AN APPROXIMATE METHOD OF ANALYSING COUPLED SHEAR WALLS
SUBJECT TO TRIANGULAR LOADING

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SYNOPSIS

In multistorey buildings lateral forces due to wind and earthquakes are often resisted by shear walls linked by beams. Such a system is effectively a frame with very high column to beam stiffness ratios. This paper presents a quick approximate method of analysing single bay and symmetrical double bay frames under a triangularly distributed horizontal load. Walls of both uniform thickness and variable thickness properties are considered. Charts are provided to determine the axial forces and bending moments in the walls, shear forces in the interconnecting beams and the top deflection.

NOMENCLATURE

a = distance between the centroids of the cross sections of the walls.

A_b = shear area of the connecting beams

A_1, A_2 = cross sectional areas of walls 1 and 2 respectively

A_c, determined by $\dfrac{1}{A_c} = \dfrac{1}{A_1} + \dfrac{1}{A_2}$

A_1^*, A_2^* = composite cross sections of walls 1 and 2 respectively for the shear deformation

b = flexible length of connecting beams

d_1, d_2 = distance of mid-pt of beams from the centroids of walls 1 and 2 respectively

E = modulus of elasticity

G = shear modulus

h = interstorey height

H = total height of walls

I_b = moment of inertia of connecting beams

I_1, I_2 = moments of inertia of walls 1 and 2 respectively

$I_c = I_1 + I_2$

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M_o = total externally applied moment

M_1, M_2 = bending moments in walls 1 and 2 respectively

N = axial force in the walls

t = $\frac{x}{H}$, $(0 \leqslant t \leqslant 1)$

V = shear force in a connecting beam

W = total horizontal load distributed triangularly

x = abscissa

y = ordinate

y_b = deflection at top, neglecting shear strain of walls

y_s = deflection at top due to shear strain of walls only

y_t = total top deflection ($= y_b + y_s$).

An additional suffix to the terms I_b, I_c and A_c specifies the location at which the term is determined. e.g. I_{bo} = I_b at $t = 0$

INTRODUCTION

A coupled shear wall system is a set of coplanar walls connected by beams, and the system may be considered as a frame with very high column to beam stiffness ratios. Because of the large stiffness ratios any iterative method of analysis is long and tedious, particularly if proper allowance is made for such effects as axial strain of the walls. Such a frame can be analysed using an electronic computer but the expense and effort is hardly warranted in the early stages of design. A quick approximate solution is therefore desirable.

In this paper the methods employed by Beck (1) have been extended to include cases of importance to designers in seismic areas.

Unsymmetrical single bay and symmetrical double bay frames are considered for both uniform and variable frame properties, under a triangularly distributed horizontal load. This loading results from the common assumption of a lateral force coefficient varying linearly from zero at the base to a maximum at the top as being the statical equivalent of the dynamic earthquake effect on a building of uniform mass distribution.

(1) Beck H., "Contribution to the Analysis of Coupled Shear Walls",
            Journal of the American Concrete Institute, Aug. 1962.

Under high lateral loading, the wall thickness is often reduced with height in steps, and the corresponding variations in frame properties $(I_b, A_b, I_c$ and $A_c)$ may be approximated by parabolic functions. In this paper design charts are provided for determining axial forces and bending moments in the walls, shear forces in the beams and the top deflection, for the following conditions:-

$$\frac{I_{b1}}{I_{bo}} = \frac{I_{c1}}{I_{co}} = \frac{A_{c1}}{A_{co}} = \frac{1}{k}$$

where k = 1.0 (uniform wall thickness)
       = 2.0 (2:1 parabolic variation)
       = 3.0 (3:1   "   "   )

Intermediate cases may be interpolated with little difficulty. The parabolic function to which the properties I_b, I_c and A_c are taken to conform is

$$\frac{I_b}{I_{bo}} = (1 - \frac{1}{k}) \cdot (t^2 - 2t) + 1$$
$$= \rho(t) \qquad \text{(See Fig. 4)}$$

ASSUMPTIONS

1. The dimensions a, b, h, d_1, d_2 and the ratio $\frac{I_b}{A_b}$ are constant.

2. In the substitute system where the beams are replaced by a continuous set of laminae, the coplanar walls have the same curvature, i.e. the point of contraflexure of the beams is taken at the mid-point of the flexible length. This assumption is justified for quite large differences in wall stiffness.

3. Shear deflection of the walls is neglected. Only where the walls are widely dissimilar and, more importantly, shear deflection is significant compared with bending deflection, may a significant error result. However, in tall buildings with large height to breadth ratios, this case should not arise.

4. The walls are assumed rigidly fixed at their bases.

5. Axial deformation of the beams is neglected.

OUTLINE OF METHOD
      (See Appendix for further detail)
      The discontinuous system of interconnecting beams (Fig. 1b) is replaced by a continuous system of laminae (Fig. 2); the beams of moment of inertia I_b distance h apart being replaced by laminae of depth dx and moment of inertia $\frac{I_b dx}{h}$. Under an applied load the walls deflect and shear forces are induced in the laminae. By considering the laminae cut through their assumed points of contraflexure, the gaps incurred through (a) rotation of the walls, (b) axial strain of the walls, and (c) bending and shear deformation of the laminae, are determined, and from these a second order differential equation in N, the axial force in the walls, is formed. The differential equation and the subsequent working is considerably

simplified by defining the following parameters:

