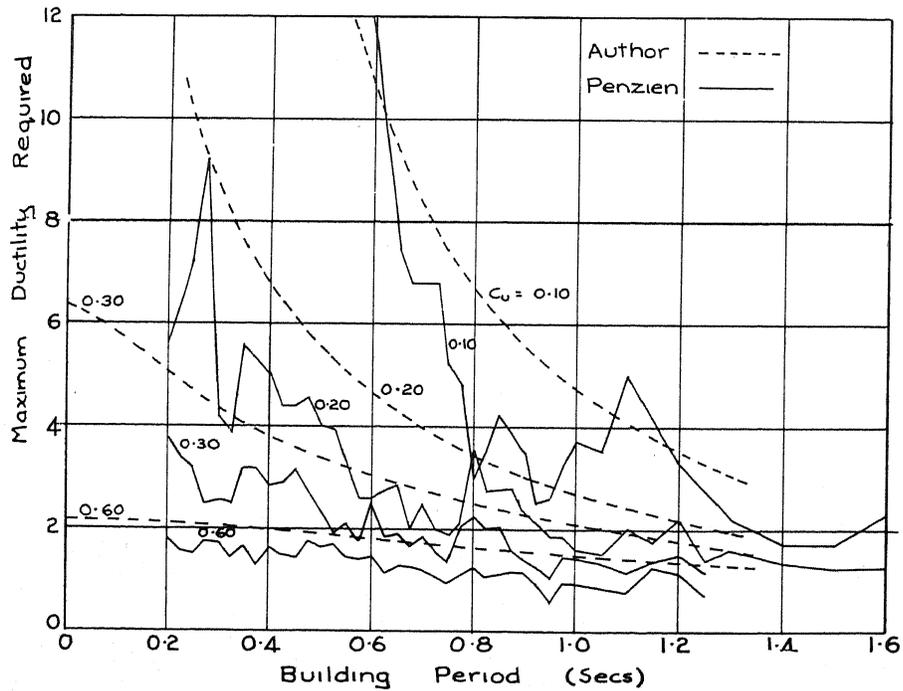


FIGURE 4 - 'EL CENTRO' DAMAGE CURVES

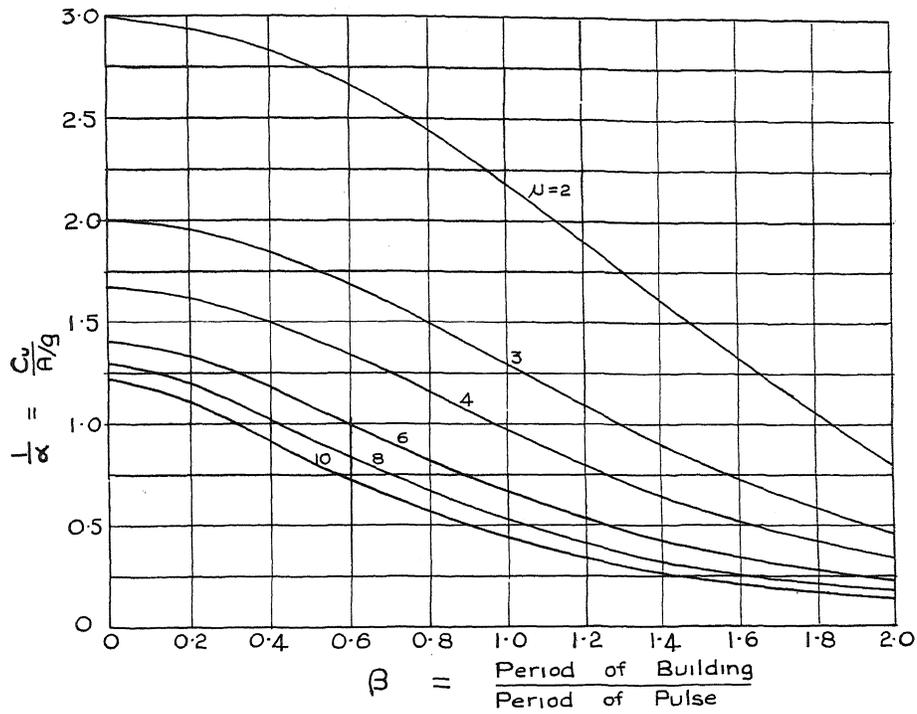


FIGURE 5 - ULTIMATE DESIGN COEFFICIENT CHART

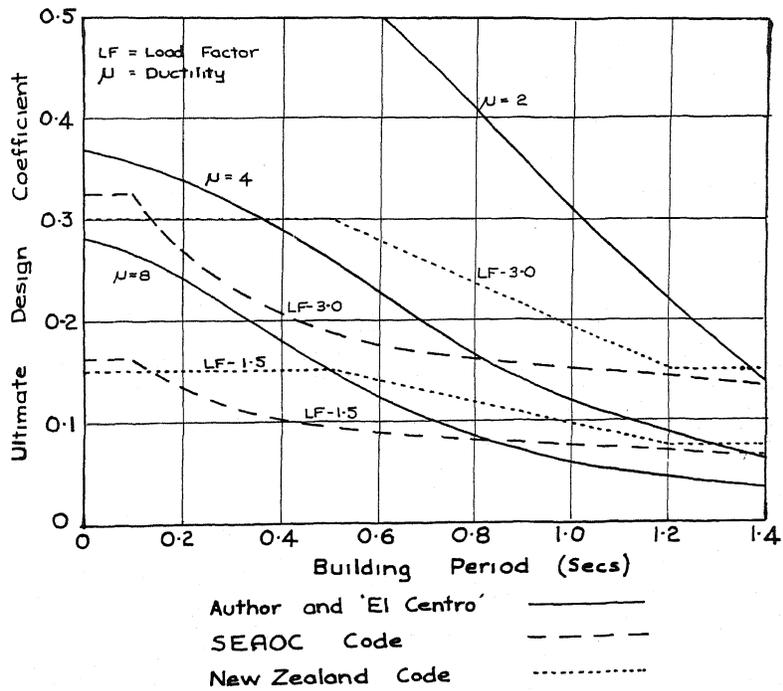


FIGURE 6 - COMPARISON OF DESIGN CURVES

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BY: G.R. WALKER

QUESTION BY:

J.I. BUSTAMANTE - MEXICO

Without commenting further into the paper content, I feel that conclusion (1) must be qualified to avoid confusion; whether a larger period structure is more likely to damage depends on what is to be considered as larger according to the ground characteristics.

AUTHOR'S REPLY:

Professor Bustamante is apparently referring to the phenomenon of a peak value in the acceleration response spectra obtained with elastic structures, indicating a type of resonant condition, and associated with the predominant period of the ground motion. The results reported by the author indicate that such a phenomenon would not be expected to occur in the response spectra for elasto-plastic structures, unless the ground motion was not sufficient to cause the structure to yield, which may happen in the case of very short period structures subjected to a predominantly long period ground motion.

N.B. A more detailed account of the investigation reported in this paper as well as its extension to more complex structures will be found in the author's Ph.D. Thesis.*

* G.R. Walker -

"A Study of the Maximum Transient Response of Simple Fully Yielding Systems".

Ph.D. Thesis. University of Auckland. February 1965.