$$\alpha^2 = \frac{12 a^2 H^2 I_{bo}}{h\, b^3\, I_{co}} \qquad \dotsb\ 1$$

$$\beta^2 = 1 + \frac{12\, I_b\, E}{b^2\, A_b\, G} \qquad \dotsb\ 2$$

$$\gamma^2 = 1 + \frac{I_{co}}{a^2\, A_{co}} \qquad \dotsb\ 3$$

$$\bar{\alpha} = \frac{\alpha\, \gamma}{\beta} \qquad \dotsb\ 4$$

The term $\bar{\alpha}$ indicates the stiffening effect of the interconnecting beams and is the ordinate of all the design charts. The greater the value of $\bar{\alpha}$ the closer the frame behaves to a homogeneous beam. The term γ^2 includes the effect of axial strain in the walls and also serves as a measure of the total moment of inertia of the system. To neglect axial strain, A_c is taken as infinite and then $\gamma^2 = 1.0$. The term β^2 allows for the effect of shear deformation of the beams. To neglect shear, A_b is taken as infinite and then $\beta^2 = 1.0$. The general equation can now be put in the form

$$\frac{d^2 N}{dt^2} - \frac{1}{\rho(t)} \cdot \frac{d\rho(t)}{dt} \cdot \frac{dN}{dt} - \bar{\alpha}^2 N = \frac{\bar{\alpha}^2}{a\gamma^2} \cdot M_o \qquad \dotsb\ 5$$

where, for triangular loading the external moment is given by

$$M_o = -\frac{WH}{3} \cdot F(t) \qquad \text{where} \qquad F(t) = t^3 - 3t + 2$$

For $k = 1.0$ (uniform wall thickness), Eq. 5 has direct solutions for continuous loading functions such as point load at top, uniform and triangular loads. For $k \neq 1.0$ there does not appear to be a general solution, but the equation may be solved by an iterative method.

Axial Force

On solving Eq. 5 for N we find

$$N = -\frac{M_o}{a\gamma^2} \cdot \mu \qquad \dotsb\ 6$$

where μ is a continuous function of $\bar{\alpha}$, k and t.

Fig. 1.1 is a chart of μ vs $\bar{\alpha}$ for $k = 1.0$.

Figs. 2.1 and 3.1 are similar but for $k = 2.0$ and 3.0 respectively. The individual curves on each chart are drawn for $t = 0, 0.2, 0.4, 0.6$

and 0.8. With $N = 0$ at $t = 1$ a smooth curve can be drawn through the six known values and the axial force at any other value of t can be found.

Beam Shear

The shear force in an interconnecting beam is the difference in magnitude of N at the mid points of adjacent storeys.

$$V_t = N_{(t - \frac{h}{2H})} - N_{(t + \frac{h}{2H})} = \frac{Wh}{a\gamma^2} \cdot \lambda \qquad \dots\dots\dots\dots\dots 7$$

The term λ is a continuous function of $\overline{\alpha}$, k and t and has a maximum value λ_m located at t_m.

Fig. 1.2 is a chart of λ_m and t_m vs $\overline{\alpha}$ for $k = 1.0$

Figs. 2.2 and 3.2 are similar but for $k = 2.0$ and 3.0 respectively.

To determine V at the fifth points the following figures are provided:

Fig. 1.3 is a chart of λ vs $\overline{\alpha}$ for $k = 1.0$

Figs. 2.3 and 3.3 are similar but for $k = 2.0$ and 3.0 respectively. The separate curves on each chart are drawn for $t = 0.2, 0.4, 0.6, 0.8$ and 1.0. With $V = 0$ at $t = 0$, and knowing V_{max} and t_m a smooth curve can be drawn from which other values of V may be found.

Wall Bending Moments

For equilibrium at any level

$$M_1 + M_2 = M_o + aN = M_o(1 - \frac{\mu}{\gamma^2}) \qquad \dots\dots\dots\dots\dots 8$$

For equal curvature of the walls

$$\frac{M_1}{I_1} = \frac{M_2}{I_2} = - \frac{E}{H^2} \cdot \frac{d^2 y}{dt^2} \qquad \dots\dots\dots\dots\dots 9$$

From Eq. 8 and Eq. 9, solving for M_1 and M_2

$$M_1 = I_1 \cdot \frac{M_o}{I_c} (1 - \frac{\mu}{\gamma^2}), \qquad M_2 = I_2 \cdot \frac{M_o}{I_c} (1 - \frac{\mu}{\gamma^2})$$

However, in the discontinuous beam system, adjustment must be made for the moments in the beams (see Fig. 3).

$$\therefore \quad M_1 = I_1 \cdot \frac{M_o}{I_c} (1 - \frac{\mu}{\gamma^2}) \mp \frac{d_1 V}{2} \qquad \dots\dots\dots\dots\dots 10$$

$$M_2 = I_2 \cdot \frac{M_o}{I_c} \left(1 - \frac{\mu}{\gamma^2}\right) \mp \frac{d_2 V}{2} \qquad \ldots\ldots\ldots\ldots \; 11$$

The sign of the last term depends on whether the moment is above (-) or below (+) the beam centre-line. The moment in the top beam must be resisted entirely by the walls immediately below that beam.

Therefore, at $t = 1$, $M_1 = Vd_1$ and $M_2 = Vd_2$

Top Deflection

From Eq. 8 and Eq. 9 above

$$\frac{d^2 y}{dt^2} = - \frac{H^2}{EI_c} \cdot (M_o + aN)$$

By substituting for M_o, N and I_c and integrating twice the deflected shape of the walls can be determined. The deflection at the top is given by

$$y_b = \frac{KWH^3}{EI_{co}} \cdot \mathfrak{z} \qquad \ldots\ldots\ldots\ldots\ldots \; 12$$

where $\mathfrak{z}$ is a function of $\bar{x}$, γ and k.

Fig. 1.4 is a chart of $\mathfrak{z}$ vs $\bar{x}$ for $k = 1.0$

Similarly Figs. 2.4 and 3.4 refer to $k = 2.0$ and 3.0 respectively. The separate curves on each chart are drawn for $\gamma = 1.0$, 1.1 and 1.2. This range should cover all cases found in practice and $\mathfrak{z}$ may be interpolated for intermediate values of γ .

Note that the constant term $\frac{KWH^3}{EI_{co}}$ is the end deflection when the two walls act as simple cantilevers, ($\bar{x} = 0$).

To determine the top deflection due to shear strain of the walls only, the following formula is provided:

$$y_s \cong \frac{C W H}{G(A_1^* + A_2^*)} (t = 0) \qquad \ldots\ldots\ldots\ldots\ldots \; 13$$

where $C = 0.667$ for $k = 1.0$
$\quad\quad\quad = 0.956$ for $k = 2.0$
$\quad\quad\quad = 1.161$ for $k = 3.0$

Eq. 13 is accurate when applied to a symmetrical frame. The total deflection at the top is given by $y_t = y_b + y_s$

APPLICATION TO SYMMETRICAL TWO BAY FRAMES

A symmetrical two bay frame (Fig. 5a) whether of constant or variable wall thickness may be analysed, using the design charts, as simply as the

single bay frame. Due to symmetry there is no resultant axial force in the centre wall and only half of the frame (one bay) need be considered, (Fig. 5b).

In this case take

$$I_c = I_1 + \frac{I_2}{2} \quad \text{and} \quad A_c = A_1, \quad (A_2 = \infty)$$

Determine the parameters α, γ, β and $\bar{a}$ as before and taking $\frac{W}{2}$ as the load, calculate the moments, shears and deflection. The moments in the centre wall must be doubled when converting back to the two-bay frame.

EXAMPLE (Single bay frame)
    A 10 storey unsymmetrical frame (Fig. 6a) with wall thickness reducing from 18" at the base to 10" at the top by 4" steps at the 2nd and 5th floor levels, is analysed for point loads applied at the floor levels. The point loads increase linearly from zero at the base to a maximum of 260 kips at the roof level.

Frame Properties

a = 25.84'	b = 10.0'	h = 10.0'
H = 100'	d_1 = 10.0'	d_2 = 15.84'
W = 1500 kip.	G = 0.4.E	

The load W is the triangularly distributed load giving the same base over-turning moment as the actual point loads. In this example the properties I_b, A_b, I_c and A_c are directly proportional to the wall thickness.

At the base (t = 0)

$A_1 = 15ft^2$	$A_2 = 33ft^2$	$A_c = 10.03ft^2$
$I_1 = 125ft^4$	$I_2 = 1233ft^4$	$I_c = 1358ft^4$
$I_b = 3.38ft^4$	$A_b = 4.40ft^2$	$I_b/A_b = 0.75ft^2$

From a plot of wall thickness against height the best fitting parabola is found (Fig. 6b). In this case the 2:1 parabola (k = 2) is satisfactory with the resulting base adjustment of $\frac{1}{0.925}$ = 1.08.

$\therefore A_{co} = 11.15ft^2$, $I_{co} = 1468ft^4$ $I_{bo} = 3.65ft^4$

From Eqs. 1, 2, 3 and 4 determine the frame parameters.

$$\alpha^2 = 19.92 \qquad \gamma^2 = 1.197 \qquad \beta^2 = 1.225$$
$$\bar{a} = 4.41$$

Beam Shear
    The shear force in the beams is given by

$$V = \frac{Wh}{a\gamma^2} \cdot \lambda \qquad = 485.\lambda$$

From Fig. 2.2, for $\bar{\alpha} = 4.41$, $\lambda_m = 0.60$ and $t_m = 0.37$

$$\therefore V_{max} = 291 \text{ kip.}$$

From Fig. 2.3 the values of λ at the fifth points are found. These values and the subsequent calculations are contained in TABLE 1.

Wall Bending Moments and Axial Forces
The axial force in the walls is given by

$$N = -\frac{M_o}{a\gamma^2} \cdot \mu \qquad = \frac{WH}{3a\gamma^2} \cdot F(t) \cdot \mu$$

$$= 1617.F(t).\mu$$

From Fig. 2.1, for $\bar{\alpha} = 4.41$ the values of μ at the fifth points are found. The calculation of N is contained in TABLE 1.

From Eq. 10 and Eq. 11, the wall bending moments are given by

$$-M_1 = \frac{I_1}{I_c} \cdot \frac{WH}{3} \cdot F(t) \cdot (1 - \frac{\mu}{\gamma^2}) \pm \frac{d_1 V}{2}$$

$$= 4600 \, F(t).(1 - \frac{\mu}{1.197}) \pm 5V$$

Similarly $\quad -M_2 = 45400 \, F(t). \, (1 - \frac{\mu}{1.197}) \pm 7.92V$

The calculation of M_1 and M_2 at the fifth points is also contained in TABLE 1.

Top Deflection
From Fig. 2.4, for $\gamma = 1.093$ and $\bar{\alpha} = 4.41$, $\beta = 0.30$

$$\therefore \quad y_b = 0.228 \frac{WH^3}{EI_{co}} \cdot \beta \qquad = \frac{69,900}{E}$$

The deflection due to shear strain of the walls only, is

$$y_s = \frac{0.956 \, WH}{G(A_1^* + A_2^*)}(t = 0) \qquad = \frac{7370}{E}$$

(where $(A_1^* + A_2^*)_{(t = 0)} = (15 + 30) \times 1.08 = 48.5 \text{ft}^2$)

The total deflection is given by

$$y_t = y_b + y_s \qquad = \frac{77,270}{E}$$

COMPARISON WITH AN "EXACT" ANALYSIS
    This same problem with the stepped variation in wall thickness was analysed using the strain energy method of influence coefficients, with the aid of an electronic computer. Both axial and shear strain of the walls were considered but axial strain of the beams was neglected.

    The results of the two methods may be compared in Fig. 7 and Fig. 8. The graph of beam shears (Fig. 7a) shows that the largest errors occur near the points of change in wall thickness and are approximately of the order of the difference in actual and assumed variation in properties (Fig. 6b). In the "exact" solution the top deflection due to bending only is $\frac{72,200}{E}$ and that due to shear only is $\frac{7150}{E}$, giving a total deflection of $\frac{79350}{E}$.

NOTES ON THE USE OF CHARTS
    In the design of this type of structure, a trial and error approach is usually used to balance -
    a) natural period with base shear coefficient
and b) beam size with maximum beam shear.

    A close estimate of the fundamental period of the structure may be made from the magnitude of the top deflection.

$$ T = 0.32 . \sqrt{D} $$

            where   T = fundamental period in seconds

                     D = deflection at the top, in inches, due to a
                         lateral force coefficient varying linearly
                         from zero at the base to 1.0g at the top (2).
The value of D may be found from Eq. 13 and the appropriate chart.

    On determining the appropriate base shear coefficient (and hence W), and the parameter α the location and magnitude of the maximum beam shear can be obtained directly from the appropriate chart. At this stage the adequacy of the beams can be checked.

    Generally, the interconnecting beams are the critical members of the frame. It is worthwhile to note that for α greater than four, a decrease in the beam stiffness (I_b) will result in increased shear forces in the

upper beams but a decrease in the more critical lower beams. However the reduction in shear force may not mean a reduction in shear stress. By reducing the beam stiffness the top deflection increases giving a higher period with a possible reduction in base shear coefficient. This will depend on the lateral force requirement of the building code.

(2) Skinner, R.I. "Revised Building Period Formulae for Draft-Code", unpublished.

The effect of using methods that neglect axial strain in the walls can be measured by putting $\gamma = 1.0$. Often the effect will be found to be considerable and if neglected the beams will be over-designed in shear and flexure and the walls under-designed in flexure. Neglect of axial strain will also lower the apparent period and a higher base shear co-efficient may be applied than is necessary.

In buildings with a number of irregular storey heights the shear in the beams adjacent to these storeys can be found from a graph of N; V being the difference in N at the mid-points of adjacent storeys.

As with other methods, it may be advisable to modify the physical dimensions of the structure, prior to analysis. For example, the flexible length of the beams (b) may be taken as the clear span plus half the beam depth (3).

Full fixity at the base is assumed. However, rotation of each wall at the foundation, about its own centre-line will increase the beam shears whereas translation will tend to decrease them. These effects may possibly be accounted for by lowering the base by one imaginary storey.

CONCLUSIONS

This approximate method will give good results for frames of five storeys or more in height, the accuracy improving as the number of storeys increases. Also with walls of variable thickness the smaller the discontinuities (steps in wall thickness) the greater the accuracy. Where the term αz is greater than 20 this method need not be used as the walls then act as one, and the shear forces in the beams can be determined from a stress analysis.

Similar design charts can be drawn for other loading conditions such as a point load at the top and a uniformly distributed load. A combination of such charts would then cover most likely code requirements.

ACKNOWLEDGMENT

(3) Muto K, and Butler D.W. "Lateral Force Distribution Coefficients and Stress Analysis of Walled Frames", 1951

APPENDIX

Derivation of Differential Equation

The beams are replaced by laminae,(Fig. 2), each of which has moment of inertia $\dfrac{I_b\ dx}{h}$ and cross sectional area $\dfrac{A_b\ dx}{h}$. In order to have a continuous set of laminae over the full height it is necessary to assume that the top beam has half the stiffness of the beam immediately below it. However the curves for V at t = 1 in Figs. 1.3, 2.3 and 3.3 are drawn for full stiffness in this beam.

Under an applied load the walls deflect and shear forces $q(x)$ are induced in the laminae. By cutting the laminae through their points of contraflexure (assumed at mid-point of b) determine the gap incurred through:-

(a) rotation of the walls

$$\delta_a = (d_1 + d_2)\frac{dy}{dx} \;=\; a\frac{dy}{dx}$$

(b) bending and shear of the laminae

$$\delta_b = -\frac{b^3 h \beta^2}{12 E I_b}\cdot q(x)$$

(c) axial strain of the walls

$$\delta_c = -\iint_o^x\!\left[\frac{1}{EA_1}\int_x^H q(x)\,dx\right]dx \;-\;\iint_o^x\!\left[\frac{1}{EA_2}\int_x^H q(x)\,dx\right]dx$$

$$= -\frac{1}{E}\iint_o^x\!\left[\frac{1}{A_c}\int_x^H q(x)\,dx\right]dx$$

For continuity

$$\delta_a + \delta_b + \delta_c = 0$$

$$a\frac{dy}{dx} - \frac{b^3 h \beta^2}{12 E I_b}\cdot q(x) \;-\; \frac{1}{E}\iint_o^x\!\left[\frac{1}{A_c}\int_x^H q(x)\,dx\right]dx \;=\; 0$$

Differentiate with respect to x

$$a\frac{d^2 y}{dx^2} - \frac{b^3 h \beta^2}{12 E}\cdot\frac{d}{dx}\left(\frac{q(x)}{I_b}\right) \;-\; \frac{1}{EA_c}\int_x^H q(x)\,dx \;=\; 0 \quad\ldots\ldots\ldots 1.$$

The axial force in the walls is given by

$$N = \int_x^H q(x)\,dx$$

For equal curvature of the walls

$$\frac{M_1}{I_1} = \frac{M_2}{I_2} \;=\; -E\frac{d^2 y}{dx^2}$$

For equilibrium

$$M_1 + M_2 \;=\; M_o + aN$$

$$\frac{d^2 y}{dx^2} \;=\; -\frac{1}{EI_c}(M_o + aN)$$

Substitute for $\dfrac{d^2 y}{dx^2}$ and q(x) in Eq. 1

$$\frac{b^3 h \beta^2}{12}\frac{d}{dx}\left(\frac{1}{I_b}\cdot\frac{dN}{dx}\right) - \left(\frac{a^2}{I_c} + \frac{1}{A_c}\right)N \;=\; \frac{a M_o}{I_c} \quad\ldots\ldots\ldots\ldots 2$$

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Now make the transformation, $t = \frac{x}{H}$, ($o \leq t \leq 1$) and, to allow for variable wall properties assume I_b, I_c and A_c vary such that -

$$I_b = I_{b,o} \cdot \rho(t) \qquad \text{where} \qquad \rho(t) = \delta\rho(t^2 - 2t) + 1$$

similarly

$$\delta\rho = 1 - \frac{I_{b,1}}{I_{b,0}}$$

$$I_c = I_{c,o} \cdot \theta(t)$$

$$A_c = A_{c,o} \cdot \phi(t)$$

Substitute in equation 2

$$\frac{b^3 h \beta^2}{12 H^2 I_{b,o}} \cdot \frac{d}{dt}\left(\frac{1}{\rho(t)} \cdot \frac{dN}{dt}\right) - \frac{a^2}{I_{c,o}\theta(t)}\left(1 + \frac{I_{c,o}\theta(t)}{a^2 A_{c,o}\phi(t)}\right)N = \frac{a M_o}{I_{c,o}\theta(t)} \quad \cdots \cdots 3.$$

To simplify Eq. 3 make the following abbreviations

$$\alpha^2 = \frac{12 a^2 H^2 I_{b,o} \rho(t)}{h b^3 I_{c,o} \theta(t)}$$

$$\gamma^2 = 1 + \frac{I_{c,o}\theta(t)}{a^2 A_{c,o}\phi(t)}$$

$$\overline{\alpha} = \frac{\alpha \gamma}{\beta}$$

Eq. 3 now becomes

$$\frac{d^2 N}{dt^2} - \frac{1}{\rho(t)} \cdot \frac{d\rho(t)}{dt} \cdot \frac{dN}{dt} - \overline{\alpha}^2 N = \frac{\overline{\alpha}^2}{a\gamma^2} \cdot M_o \quad \cdots \cdots \cdots 4.$$

Uniform Wall Thickness

The properties I_b, I_c and A_c are constant and the parameters $\overline{\alpha}$ and γ are also constant. Eq. 4 now becomes

$$\frac{d^2 N}{dt^2} - \overline{\alpha}^2 N = \frac{\overline{\alpha}^2}{a\gamma^2} M_o$$

Solving for N

$$N = D_1 \cosh\overline{\alpha}t + D_2 \sinh\overline{\alpha}t - \frac{1}{a\gamma^2}\left(M_o + \frac{M_o''}{\overline{\alpha}^2} + \frac{M_o''''}{\overline{\alpha}^4} + \cdots\right)$$

The boundary conditions are

$$N = 0 \text{ at } t = 1$$

$$\frac{dN}{dt} = 0 \text{ at } t = 0 \text{ (no rotation at the base)}$$

For triangular loading the external moment is given by

$$M_o = -\frac{WH}{3} \cdot F(t) \qquad \text{where} \qquad F(t) = t^3 - 3t + 2$$

$$\therefore N = \frac{WH}{3a\gamma^2} \cdot F(t)\left[1 + \frac{1}{F(t)} \cdot \left(C_1 \cosh\overline{\alpha}t + C_2 \sinh\overline{\alpha}t + \frac{6t}{\overline{\alpha}^2}\right)\right]$$

where $\quad C_2 = \frac{3}{\overline{\alpha}}\left(1 - \frac{2}{\overline{\alpha}^2}\right), \qquad C_1 = -C_2 \tanh\overline{\alpha} - \frac{6}{\overline{\alpha}^2 \cosh\overline{\alpha}}$

$$\therefore N = -\frac{M_o}{a\gamma^2} \cdot \mu$$

where μ is the term in brackets and is a function of $\overline{\alpha}$ and t.

The beam shear force is given by

$$V_t = N_{(t - \frac{h}{2H})} - N_{(t + \frac{h}{2H})} = \int_{t + \frac{h}{2H}}^{t - \frac{h}{2H}} Q(t)dt$$

$$\therefore \; V = \frac{Wh}{\alpha \gamma^2}\left[(t^2-1) + \frac{h^2}{12H^2} + \frac{2}{\alpha^2} + \frac{3H}{2h}\left(C_1\sinh\alpha t + C_2\cosh\alpha t\right)\sinh\frac{\alpha h}{2H}\right]$$

$$= \frac{Wh}{\alpha \gamma^2}\cdot\lambda$$

where λ is the term in brackets and is approximately a function of α and t.

Equations for slope and deflection can be found from the equation

$$\frac{d^2 y}{dt^2} = \frac{H^2}{EI_c}(M_o + aN)$$

Substitute for M_o and N and integrate, using the boundary conditions

$$y = \frac{dy}{dt} = 0 \quad at \; t = 0$$

The deflection at the top is found to be

$$y_b = \frac{WH^3}{EI_c}\left\{\frac{11}{60}\left(1-\frac{1}{\gamma^2}\right) - \frac{1}{3\alpha^2\gamma^2}\left[1 - C_1(1-\cosh\alpha) - C_2\left(\alpha - \sinh\alpha\right)\right]\right\}$$

$$= \frac{11WH^3}{60EI_c}\cdot\zeta$$

where ζ is a function of α and γ.

For α greater than six, $C_1 \doteq - C_2$, and, by converting to exponential form the above equations can be considerably simplified.

Variable Wall Thickness

The functions $\rho(t)$, $\Theta(t)$ and $\emptyset(t)$ are variable and a general algebraic solution to Eq. 4 could not be found for this case. However the equation can be solved using an iterative method with the aid of an electronic computer and it was from this source that data was obtained to draw the design charts for $k = 2$ and 3.

It was found that the results for the variable case were similar in form to those of the uniform case, i.e.

$$N = -\frac{M_{o_2}}{\alpha\gamma^2}\cdot\mu$$

$$V = \frac{Wh}{\alpha\gamma^2}\cdot\lambda$$

$$y_b = \frac{KWH^3}{EI_{c,o}}\cdot\zeta$$

where μ and λ are functions of t, α, and k, and ζ is a function of α, γ and k.

An approximate solution to Eq. 4 for α greater than six and $\rho(t) = \Theta(t) = \emptyset(t)$ is given by

$$N = -\frac{M_{o_2}}{\alpha\gamma^2}\left[1 + \frac{F'(o)\sqrt{\rho(t)}\; e^{-\alpha t}}{F(t)(6\rho + \alpha)}\right]$$

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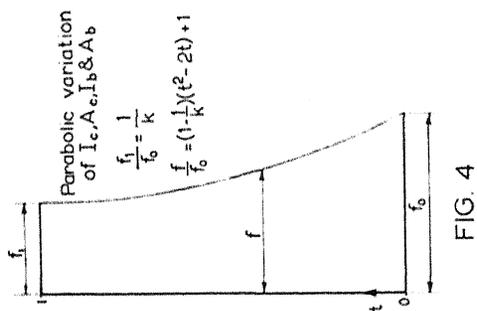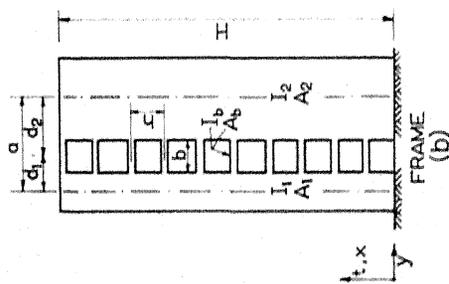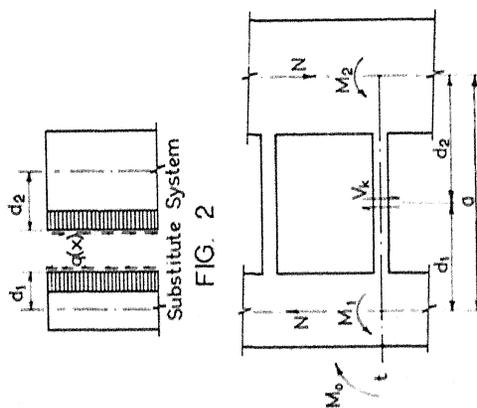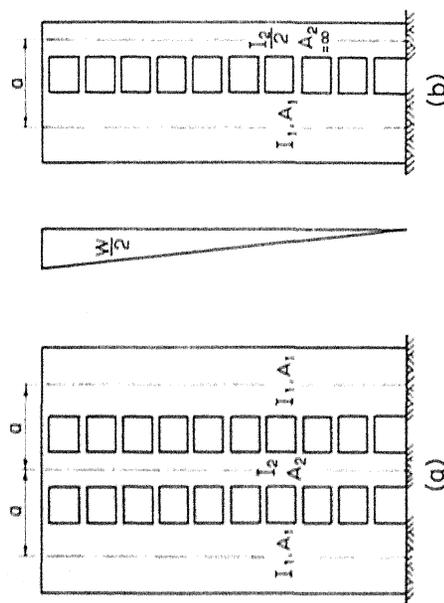
Parabolic variation of I_c, A_c, I_b & A_b

$$\frac{f_1}{f_o} = \frac{1}{k}$$

$$\frac{f}{f_o} = (1-\frac{1}{k})(t^2-2t)+1$$

FIG. 4

Substitute System

FIG. 2

FIG. 3

(b)

(a)

SYMMETRICAL TWO-BAY FRAME

FIG. 5

FRAME (b)

FIG. 1

LOAD (a)

TABLE 1. Calculation of V, N, M₁ & M₂

t	0	0·2	0·4	0·6	0·8	1·0
λ from fig. 2·3	0	0·50	0·595	0·510	0·380	0·295
$V(\text{kips}) = 485\lambda$	0	243	289	247	184	143
$F(t) = t^2 - 3t + 2$	2·0	1·408	0·864	0·416	0·112	0
μ from fig 2·1	0·67	0·81	0·93	1·11	1·72	—
$N(\text{kips}) = 1617 F(t)\mu$	2167	1844	1299	747	311	0
$1 - \frac{\bar{t}}{1·197}$	0·440	0·323	0·223	0·073	-0·437	—
$4600 F(t)(1 - \frac{\bar{t}}{1·197})$	4048	2092	886	140	-225	0
$5V$	0	1215	1445	1235	920	715
$-M_1(\text{kip ft})$ above joint	4048	3307	2331	1375	695	—
below joint	—	877	-559	-1095	-1145	-1430
$45400 F(t)(1 - \frac{\bar{t}}{1·197})$	39952	20,647	8747	1379	-2222	0
$7·92 V$	0	1925	2289	1956	1457	1133
$-M_2(\text{kip ft})$ above joint	39952	22,572	11,036	3335	-765	—
below joint	18,722	6,458	-577	-3679	-2266	

(a) FIG. 6 EXAMPLE

SECTION A-A
$H = 100'$
$a = 258.4'$
$\omega = 260$ kip

(b)
$\frac{I}{I_0} = 0.5(t^2 - 2t) + 1$
Actual variation of I, A_b & A_b

WALL 1 (a)
WALL 2 (b)
Values in brackets are those from "EXACT" method
WALL BENDING MOMENTS (kip. ft.)
FIG. 8

Wall 1 values: -1430 (-1325), -1145 (-1246), -1095 (-916), -559 (-600), 877 (745), 39,952 (38,100); 595 (595), 1375 (1320), 2331 (2350), 3307 (3350), 4048 (5300)

Wall 2 values: -2266 (-2140), -765 (-588), -3679 (-3510), 3335 (4410), 6458 (7150), -577 (857), 11036 (11840), 18722 (18500), 22572 (22650), 39,952 (38,800)

(a) BEAM SHEAR (kip)
143 (134), 184 (184), 247 (223), 289 (295), 243 (260)

(b) AXIAL FORCE IN WALLS (kip)
311 (292), 747 (685), 1299 (1216), 1844 (1774), 2167 (2200)

— Approx. method
--- "Exact" method

FIG. 7

BEAM SHEAR
MAGNITUDE AND POSITION OF MAXIMUM SHEAR

$V_{max} = \dfrac{Wh}{a\delta^2} \cdot \lambda_m$

t_m = location of V_{max}

λ_m

t_m

FIG. 1·2

TOP DEFLECTION

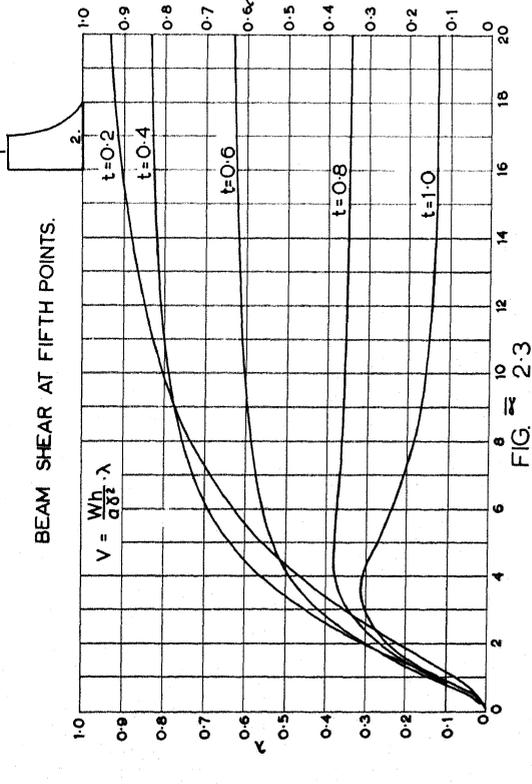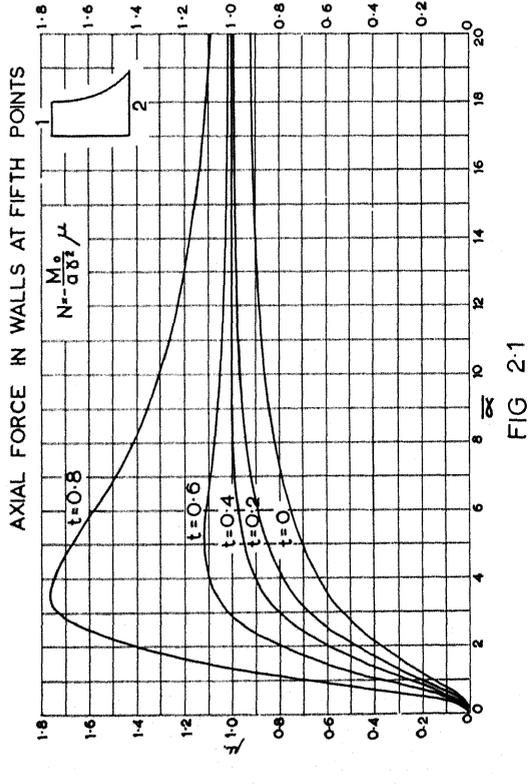$y_b = 0.183 \dfrac{WH^3}{ET_{co}} \cdot \eta$

$\delta = 120$

$\delta = 10$

$\delta = 1.0$

FIG. 1·4

AXIAL FORCE IN WALLS AT FIFTH POINTS

$N = \dfrac{M_c}{a\delta^2} \cdot \mu$

t=0·8   t=0·6   t=0·4   t=0·2   t=0

FIG. 1·1

BEAM SHEAR AT FIFTH POINTS

$v = \dfrac{Wh}{a\delta^2} \cdot \lambda$

t=0·2   t=0·4   t=0·6   t=0·8   t=1·0

FIG. 1·3

BEAM SHEAR
MAGNITUDE & POSITION OF MAXIMUM SHEAR

$V_{max} = \dfrac{Wh}{a\delta^2} \cdot \lambda_m$

t_m = location of V_{max}.

FIG. 2·2

TOP DEFLECTION

$y_b = 0.228 \dfrac{WH^3}{EI_{\infty}} \cdot \gamma$

FIG. 2·4

AXIAL FORCE IN WALLS AT FIFTH POINTS

$N = \dfrac{M_o}{a\delta^2} \cdot \mu$

t = 0·8
t = 0·6
t = 0·4
t = 0·2
t = 0

FIG 2·1

BEAM SHEAR AT FIFTH POINTS.

$V = \dfrac{Wh}{a\delta^2} \cdot \lambda$

t = 0·2
t = 0·4
t = 0·6
t = 0·8
t = 1·0

FIG. 2·3

BEAM SHEAR MAGNITUDE & POSITION OF MAXIMUM SHEAR

$V_{max} = \dfrac{Wh}{a g^2} \cdot \lambda_m$

t_m = location of V_{max}

λ_m

t_m

FIG 3·2

TOP DEFLECTION

$y_b = 0·253 \dfrac{WH^3}{EI_{co}} \cdot \gamma$

$\delta = 1·20$

$\delta = 1·10$

$\delta = 1·0$

FIG. 3·4

AXIAL FORCE IN WALLS AT FIFTH POINTS

$N = \dfrac{M_e}{a g F} \cdot \mu$

t = 0·8

t = 0·6

t = 0·4

t = 0·2

t = 0

FIG. 3·1

BEAM SHEAR AT FIFTH POINTS

$V = \dfrac{Wh}{a g^2} \cdot \lambda$

t = 0·2

t = 0·4

t = 0·6

t = 0·8

t = 1·0
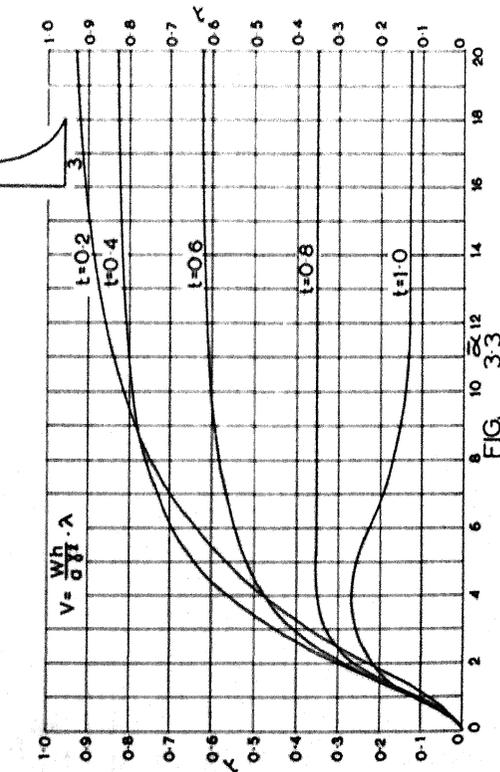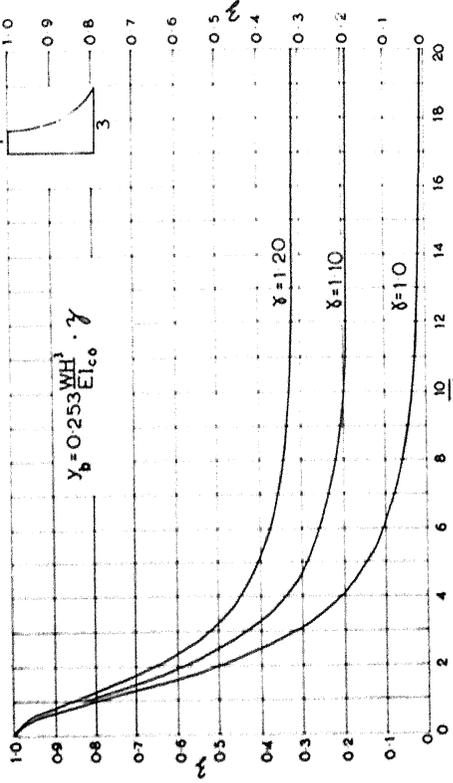
FIG. 3·3

AN APPROXIMATE METHOD OF ANALYSING COUPLED SHEAR WALLS

SUBJECT TO TRIANGULAR LOADING

BY R.J. BURNS

QUESTION BY:

T. PAULAY - NEW ZEALAND

The suggested approximate analysis is an efficient one
for isolated shear walls subject to a known external
load. In the opinion of the discusser its accuracy is
quite sufficient also for final design purposes
provided that the assumptions upon which this approach
is based are met. Similar studies by Albiges (1)
Rosman (2) and Arcan(3) have been verified by photo-
elastic model studies. The limitations of this method
arise from the facts that:

1. The analysis is based on the linear elastic
   behaviour of a homogeneous, isotropic material.
   The two wall elements of the single bay system,
   for example, are subject to vastly differing stress
   conditions when axial forces are large. The
   effective moment of inertia of the wall subject to
   axial tension is greatly reduced unless tensile
   stresses are supressed by prestressing.

   The beams, as the author pointed out, may be sub-
   ject to rather high shearing forces. The shear
   deformation of such cracked reinforced concrete
   beams is likely to be in excess of the value pre-
   dicted by the elastic analysis. A radical reduction
   of the shear stiffness of the affected laminae seems
   to be warranted.

2. The assumption of full base fixity can seldom be
   satisfied. Shear walls are particularly sensitive
   with respect to base displacements. From subsoil
   studies an appropriate dynamic modulus of subgrade
   reaction should be estimated. Having assessed the
   likely behaviour of the foundation material it is
   relatively easy to adjust the mathematical model
   and to solve the differential equation for the
   appropriate new boundary conditions (4).

3. Shear walls often resist the total lateral load
   together with other shear walls and rigid-jointed
   frames. A more general approach is required to
   solve this type of problem. Whatever manual
   analysis is adopted to do this it is bound to be
   tedious. Contrary to the author's opinion the

discusser believes that iterative methods can be
efficiently employed in such and more complex situat-
ions. The efficiency of any relaxation process de-
pends largely upon the rate of convergence. It is
possible to formulate computer programmes for over-
relaxations or extra-polations by which the convergence
of ill-conditioned frames, such as coupled shear walls,
can be greatly accelerated.

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(2)   Rosman R. "Spannungsoptische Untersuchung einer
      waagrecht belasteten Querwand eines Hochbaus".
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(3)   Arcan M. "Berechnungsverfahren für Wandscheiben
      mit einer Reihe von Oeffnungen. Spannungsoptische
      Untersuchung". Die Bautechnik Vol. 41 (1964),
      pp. 95 - 100.

(4)   Rosman R. "Approximate Analysis of Shear Walls
      Subject to Lateral Loads". Proc. A.C.I. Vol.
      v.61 (1964) pp. 717 - 733.

AUTHOR'S REPLY: The analysis is based on the behaviour of an elastic
homogeneous material which is the common assumption
for evaluating statical values of reinforced concrete
structures (1). To allow for cracking in the wall
subject to axial tension and in the beams subject to
high shear and bending stresses the properties of the
cracked sections could be assumed in the analysis.
Under a given load the effect of cracking of the walls
will be an increase in the forces induced in the beams
whereas cracking of the beams will result in lower
shear forces in the beams.

Use has been made of prestress to minimise cracking of
walls due to axial and bending stresses (2).

Admittedly full base fixity can seldom be realised. In
practice the walls will be connected by a foundation
beam and the author's experience on a number of such
cases has shown that provided the α term is greater
than five and the stiffness of this beam is of the
order of five times that of the beams above, the
assumption of full base fixity is justified. With such
a foundation beam, distortions in the subgrade cause

only a rotation of the walled frame as a rigid body and have no influence on the state of stresses in the frame (3). However, such rotation will increase the flexibility of the system as a whole and may justify a reduction in base shear coefficient. In the formulae for estimating the fundamental period $(T = 0.32 \sqrt{D})$, deflection at the top due to foundation rotation should be included.

I agree with the writer in that a more general approach is required where walls and rigid frames act together in resisting lateral load. In this case rotation of the walls due to subgrade distortions may significantly effect the distribution of load between walls and frames (4).

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